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SU(2) Higher-order effective quark interactions from polarization

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ABSTRACT

Higher order quark effective interactions are found for SU(2) flavor by departing from a non-local quark–quark interaction. By integrating out a component of the quark field, the determinant is expanded in chirally symmetric and symmetry breaking effective interactions up to the fifth order in the quark bilinears. The resulting coupling constants are resolved in the leading order of the longwavelength limit and exact numerical ratios between several of these coupling constants are obtained in the large quark mass limit. In this level, chiral invariant interactions only show up in even powers of the quark bilinears, i.e. $\mathcal{O}(\bar{\psi}\psi)^{2n}$ $(n = 1, 2, 3, ..)$, whereas (explicit) chiral symmetry breaking terms emerge as $\mathcal{O}(\bar{\psi}\psi)^n$ being always proportional to some power of the Lagrangian quark mass.

1. Introduction

The understanding of the effects and mechanisms by which quarks interact among themselves is a necessary step to provide a complete description of hadron structure and dynamics and the phase diagram of Strong Interactions. In low and intermediary energies these interactions can be parametrized in terms of realistic effective quark interactions that usually provide important information to establish the needed relations between QCD and hadron dynamics [1,2]. The basic and fundamental mechanisms that give rise to each of the effective interactions and parameters present in effective models and theories should be expected to be well understood, although a quite large amount of different quark effective interactions are expected to emerge due to the intricate structure of QCD. The Nambu Jona Lasinio (NJL) model is known to describe qualitatively well several important effects in hadron phenomenology [3,4] in spite of its known limitations. A large variety of possible corrections to the NJL coupling can be expected to emerge from QCD, and higher order quark interactions were shown to provide relevant effects for the ground state [5–7], chiral phase transition (flavor SU(2) and SU(3)) and higher energies [8–12] and eventually they might contribute to multiquark structures [13]. In FAIR-GSI the high density phase diagram will be tested eventually providing relevant information also about the role of multiquark interactions in different regions of the phase diagram. Few mechanisms have been shown to drive quark effective interactions by gluon exchange [14–24]. Instanton mediation have been shown to provide one of the most investigated mechanisms for effective quark interactions for example by means of the Kobayashi–Maskawa–'t Hooft interaction or instanton gas model. It depends strongly on flavor and, for flavor SU(2), it yields a second order quark interaction different from the usual chiral NJL interaction, producing the axial anomaly and its phenomenological consequences [14,25,3,4]. Polarization effects were shown to produce low energy and higher order effective interactions [22].

In the present work, flavor SU(2) higher order quark effective interactions are calculated from polarization effects by departing from a dressed one gluon exchange (i.e. a global color model) along the lines of Refs. [22]. Simple gluon exchange is a basic mechanism that cannot describe low energy hadron properties, including dynamical breakdown of chiral symmetry (DχSB), although it can be dressed by gluon interactions producing enough strength for DχSB [26–28]. This work is organized as follows. In the next section the method is shortly described according to which the quark bilinears are separated into two components, i.e. $\bar{\psi}\Gamma\psi \to (\bar{\psi}\Gamma\psi)_1 + (\bar{\psi}\Gamma\psi)_2$, as done in the background field method [29]. The background field (ψ_1) remains as interacting quarks and the field ψ_2 is integrated out. Instead of introducing auxiliary fields (a.f.) for the component that is integrated out, a weak field approximation is considered such that: $(\bar{\psi}\psi)_1^2 >> (\bar{\psi}\psi)_2^2$. Results are the same as by introducing a.f. in the leading order since the a.f., for example as shown in Ref. [22,30,31], play no role in the resulting leading quark–quark effective interactions. The quark determinant is expanded in powers of quark bilinears yielding chiral invariant and also symmetry breaking terms proportional to the Lagrangian

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quark mass. The corresponding effective couplings are resolved. This expansion is performed up to the eighth order for all the bilinears and up to the tenth order for the scalar-pseudoscalar ones. Some ratios between the effective coupling constant are shown to provide simple numerical values. Some numerical estimations are also shown.

