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# Invariant vacuum

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ABSTRACT

We apply the Lewis–Riesenfeld invariant method for the harmonic oscillator with time dependent mass and frequency to the modes of a charged scalar field that propagates in a curved, homogeneous and isotropic spacetime. We recover the Bunch–Davies vacuum in the case of a flat DeSitter spacetime, the equivalent one in the case of a closed DeSitter spacetime and the invariant vacuum in a curved spacetime that evolves adiabatically. In the three cases, it is computed the thermodynamical magnitudes of entanglement between the modes of the particles and antiparticles of the invariant vacuum, and the modification of the Friedmann equation caused by the existence of the energy density of entanglement. The amplitude of the vacuum fluctuations are also computed.



## 1. Introduction

All the machinery of a quantum field theory is ultimately rooted on the definition of the vacuum state. Once this is defined a Fock space can be generated from the number eigenstates of the corresponding representation and the general quantum state of the field can be written as a vector of such space. The field can then be interpreted as composed of many particles propagating along the spacetime.

However, the definition of the vacuum state and the associated definition of particle cannot be always unambiguously stated in a curved spacetime. The most appropriate definition of the vacuum state in a local region of the spacetime may not correspond to the vacuum state in another local region, and that may lead to the creation of particles [1–6]. The question is then which vacuum state has to be selected from the set of possible vacuum states, with a twofold consideration: which quantum representation can determine the appropriate boundary condition for the field; and, which one can represent the observable particles.

A customary approach [7,8] is to define the vacuum state in an "IN" and "OUT" regions that asymptotically behave like Minkowski spacetime, where the vacuum state is therefore well defined. The corresponding "IN" vacuum is assumed to supply the initial boundary condition for the field and the "OUT" vacuum is expected to define the kind of measurable particles. Generally, the result is that the initial vacuum state turns out to be full of particles of the "OUT" representation. A problem with this approach is that it is not always possible to find in a curved spacetime two asymptotically flat regions where to define these vacuum states. That might wrongly induce us to think that a well defined vacuum state cannot be then given.

In this paper we shall adopt a different point of view. On the one hand, one would expect that the appropriate boundary condition for a cosmological field should be global, i.e. not tied to a local initial state, and such that the field should remain in the same state along the entire evolution of the field if no external force is present. In that case the state of the field should be invariant under time evolution. Furthermore, in cosmology there is no external element to the universe[1] so in particular, one would expect the field to stay in the ground state or the state of minimal excitation of some invariant representation.

In most cases of interest the wave equation of the field modes in a curved spacetime turns out to be the wave equation of a harmonic oscillator with time dependent mass and frequency. Then, we can apply the method of the invariants of the harmonic oscillator, developed by Lewis–Riesenfield [9,10] and others [11–16], to find the invariant representation of the field modes. The important property of the invariant representation is that the associated number operator turns out to be a constant of motion. It means that once the field is in a given quantum superposition of the number eigenstates of the invariant representation it remains in

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[1] We are not considering a multiverse scenario here. If that would be the case the same would apply to the multiverse as a whole instead of a single universe.

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the same state along the entire evolution of the field. In particular, if the field is in the vacuum state of the invariant representation at a given moment of time it will remain in the same vacuum state along the entire evolution of the field.

Then, we shall assume that the field is in the vacuum state of the invariant representation. Furthermore, instead of imposing an *initial* condition on the state of the field at some given time $t_0$, we shall impose the *boundary* condition that the largest modes of the field must be the positive frequency modes of a field that propagates in a Minkowski spacetime. This is a boundary condition that is ultimately rooted in the equivalence principle of the theory of relativity. For a sufficiently closed neighborhood, the spacetime looks always like a flat spacetime and, therefore, the largest modes of the field must not feel the curvature of the spacetime. This boundary condition will fix the invariant representation to be used and, thus, it will fix the invariant vacuum state.

In terms of the invariant representation the invariant vacuum state will then represent the ground state along the entire evolution of the field. However, in terms of the number states of any other representation the vacuum state of the invariant representation may contain particles. Let us notice that the concept of particle is a local concept that is based on the definition of the particle detector and, thus, the number of detected particles is an observer-dependent quantity. In particular, for an observer that is making measurements in a local region of the spacetime, the most appropriate representation of the vacuum seems to be the vacuum of instantaneous Hamiltonian diagonalization [8], which represents the state of minimal excitation at a given moment of time. More concretely, an actual detector will only detect particles with wavelength smaller than the characteristic length of the detector. We shall then show that such a detector will in practice detect no particles in a small local region of the spacetime because, as a consequence of the boundary condition, the field modes remain there in the vacuum state along the entire evolution of the field. However, on cosmological grounds, the invariant vacuum turns out to be full of particle-antiparticle pairs of the diagonal representation, which are created in entangled states. We can then analyze the quantum state of each component of the entangled pair and their evolution separately.

The paper is outlined as follows. In Sect. 2 we briefly review the customary procedure of canonical quantization of a charged scalar field. In Sect. 3 we obtain the invariant representation of the associated Hamiltonian and define the invariant vacuum state. In Sect. 4 we apply the results to the case of a DeSitter spacetime and in Sect. 5 the same is done for a homogeneous and isotropic spacetime that evolves adiabatically. Finally, we summarize and draw some conclusions in Sect. 6.

