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Imprint of quantum gravity in the dimension and fabric of spacetime

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ABSTRACT

We here conjecture that two much-studied aspects of quantum gravity, *dimensional flow* and *spacetime fuzziness*, might be deeply connected. We illustrate the mechanism, providing first evidence in support of our conjecture, by working within the framework of multifractional theories, whose key assumption is an anomalous scaling of the spacetime dimension in the ultraviolet and a slow change of the dimension in the infrared. This sole ingredient is enough to produce a scale-dependent deformation of the integration measure with also a fuzzy spacetime structure. We also compare the multifractional correction to lengths with the types of Planckian uncertainty for distance and time measurements that was reported in studies combining quantum mechanics and general relativity heuristically. This allows us to fix two free parameters of the theory and leads, in one of the scenarios we contemplate, to a value of the ultraviolet dimension which had already found support in other quantum-gravity analyses. We also formalize a picture such that fuzziness originates from a fundamental discrete scale invariance at short scales and corresponds to a stochastic spacetime geometry.

1. Introduction and main goal

The landscape of quantum gravity (QG) looks like a variegated compound of approaches that start from different conceptual premises and use different mathematical formalisms (see, e.g., Refs. [1–21]). Rather surprisingly, despite this heterogeneity, over the past few years a generic prediction has emerged: *dimensional flow* [22–39], i.e., a change of spacetime dimension with the scale of the observer. In all QG models, the dimensionality of spacetime exhibits a dependence on the scale, changing (or "flowing") from the topological dimension D in the infrared (IR) to a different value in the ultraviolet (UV). So far, there has been no deep explanation for this universal property. Understanding its origin is just as important as looking for its physical characterization, needed to relate the flow of dimensions to physical observables.

We here put forward and motivate the conjecture that dimensional flow is directly related to the presence of limitations on the measurability of distances close to the Planck length $\ell_{\rm Pl} = \sqrt{G\hbar/c^3}$, a feature (*spacetime fuzziness*) which has been of interest for QG research for decades [40–47]. More precisely, we shall provide preliminary "theoretical evidence" in support of a connection between the number of spacetime dimensions in the UV and the form of the uncertainty on spacetime distances. Important from our perspective is the fact that such a connection might set the stage for a role for dimensional flow in QG phenomenology [48]. Indeed, it has been shown that, in some cases, spacetime fuzziness could be investigated in ongoing and forthcoming experiments, even if the fuzziness is introduced at the Planck scale. This was first explored in analyses of the interferometers used for gravity-wave searches [48–50], and more recently is focusing mainly on the implications of fuzziness for the formation of halo structures in the images of distant quasars [48,51].

2. Example: multifractional theories

We provide preliminary support for our conjecture within the context of multifractional theories [25,52] fully reviewed in [53]. These are a class of field theories of matter and gravity where spacetime is "anomalous" and changes properties with the probed scale, in a way similar to a multifractal. While in other quantum gravities dimensional flow is a derived property not required *a priori*, here it is part of the definition of the framework. Thanks to their peculiar properties, these field theories living on a multifractal spacetime reproduce a wealth of phenomena found in QG. In

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particular, the running of dimensions is produced by an integration measure of the type $d^D q(x) := dq^0(x^0)dq^1(x^1)\cdots dq^{D-1}(x^{D-1}) = \partial_0 q^0 dx^0 \partial_1 q^1 dx^1 \cdots \partial_{D-1} q^{D-1} dx^{D-1}$. The factorizable form is assumed for technical reasons [53] not especially important here, while the specific form of the distributions $q^\mu(x^\mu)$ is obtained by requiring that dimensional flow is slow at large scales. This assumption (spacetime dimension almost constant in the IR), true in all quantum gravities without known exception, is at the core of a result we will invoke often later, the second flow-equation theorem [52] (a "first" version holds for nonfactorizable measures). An approximation of the full measure, which is physically nonrestrictive but will be refined later, is the binomial space-isotropic profile

$$q^\mu(x^\mu) \simeq (x^\mu - \bar{x}^\mu) + \frac{\ell_*}{\alpha_\mu}\left|\frac{x^\mu - \bar{x}^\mu}{\ell_*}\right|^{\alpha_\mu}, \tag{1}$$

where the index μ is not summed over and takes values $0, 1, 2, \ldots, D-1$. For simplicity, we assume $\alpha_\mu = \delta_{0\mu}\alpha_0 + (1-\delta_{0\mu})\alpha$, i.e., the exponents $\alpha_{\mu\neq 0}$ associated with spatial directions have all the same value α; moreover, we also enforce $0 < \alpha_0, \alpha < 1$, to avoid negative dimensions and obtain the correct IR limit [53]. Note that (1) is uniquely determined parametrically as soon as dimensional flow is switched on and is slow (almost constant spacetime dimension) in the IR [52]. This means that different models of quantum gravity can predict different values of the parameters α_μ and ℓ_* (plus other parameters that appear in the full expression at mesoscopic scales [52]), but the general form of the measure as a parametric profile are the same and given by (1). The only ambiguity left undecided by the second flow-equation theorem is a shift in the coordinates, represented by the given point $\bar{x}^\mu$. This shift ambiguity is a puzzling aspect from the viewpoint of interpretation, since it is a sort of preferred point in the universe. However, our results will neutralize this feature and embed it into a more amenable physical interpretation. We will comment on this shortly.

Depending on the symmetries of the Lagrangian, there are four possible multifractional theories, classified according to the derivative operators appearing in kinetic terms. Here we will concentrate on two theories with the same asymptotic expression for lengths, with so-called q- and fractional derivatives. For the purposes of this paper, suffice it to say that q-derivatives are defined as $\partial_{q^\mu} = (dq^\mu/dx^\mu)^{-1}\partial_\mu$. Details on fractional derivatives are discussed in [53].

