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Power corrections from decoupling of the charm quark

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ABSTRACT

Decoupling of heavy quarks at low energies can be described by means of an effective theory as shown by S. Weinberg in Ref. [1]. We study the decoupling of the charm quark by lattice simulations. We simulate a model, QCD with two degenerate charm quarks. In this case the leading order term in the effective theory is a pure gauge theory. The higher order terms are proportional to inverse powers of the charm quark mass M starting at M^{-2}. Ratios of hadronic scales are equal to their value in the pure gauge theory up to power corrections. We show, by precise measurements of ratios of scales defined from the Wilson flow, that these corrections are very small and that they can be described by a term proportional to M^{-2} down to masses in the region of the charm quark mass.

1. Introduction

In a field theory which contains light (mass-less) fields and fields of a heavy mass M, the functional integral over the latter can be performed resulting in an effective theory for the light fields which was formulated by Weinberg [1]. The action of the effective theory contains the action of the light fields (without the heavy fields) and an infinite number of non-renormalizable terms. The latter are suppressed by powers of E/M at low energies $E \ll M$. Moreover, the non-renormalizable couplings do not contribute to the renormalization group equations of the renormalizable couplings of the light fields. This property holds for mass-independent renormalization schemes like the $\overline{\text{MS}}$ scheme as shown in [1]. The heavy fields still affect the value of the renormalized couplings of the light fields through the decoupling relations, which result from the matching of the effective and the fundamental theory at low energies.

Assuming the validity of perturbation theory at the matching scale, the decoupling relations can be computed perturbatively. In the case of QCD and one heavy quark, such as the charm or the bottom quark, the decoupling relation for the renormalized strong coupling is known to four loops [1–4]. The strong coupling of the five-flavor theory can be extracted in this way from the coupling computed non-perturbatively in the three-flavor theory using lattice simulations [5]. We remark that the decoupling relation for the strong coupling can be equivalently expressed as a relation between the Λ parameters of the effective and the fundamental theory [6].

Simulations of QCD on the lattice are often carried out with three light sea quarks [7–12]. The inclusion of a charm sea quark increases significantly the computational cost and introduces additional tuning to set the bare quark masses on a line of constant physics. Moreover, in the case of simulations with Wilson fermions, Symanzik O(a) improvement requires the computation of coefficients which multiplies terms proportional to the bare quark masses in lattice units am [13,14]. The contribution of these terms is significant for the charm quark $am_c > 0.3$ at the affordable lattice spacings $a > 0.05$ fm. Some of these coefficients, like the one of the gluon action, are difficult to extract non-perturbatively. Relying on decoupling of the charm quark at low energies allows to simulate the cheaper and simpler effective theory with three flavors only.

The applicability of decoupling for the charm quark has to be justified. In [6] this was studied in a model, QCD with two heavy mass-degenerate quarks and no light quarks. The decoupling of

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the heavy quarks leave a pure gauge theory[1] up to power corrections (which are due to the non-renormalizable interactions) at low energies. The latter were extracted by computing low energy quantities related to the Wilson flow [16–19]. Ratios of two such quantities are insensitive to the matching of the gauge couplings, and after taking the continuum limit, can be compared to their counterparts in the pure gauge theory. The differences are due to the power corrections. By interpolating data obtained from simulations at quark masses ranging from one eighth up to one half of the charm quark mass with data from simulations of the pure gauge theory, the size of the power corrections due to one sea charm quark was estimated to be at the sub-percent level [6].

In [6] it was noted that the simulated masses were not large enough to see the leading behavior of the power corrections which start at $1/M^2$ in the effective theory. Instead a behavior more like $1/M$ was observed. In this article we study the same model as in [6] but extend the simulated quark masses to the charm quark mass and slightly above. Thus we can directly compute the size of the power corrections from decoupling of the charm quark. Furthermore we perform a non-perturbative test of the validity of the effective theory of decoupling for the charm quark. Our goal is to determine whether the leading power corrections in the inverse heavy quark mass behave in the charm region as $1/M^2$.

The article is organized as follows. In Sect. 2 we briefly review the theoretical framework of the effective theory of decoupling for QCD with two heavy mass-degenerate quarks (in the continuum). Sect. 3 presents the details of the Monte Carlo simulations of this model formulated on the lattice. The results for the ratios of low energy quantities are presented in Sect. 4 and their dependence on the heavy quark mass is compared to the effective theory prediction. The conclusions of our work are drawn in Sect. 5.

2. Decoupling

To avoid a multi-scale problem, we consider a simplified version of QCD, namely an $SU(3)$ Yang–Mills theory coupled to two degenerate heavy quarks. This allows us to perform simulations in relatively small volumes with very small lattice spacings, as we describe in Sect. 3. We briefly review the theoretical framework of decoupling specifically for our model. The fundamental theory is QCD with $N_\mathrm{f} = 2$ mass-degenerate quarks. Λ is the Lambda parameter in the $\overline{\mathrm{MS}}$ scheme and M is the renormalization group invariant (RGI) mass of the heavy quarks.[2] After decoupling of the heavy quarks, what is left is a pure gauge theory. Therefore, the Lagrangian of the effective theory valid at energies $E \ll M$ is given by [1,20]

$$\mathcal{L}_\mathrm{dec} = \mathcal{L}_\mathrm{YM} + 1/M^2 \sum_i \omega_i \Phi_i + \mathrm{O}(\Lambda^4/M^4)\,. \quad (1)$$

$\mathcal{L}_\mathrm{YM}$ is the Lagrangian of the $SU(3)$ Yang–Mills (pure gauge) theory. Due to gauge invariance there are no fields of mass dimension equal to five. A complete set of fields of mass dimension equal to six is $\Phi_1 = \mathrm{tr}\{D_\mu F_{\nu\rho} D_\mu F_{\nu\rho}\}$ and $\Phi_2 = \mathrm{tr}\{D_\mu F_{\mu\rho} D_\nu F_{\nu\rho}\}$, where $F_{\mu\nu}$ is the $SU(3)$ field strength tensor and $D_\mu F_{\nu\rho}$ its covariant derivative.

