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# Elastic scattering in geometrical model

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A B S T R A C T

The experimental data on proton–proton elastic and inelastic scattering emerging from the measurements at the Large Hadron Collider, calls for an efficient model to fit the data. We have examined the optical, geometrical picture and we have found the simplest, linear dependence of this model parameters on the logarithm of the interaction energy with the significant change of the respective slopes at one point corresponding to the energy of about 300 GeV. The logarithmic dependence observed at high energies allows one to extrapolate the proton–proton elastic, total (and inelastic) cross sections to ultra high energies seen in cosmic rays events which makes a solid justification of the extrapolation to very high energy domain of cosmic rays and could help us to interpret the data from an astrophysical and a high energy physics point of view.



## 1. Introduction

The process of elastic scattering of hadrons has been studied experimentally in a wide energy region for more than half a century. In the 1960's with the available center of mass (c.m.s.) energies of $\sqrt{s} = 4$–$6$ GeV it was found that the conventional "diffraction cone" mechanism failed what was clearly visible at larger transferred momenta. Additional data at the energies of $\sqrt{s} = 19$, 20, 23, 28, 31, 45, 53, 62 GeV were published in the middle of 70's. At the end of the previous millennium the range of available energies ends around 2 TeV. Only recently the results of the TOTEM collaboration at the LHC on elastic pp scattering processes at $\sqrt{s} = 7$ TeV were published [9,12].

The measurements at the LHC at 7 TeV c.m.s. collision energy set the next point on an energy scale where the optical model of hadrons can be examined. The observed so far evolution of the proton shadow profile and the energy dependence of the parameters describing its shape could be extended towards the limit of the ultra high-energy cosmic rays (UHECR), where important questions of physics and astrophysics are still unanswered. It is expected that the answers could be linked (also) to some extent to the value of the proton–proton cross sections at around $10^{20}$ eV of laboratory energy.

Many phenomenological models of proton have been proposed. As it is said by Dremlin in Ref. [39] *[....] "Most of them aspire to be 'a phenomenology of everything' related to elastic scattering of hadrons in a wide energy range. Doing so in the absence of applicable laws and methods of the fundamental theory, they have to use a large number of adjustable parameters. The free parameters have been determined by fitting the model results to the available experimental data." [...] Independent of their success and failure, we are sure that, "in the long run, the physical picture may be expected to be much more important than most of the detailed computations".* (the last citation is from the 1969 paper by Cheng and Wu published in the first volume of Phys. Rev. D [36]).

## 2. Phenomenology of the scattering process

The elastic scattering amplitude $F(s,t)$ describing the proton–proton scattering

$$\frac{d\,\sigma_{el}}{d|t|} = \pi\;|F(t)|^2 \quad , \tag{1}$$

could be parameterized in many ways starting from the simple exponential $\exp(Bt)$ proposed already in 1964 by Orear in Ref. [54]. New data allows for more sophisticated form. It was proposed by Barger and Phillips [56] in 1973 in the form

$$F(s,t) = i\left[\sqrt{A(s)}e^{\frac{1}{2}B(s)t} + \sqrt{C(s)}e^{\phi(s)}e^{\frac{1}{2}D(s)t}\right] \quad , \tag{2}$$

which can be used for 7 TeV LHC scattering data explicitly [45,50], or modified, as proposed, e.g., in Ref. [42]

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**Table 1**
The extrapolated cross sections in mb at higher energies.

| Energy ($\sqrt{s}$) | 14 TeV | 24 TeV | 30 TeV | 57 TeV | 95 TeV |
|---|---|---|---|---|---|
| Fagundes et al. [41] | 108.6 ± 1.2 | | | | |
| Bourelly et al. [27] | 103.63 ± 1.0 | | | | |
| Petrov et al. [55] | 106.73 | | | | |
| Block, Halzen et al. [26] | 107.30 | | | | |
| Islam et al. [48] | 110.00 | | | | |
| Jenkovszky et al. [50] | 111.00 | | | | |
| Block [22] | | | | 133.40 ± 1.6 | |
| AKENO [53] | 104 ± 26 | 124 ± 34 | | | |
| Fly's Eye [18] | | | 120 ± 15 | | |
| AUGER [3] | | | | 133.20 ± 13 | |
| Telescope Array [1] | | | | | 170.00 ± 50 |
| **This work** | **105.56** | **115.8** | **120.33** | **132.66** | **143.09** |

$$F(s,t) = i\left[\sqrt{A(s)}e^{\frac{1}{2}B(s)t}G(s,t) + \sqrt{C(s)}e^{\phi(s)}e^{\frac{1}{2}D(s)t}\right] \ , \tag{3}$$

or in the number of possibilities inspected by Khoze, Martin and Ryskin in Ref. [51].