## 2. Field quantization

Let us briefly summarize the standard procedure of canonical quantization for a charged scalar field, $\phi(x) = \phi(\mathbf{x}, t)$, by starting from the action integral

$$S = \int dtd^3\mathbf{x}\, \mathcal{L} = \int dt\, L, \tag{1}$$

with the Lagrangian density $\mathcal{L}$ given by [5,7,17,18]

$$\mathcal{L}(x) = \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - \left(m^2 + \xi R(x)\right)\phi(x)\phi^*(x)\right), \tag{2}$$

where $m$ is the mass of the field and $g_{\mu\nu}$ is the metric tensor, with $g \equiv \det(g_{\mu\nu})$. The coupling between the scalar field and the gravitational field is represented by the term $\xi R\phi^2$, where $R(x)$ is the Ricci scalar. The value $\xi = 0$ corresponds to the so-called minimal coupling and the value $\xi = \frac{1}{6}$ corresponds to the conformal coupling. Unless otherwise indicated, we shall assume minimal coupling ($\xi = 0$) but a similar procedure can be followed with any other value of $\xi$. The variational principle of the action (1) yields the field equation

$$\left(\Box_x + m^2 + \xi R(x)\right)\phi(x) = 0, \tag{3}$$

where the d'Alembertian operator $\Box_x$ is given by [7]

$$\Box_x\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right). \tag{4}$$

In particular, let us consider a homogeneous and isotropic spacetime with metric element given by

$$ds^2 = dt^2 - a^2\, dl^2, \tag{5}$$

where, $a = a(t)$ is the scale factor and $dl^2 = h_{ij}dx^i dx^j$, is the metric element of the three dimensional space with the constant curvature $\kappa = 0, \pm 1$. It is customary to work in conformal time $\eta$, and to scale the scalar field according to, $\phi = a^{-1}\chi$. In that case, the modes of the field $\chi$ satisfy the wave equation of a harmonic oscillator with constant mass and time dependent frequency. However, we shall work in cosmic time $t$ and retain the charged scalar field $\phi(\mathbf{x}, t)$ for at least for three reasons: i) the scaling is unnecessary for obtaining the invariant representation of the scalar field $\phi(x)$; ii) unlike in the wave equation of $\chi$, the frequency of the wave equation of $\phi$ is always real, so we shall avoid imaginary values of the frequency of the modes; and, iii) the invariant representation of any two field variables is the same provided that they are related by a canonical transformation, i.e. the invariant representation of the field $\chi(x)$ is also the invariant representation of the field $\phi(x)$, so the vacuum state of the invariant representation is the same for both fields.

The isotropy of the spacetime described by the metric (5) allows us to expand the field in Fourier modes

$$\phi(\mathbf{x}, t) = \int d\mu(k)\psi_{\mathbf{k}}(\mathbf{x})\phi_{\mathbf{k}}(t), \tag{6}$$

where $\psi_{\mathbf{k}}$ are the eigenfunctions of the three-dimensional Laplacian,

$$\Delta^{(3)}\psi_{\mathbf{k}}(\mathbf{x}) = -(k^2 - \kappa)\psi_{\mathbf{k}}(\mathbf{x}), \tag{7}$$

and, $k = |\mathbf{k}|$ with $\mathbf{k} = \{k_x, k_y, k_z\}$ with $-\infty < k_i < \infty$ in the flat case, or just $k$ in $\mathbf{k} = \{k, l, m\}$ with $0 < k < \infty$, $l = 0, 1, 2, \ldots$ in the open case, $k = 1, 2, \ldots$ and $l = 0, 1, \ldots, k-1$ in the closed case, with $-l \le m \le l$ in both cases, and $d\mu(k)$ is the measure of the Fourier space (see Refs. [4,5,7] for the details). With (6) and (7), integrating by parts and using the orthogonality properties of the functions $\psi_{\mathbf{k}}(\mathbf{x})$ [7], the Lagrangian in (1) turns out to be

$$L = \int d\mu(k)M(t)\left\{\dot{\phi}_{\mathbf{k}}\dot{\phi}^*{}_{\mathbf{k}} - \omega_k^2(t)\,\phi_{\mathbf{k}}\phi^*_{\mathbf{k}}\right\}, \tag{8}$$

where, $M(t) = a^3(t)$,

$$\omega_k^2(t) = \frac{k^2 - \kappa}{a^2} + m^2 + \xi R. \tag{9}$$

The Lagrangian (8) is the Lagrangian of a set of harmonic oscillators with time dependent mass and frequency. Let us now proceed to quantize the field modes by writing [5,7,8]

$$\phi_{\mathbf{k}}(t) = \frac{1}{\sqrt{2}}\left(v_k(t)a_{\mathbf{k}} + (-1)^{\kappa m}v_k^*(t)b^\dagger_{-\mathbf{k}}\right), \tag{10}$$

where, $-\mathbf{k} = \{-k_x, -k_y, -k_z\}$, in the flat case and, $-\mathbf{k} = \{k, l, -m\}$ in the open and closed cases and, $\psi^*_{\mathbf{k}} = (-1)^{\kappa m}\psi_{-\mathbf{k}}$, for $\kappa = 0, \pm 1$.