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Magnetic polarizability of pion

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ABSTRACT

We explore the energy dependence of π mesons off the background Abelian magnetic field on the base of quenched SU(3) lattice gauge theory and calculate the magnetic dipole polarizability of charged and neutral pions for various lattice volumes and lattice spacings. The contribution of the magnetic hyperpolarizability to the neutral pion energy has been also found.

1. Introduction

Quantum Chromodynamics in strong magnetic fields is a promising topic for research. Magnetic fields of hadronic scale could exist in the Early Universe [1] and could be formed in cosmic objects like magnetars and neutron stars. They can also be achieved in terrestrial laboratories (RHIC, LHC, FAIR, NICA) [2]. The external electromagnetic fields can be utilized as a probe of QCD properties, the recent progress obtained on this way in lattice gauge theories is discussed in [3]. The energy levels of hadrons in external magnetic field can be useful for the calculation of the cross sections [4]. The energies of mesons at the nonzero magnetic field were calculated in various phenomenological approaches [5–9], within the QCD sum rules [10,11] and in the lattice gauge theories [12–14].

Background magnetic Abelian fields also enable to calculate the magnetic polarizabilities of hadrons. In order to get the dipole magnetic polarizability and the hyperpolarizability we measure the energy of a meson as a function of the magnetic field. The magnetic polarizabilities are important physical characteristics describing the distribution of quark currents inside a hadron in an external field. For the first time the concept of polarizability for the nuclear matter was used by A.B. Migdal in the analysis of the scattering of low energy gamma quanta by atomic nuclei [15]. For hadrons the notion of the polarizability was discussed in papers [16,17].

There were some discrepancies between the experimental obtained value of the magnetic and the electric polarizabilities of the charged π mesons and some theoretical predictions based on the chiral perturbation theory [18,19].

Measurement of the electrical and the magnetic polarizabilities of π mesons was performed on the spectrometer SIGMA-AJAX in Serpukhov, on the electron synchrotron Pakhra (LPI) in Moscow, on the MarkII detector at SLAC, at COMPASS (CERN) and other experiments.

According to the obtained data from these experiments, the value of the polarizability of the charged π mesons is positive. The most precise value of the charged pion electric polarizability has been obtained experimentally by the COMPASS experiment $\alpha_\pi = (2.0 \pm 0.6_{stat} \pm 0.7_{sist}) \times 10^{-4}$ fm^3 [20] under the assumption $\alpha_\pi = -\beta_\pi$. Comparison with the successful predictions of the experiments and the chiral perturbation theory [18,19] is interesting for fundamental science.

In this work we consider a behaviour of the ground state energy of pions in the strong magnetic fields on the base of the SU(3) lattice gauge theory. The details of the calculations are briefly sketched in section 2. We discuss the dipole magnetic polarizabilities of $\pi^\pm$ and π^0 mesons in sections 3 and 4 accordingly. The

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Table 1
The lattice simulations details. The lattice volume is shown in the second column, the lattice spacing and number of configurations used are presented in the fourth and the firth columns respectively.

Ensemble	$N_t \times N_s^3$	β_{imp}	a, fm	N_{conf}
A_{16}	16^4	8.20	0.115	245
A_{18}	18^4	8.10	0.125	285
B_{18}	18^4	8.20	0.115	200
C_{18}	18^4	8.30	0.105	235
D_{18}	18^4	8.45	0.095	195
E_{18}	18^4	8.60	0.084	180
A_{20}	20^4	8.20	0.115	275

magnetic hyperpolarizability of the neutral pion is calculated in section 5.

2. Details of calculations

2.1. The improved gauge action

For the generation of quenched $SU(3)$ lattice configurations we used the tadpole improved Lüscher–Weisz action [21], which reduces the ultraviolet lattice artifacts. The action has the form

$$S = \beta_{imp} \sum_{\text{pl}} S_{\text{pl}} - \frac{\beta_{imp}}{20u_0^2} \sum_{\text{rt}} S_{\text{rt}}, \tag{1}$$

where $S_{\text{pl,rt}} = (1/2)\text{Tr}\,(1 - U_{\text{pl,rt}})$ is the plaquette (denoted by pl) or 1×2 rectangular loop term (rt), $u_0 = (W_{1\times1})^{1/4} = \langle(1/2)\text{Tr}\,U_{\text{pl}}\rangle^{1/4}$ is the input tadpole factor computed at zero temperature [28]. Our simulations have been carried out on the symmetrical lattices. The parameters of the lattice ensembles and number of the configurations are listed in the Table 1.

2.2. Fermionic spectrum

We solve the Dirac equation numerically

$$D\psi_k = i\lambda_k\psi_k, \quad D = \gamma^{\mu}(\partial_{\mu} - iA_{\mu}) \tag{2}$$

and found the eigenfunctions ψ_k and the eigenvalues λ_k for a test quark in an external gauge field A_{μ}.