A different modification was proposed by Menon and collaborators in Ref. [35] who consider the parameterization of the scattering amplitude as a sum of Orear exponentials [54]:

$$F(s,t) = \sum_{i=1}^{n} \alpha_i e^{\beta_i t} \ . \tag{4}$$

They obtained, with the summation of up to six components, perfect fits to the ISR data from 19.4 GeV to 62.4 GeV [40]. Their 'model-independent' analysis of elastic proton–proton scattering data [16,17,40] was extended to higher energies and the parameters $\alpha_i$ and $\beta_i$ were expressed as functions of the available c.m.s. energy. Predictions for LHC were given there and are listed in the Table 1.

On the other hand the absorption processes can be naturally studied in a geometrical framework. The correspondence between interaction geometry and the momentum transfer space is defined with the Fourier transform with the help of the profile function $\Gamma(s,b)$ (or the eikonal $\Omega$)

$$\begin{aligned} F(s,t) &= i\int_0^\infty J_0\left(b\sqrt{-t}\right)\ \Gamma(s,b)\, b\, db\ = \\ &= i\int_0^\infty J_0\left(b\sqrt{-t}\right)\ \{1-\exp[-\Omega(s,b)]\}\, b\, db \ . \end{aligned} \tag{5}$$

This gives the possibility to apply the form-factor formalism to the hadron interaction $\Omega(s,t) = C(s)G_p(t)G_p(t)$ where $C(s)$ works for the absorption coefficient.

$$\Omega(s,b) \ = \ (1-i\alpha)\int_0^\infty J_0(qb)\, G^2_{p,E}(t)\, \frac{f(t)}{f(0)}\, q\, dq \ , \tag{6}$$

($q = \sqrt{-t}$). This formalism has been proposed and developed by Bourrely, Soffer and Wu since late 70's [28–30] using

$$\begin{aligned} G_{p,E}(t) &= \frac{1}{\left(1-t/m_1^2\right)\left(1-t/m_2^2\right)} \ , \\ f(t) &= f(0)\,\frac{a^2+t}{a^2-t} \ . \end{aligned} \tag{7}$$

The initial simple model with six free parameters (at high energies) becomes at the LHC energies much more complicated [31]. The asymptotic form has been eventually estimated and compared with the numerical results in Ref. [27].

The pure geometrical picture of proton scattering and the relation of the scattering amplitude to the transmission coefficient ($|\Omega|$) appears already in 1968 in the paper by Chou and Yang [37]. The main point there is to find the (mean) opaqueness, which may be, in general, a complex-valued function, for the given value of the impact parameter. It is quite natural to assume that the hadron has the internal structure defined by the density function $\rho(x,y,z)$. Taking $z$ as a collision axis we can define a hadron profile

$$D(\mathbf{b}) = \int_{-\infty}^{\infty} \rho(x,y,z)\, dz \ , \tag{8}$$

and for two colliding hadrons the convolution is

$$\Omega(b) = \Omega(\mathbf{b}) = iK_{pp}\iint_{-\infty}^{\infty} D(\mathbf{b}-\mathbf{b}')D(\mathbf{b}')\, d^2\mathbf{b}' \ . \tag{9}$$

Any particular model could be fully characterized by the generalized opacity: the eikonal function $\Omega$ (in the impact parameter space) as it is written in Eq. (5). Its particular shape can be obtained using dipole electromagnetic form factors like it is done, for example, in Ref. [37] similar to the one given in Eq. (7).

Another interesting way of introducing $\Omega$ is to use the evolution of the imaginary part of the profile function $\Gamma(s,b) = 1 - \exp[-\Omega(s,b)]$ which could be, according to Ref. [34], determined using the nonlinear differential logistic equation. The concept is that it includes, in a natural way, saturation effects expected as energy grows. This assumption leads to

$$\Gamma(s,b) \ = \ \frac{1}{e^{(b-b_0)/\gamma}+1} \ , \tag{10}$$

where $b_0$ and $\gamma$ are proton radial scale parameters which define the cross section scaling properties. A very similar profile function was found as a special case of the model of Rybczyński and Włodarczyk where shapes of colliding protons are defined by the event-by-event fluctuations of the radius of the proton in the 'black disk' picture [57]. If the fluctuations are negligible the black disk limit is retained, while for the cross section fluctuations described by the gamma distribution, another extreme is obtained: the Gaussian proton profile.

Introducing new scaling variable $\hat{b} = b/b_0(s)$ to Eq. (10) the proton profile satisfies the (modified) geometrical scaling (if $\gamma/b_0$ is constant)

$$\frac{d\sigma_{\rm el}}{dt} \sim b_0^2\left[\ f\left(|t|\, b_0^2\right)\right]^2 \ , \tag{11}$$