At leading order the effective theory, eq. (1), is a Yang–Mills theory. It has only one free parameter, the renormalized gauge coupling. This coupling is fixed by matching the effective theory to the fundamental theory. Equivalently one can fix the Λ parameter of the Yang–Mills theory, Λ_YM, which becomes a function $\Lambda_\mathrm{YM} = \Lambda_\mathrm{dec}(M, \Lambda)$, see [6,21]. Matching requires that low energy physical observables are the same in the two theories up to power corrections. Let us denote a low energy observable by m^had where, for example, it can represent a hadronic scale such as $1/\sqrt{t_0}$ [22] or $1/r_0$ [23]. After matching

$$m^\mathrm{had}(M) = m^\mathrm{had}_\mathrm{YM} + \mathrm{O}(\Lambda^2/M^2)\,, \quad (2)$$

where $m^\mathrm{had}(M)$ is the hadronic scale in QCD with $N_\mathrm{f} = 2$ heavy quarks of mass M and $m^\mathrm{had}_\mathrm{YM}$ is the hadronic scale in the Yang–Mills theory. Note that $m^\mathrm{had}_\mathrm{YM}$ depends on M through the matching, in particular $m^\mathrm{had}_\mathrm{YM}/\Lambda_\mathrm{YM}$ is a pure number independent of M. Therefore we consider two hadronic scales, $m^{\mathrm{had},1}(M)$ and $m^{\mathrm{had},2}(M)$, whose values in the Yang–Mills theory are $m^{\mathrm{had},1}_\mathrm{YM}$ and $m^{\mathrm{had},2}_\mathrm{YM}$ respectively. An immediate consequence of eq. (2) is

$$R(M) = \frac{m^{\mathrm{had},1}(M)}{m^{\mathrm{had},2}(M)} = \frac{m^{\mathrm{had},1}_\mathrm{YM}}{m^{\mathrm{had},2}_\mathrm{YM}} + \mathrm{O}(\Lambda^2/M^2)\,. \quad (3)$$

The matching of the coupling is irrelevant for the ratios and we have direct access to the power corrections [24]. The effective theory of decoupling predicts that the ratios like in eq. (3) are equal to their value $R(M = \infty)$ in the Yang–Mills theory with a leading power correction in the inverse heavy quark mass given by

$$R(M) = R(\infty) + k\Lambda^2/M^2\,, \quad (4)$$

where k is a parameter which depends on the hadronic scales which are taken to form the ratio. The goal of this work is to verify the behavior in eq. (4) and to establish whether it applies for masses around the charm quark mass.

3. Monte Carlo simulations

We simulate QCD with two mass-degenerate flavors of quarks ($N_\mathrm{f} = 2$). Wilson's plaquette gauge action [25] is employed in the Yang–Mills sector and a doublet of quarks is realized either as standard or as twisted mass [26] Wilson quarks. In both cases a clover term [27,13] with non-perturbatively determined improvement coefficient c_sw [28] is added. It is not needed for the $O(a)$ improvement of the twisted mass action at maximal twist, but was found to reduce the $O(a^2)$ lattice artifacts, see e.g. [29].

The bare coupling β of the gauge action was chosen such that the lattice spacings cover the range $0.023\ \mathrm{fm} \lesssim a \lesssim 0.066\ \mathrm{fm}$. The lattice spacing is determined from the hadronic scale L_1 [30,31]. The scale L_1/a is defined at vanishing quark mass, where the standard and twisted mass Wilson quark formulations are equivalent. Therefore, the lattice spacing for a given bare coupling β is the same for both formulations. In order to obtain the scale L_1 in lattice units at a particular value of β, we fitted the data in Table 13 of [31] as it is explained there. The lattice spacing in physical units is estimated by rescaling the value $a = 0.0486$ fm at $\beta = 5.5$ from [31] by the ratio of the L_1/a values.

In order to resolve the short correlation lengths associated with the large quark masses that we aim at, we are forced to simulate at very small lattice spacings. Critical slowing down becomes a major obstacle which we alleviate by the implementation of open boundary conditions in the time direction [32]. The boundary improvement coefficients are kept at their tree-level values $c_G = 1$ and $c_F = 1$. The publicly available openQCD simulation program [33,34] is used for our simulations.

We used standard O(a) improved Wilson quarks to simulate at quark masses of approximately a factor 1/8, 1/4 and 1/2 of the

[1] Perturbatively the simultaneous decoupling of two heavy quarks is known at three-loop order [15].

[2] Throughout this work, the Λ parameter is defined in the $\overline{\mathrm{MS}}$ scheme. For mass-independent schemes like the $\overline{\mathrm{MS}}$, there is an exact one-loop relation for the Λ parameters between different schemes. The RGI mass M is independent of the scheme (for mass-independent schemes).