For this goal we use the massive overlap operator [22]. It has the following form

$$M_{ov} = \left(1 - \frac{am_q}{2\rho}\right) D_{ov} + m_q, \tag{3}$$

where m_q is the quark mass, $\rho = 1.4$ is the parameter in our calculations, D_{ov} is the massless overlap Dirac operator, which preserves chiral invariance even at the finite lattice spacing a. It may be written as

$$D_{ov} = \frac{\rho}{a}\left(1 + \frac{D_W}{\sqrt{D_W^{\dagger} D_W}}\right) = \frac{\rho}{a}(1 + \gamma_5 \text{sign}\,(H)), \tag{4}$$

where $D_W = M - \rho/a$ is the Wilson–Dirac operator with the negative mass term ρ/a, M is the Wilson hopping term, $H = \gamma_5 D_W$ is the Hermitian Wilson–Dirac operator.

We construct the polynomial approximation for the function

$$\text{sign}\,(H) = H/\sqrt{H^{\dagger}H}. \tag{5}$$

This approximation should be valid on the entire spectrum of H matrix $[\lambda_{min}, \lambda_{max}] \in \mathcal{R}$. Since

$$\text{sign}\,(H) \equiv \text{sign}\,(H/\|H\|) = \text{sign}\,(W) \tag{6}$$

and $\|H\| = \lambda_{max}$, then spec$(W) \in [\lambda_{min}/\lambda_{max}; 1] \in \mathcal{R}$. The MinMax polynomial approximation is used for the function $1/\sqrt{H^2}$ at $\sqrt{\epsilon} \leq H < 1$, where $\epsilon = \lambda_{min}^2/\lambda_{max}^2$.

The polynomial $P_n(H^2)$, $H^2 \in [\epsilon; 1]$ of a degree n can be the best approximation, if it minimizes the maximal relative error

$$\delta = max|h(H^2)| \tag{7}$$

where $h(H^2) = 1 - \sqrt{H^2}P_n(H^2)$. The polynom is represented by the series

$$P_n(H^2) = \sum_{k=0}^{n} c_k T_k(z), \quad z = \frac{2H^2 - 1 - \epsilon}{1 - \epsilon}, \tag{8}$$

where $T_n(z)$, $n = 0, 1, 2, \ldots$ are the Chebyshev polynomials defined in the range $[-1; 1]$. The detailed description of the algorithm used for the calculation of P_n can be found in [29]. The resulting polynomial of the matrix has the same eigenfunctions ψ_k as the original matrix. The eigenvalues can be found from the eigenfunctions using the formula $\lambda_k = \frac{\psi_k^{\dagger}|Q|\psi_k}{\psi_k^{\dagger}\psi_k}$ for a some operator Q [30]. From the polynomial approximation for the sign function we get the approximation for the overlap Dirac operator. We find its eigenfunctions and eigenvalues, which are used for the calculation of the propagators and the correlators.

As we consider pure lattice gauge theory, the Abelian magnetic field is introduced only into the overlap Dirac operator. In the symmetric gauge the magnetic field parallel to 'z' axis has the form

$$A_{\mu}^{B} = \frac{B}{2}(x\delta_{\mu,2} - y\delta_{\mu,1}). \tag{9}$$

The total gauge field is the sum of the non-Abelian $SU(3)$ gluonic field and the Abelian $U(1)$ field of magnetic photons

$$A_{\mu\,ij} = A_{\mu\,ij}^{gl} + A_{\mu}^{B}\delta_{ij}, \tag{10}$$

where $i, j = 1, ..N_c^2 - 1$ are the colour indices, $\mu = 1, 2, 3, 4$ are the Lorentz indices. Quark fields obey periodic boundary conditions in space and antiperiodic boundary conditions in time. In order to match (10) with the periodic boundary conditions we apply the additional x-dependent boundary twist for fermions [31].

In the finite lattice volume the magnetic flux trough any two-dimensional face of the hypercube is quantized. So the magnetic field value is

$$eB = \frac{6\pi n_B}{(aN_s)^2}, \quad n_B \in \mathbb{Z}, \tag{11}$$

where e is the elementary charge and N_s is the numbers of lattice sites in spatial directions.

2.3. Calculation of correlation functions

To observe the ground state energy for a meson we construct the interpolating operator creating the state with the corresponding quantum numbers. In case the pseudoscalar charged π meson the interpolating operator is described by the equations

$$O(\pi^{+}) = \bar{\psi}_d(n)\gamma_5\psi_u(n), \quad O(\pi^{-}) = \bar{\psi}_u(n)\gamma_5\psi_d(n) \tag{12}$$

The interpolating operator for the neutral pion is

$$O(\pi^{0}) = (\bar{\psi}_u(n)\gamma_5\psi_u(n) - \bar{\psi}_d(n)\gamma_5\psi_d(n))/\sqrt{2}. \tag{13}$$

It should be mentioned that in Eucledean space $\bar{\psi} = \psi^{\dagger}$. We are interested in the 2-point lattice correlation function of the interpolating operators