To get the Hausdorff dimensions d_H of spacetime, one computes the volume $\mathcal{V}$ of a D-cube with size edge ℓ, leading to the result that, if $\alpha_0 = \alpha$ (as fixed by the arguments below), then $\mathcal{V} = \int_\mathrm{cube} d^D q(x) \simeq \ell_*^D[(\ell/\ell_*)^D + (\ell/\ell_*)^{D\alpha}]$. Thus, we have $d_\mathrm{H} \simeq D\alpha$ in the UV ($\ell < \ell_*$). Here we have neglected mesoscopic contributions to $\mathcal{V}$, which are not relevant to get the number of dimensions in the far UV [54]. For the two multifractional theories considered here, it is not difficult to prove that, in the UV, the spectral dimension (the scaling of the return probability $\mathcal{P} \sim \ell^{-d_\mathrm{S}}$ measuring how likely it is to find a test particle in a neighborhood of its actual position when probing spacetime with an apparatus with resolution $1/\ell$) coincides with the Hausdorff dimension, $d_\mathrm{S} \simeq D\alpha \simeq d_\mathrm{H}$, for $\alpha_0 = \alpha$ [53]. Both α and ℓ_* are free parameters of the theory with the only requirement that ℓ_* must be small enough to comply with experimental constraints [53]. As said above, the measure $q^\mu(x^\mu)$ is fixed by the second flow-equation theorem [52], but there remains an ambiguity related to the choice of a preferred frame, which amounts to the choice of $\bar{x}^\mu$ in Eq. (1). In fact, physical observables have to be compared in the picture with x^μ coordinates representing clocks and rods that do not adapt to the scale. This poses the so-called presentation problem [29,53], which consists in the choice of the physical frame where Eq. (1) is defined and observables are calculated.

3. Connecting dimensional flow and fuzziness: first glimpse

As announced, we shall use multifractional theories as a testing ground for our conjecture. We shall seek a connection between dimensional flow in multifractional theories and the limitations on the measurability of spacetime distances obtained by many authors heuristically combining aspects of quantum mechanics (QM) and general relativity (GR) [41,44–47]. It is noteworthy that the presence of these distance-measurement uncertainties, though originally discussed exclusively with heuristic reasoning, has found confirmation in concrete QG theories in recent years (see, e.g., Refs. [1,2]), each of which realizes the corresponding UV features in very different ways [22,23,25,53]. The observations we here report can also be viewed as an explanation of why one gets a correct intuition about distance fuzziness even just resorting to the qualitative interplay of QM and GR. The link is provided by the fact that limitations on geometric measurements are intimately related to dimensional flow. As a byproduct of our analysis, we will also give a physical interpretation for the ambiguities of multifractional theories and select two sets of preferred values for α and ℓ_*. Remarkably, in one of these cases, we obtain $\alpha = 1/2$ and, consequently, $d_\mathrm{H} \simeq d_\mathrm{S} \simeq 2$ in the UV, a value that has already been singled out for independent reasons in many QG studies (see Refs. [11,16,17,22,23,25,30,33–36] and references therein).

We focus on the $(1+1)$-dimensional theory with q-derivatives, a context where the analysis progresses more simply but without loss of any characteristic feature. Using Eq. (1), the reader can easily realize that the spatial distance between two points A and B is

$$L := \int_{x_\mathrm{A}}^{x_\mathrm{B}} dq^1 = \ell + \frac{1}{\alpha}\frac{\ell_*}{\ell}\left(\left|\frac{x_\mathrm{B} - \bar{x}}{\ell_*}\right|^\alpha - \left|\frac{x_\mathrm{A} - \bar{x}}{\ell_*}\right|^\alpha\right), \tag{2}$$

with $\ell = x_\mathrm{B} - x_\mathrm{A}$. Thus, different presentations (i.e., different values of $\bar{x}$ [29,53]) give different results for the distance, although they do not change the anomalous scaling, which is solely governed by α. Up to now, this has been regarded as a freedom of the model, but we here suggest that the presentation ambiguity should be viewed as a manifestation of spacetime fuzziness.

Four presentation choices have been identified as special among the others [29], but the second flow-equation theorem [52] selects only two of these: the initial-point presentation, where $\bar{x} = x_\mathrm{A}$, and the final-point presentation, where $\bar{x} = x_\mathrm{B}$. In both cases, Eq. (2) simplifies in such a way that the difference between L and the value ℓ that would be measured in an ordinary space is [29]

$$\delta L_\alpha \simeq \pm\frac{\ell_*}{\alpha}\left(\frac{\ell}{\ell_*}\right)^\alpha, \tag{3}$$

approximately valid in any space dimensions, where the plus sign is for the initial-point presentation and the minus is for the final-point presentation.

Strikingly, the multifractional contribution to distances (3) is of the same type of the lower bound on distances found by heuristically combining QM and GR arguments [41,44–47]. In particular, in Ref. [47], one of us proposed an argument leading to a minimal length uncertainty $\delta L \sim \sqrt{\ell_\mathrm{Pl}^2 \ell / s}$, where s is a length scale characterizing the measuring apparatus. Using a somewhat different line of reasoning, the authors of Ref. [46] suggested instead fluctuations of magnitude $\sim (\ell_\mathrm{Pl}^2 \ell)^{\frac{1}{3}}$. Both of these well-studied scenarios for distance fuzziness match quantitatively the multifractional contribution to distances (3) upon adopting

$$\alpha = \frac{1}{2}, \qquad \ell_* = \frac{\ell_\mathrm{Pl}^2}{s}, \tag{4}$$