2. Diquark interaction and quark field splitting

The departing point is the following quark effective interaction:

$$S_{eff}[\bar{\psi},\psi] = \int_x \left[\bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi - \frac{g^2}{2}\int_y j^b_\mu(x)\tilde{R}^{\mu\nu}_{bc}(x-y)j^c_\nu(y) \right], \quad (1)$$

where b, c stand for color indices, the color quark current is $j^\mu_b = \bar{\psi}\lambda_b\gamma^\mu\psi$, the sum in color, flavor and Dirac indices are implicit, $\int_x$ stands for $\int d^4x$, the kernel $\tilde{R}^{\mu\nu}_{bc}$ can be written in terms of transversal and longitudinal components (R_T and R_L) as: $\tilde{R}^{\mu\nu}_{ab} \equiv \tilde{R}^{\mu\nu}_{ab}(x-y) = \delta_{ab}\left[R_T\left(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\partial^2}\right) + R_L\frac{\partial^\mu\partial^\nu}{\partial^2}\right]$ with implicit Dirac delta functions $\delta(x-y)$. With a Fierz transformation [3,4,30,31], by picking up the color singlet sector only, the above effective quark interaction can be expressed in terms of bilocal quark bilinears, $j^q_i(x,y) = \bar{\psi}(x)\Gamma^q\psi(y)$ where $q = s, p, v, a$ and Γ_q stands for Dirac and flavor SU(2) operators $\Gamma_s = I$ for the 2×2 flavor and 4×4 identities, $\Gamma_p = \sigma_i i\gamma_5$, $\Gamma^\mu_v = \gamma^\mu\sigma_i$ and $\Gamma^\mu_a = i\gamma_5\gamma^\mu\sigma_i$, being σ_i are the flavor SU(2) Pauli matrices. The Fierz transformed interaction is written as: $\Omega = \alpha\sum_q j^q_i(x,y)R_q(x-y)j^q_i(y,x)$, where $\alpha = 8/9$, R_q are the kernels in each of the q channel of the interaction. Next the quark field is separated into two components, one of them associated with polarization virtual processes eventually to the formation of quark bound states such as light mesons and the chiral condensate and the other component remains as (constituent) quark. This procedure is basically the one loop background field method [29], and this will be done by rewriting the quark bilinears above as:

$$\bar{\psi}\Gamma^q\psi \to (\bar{\psi}\Gamma^q\psi)_2 + (\bar{\psi}\Gamma^q\psi)_1. \quad (2)$$

The Fierz transformed non-local interaction above can then be written as: $\Omega \to \Omega_1 + \Omega_2 + \Omega_{12}$ where Ω_1 and Ω_2 stand for the interactions of each of the quark components, and Ω_{12} for the mixed terms. The component ψ_2 will be integrated out and the fourth order terms can be eliminated in different approximated ways. Firstly by simply considering a weak field approximation and therefore by neglecting $\Omega_2 << \Omega_1$. This yields the same results as the leading terms resulting from the auxiliary field method which eliminates the fourth order interactions Ω_2, as discussed in Refs. [22,30,31]. In this case, bilocal auxiliary fields $(S, P_i, V^i_\mu, \bar{A}^i_\mu)$ are introduced which couple to the remaining quark component. These couplings encode the non-linearities of the initial model. However in this work we are interested only in the quark self-interactions and these couplings can be neglected. Even if one were interested in the effective interactions induced by these couplings to the auxiliary fields (a.f.), the resulting quark–quark effective interactions induced by the a.f. would be of higher order and numerically smaller. By integrating out the component $(\psi)_2$, and by writing the determinant as: $\det(A) = \exp(Tr\ln A)$, the following non-linear non-local effective action for quarks $(\psi)_1$ is obtained:

$$\begin{aligned} S_{eff} = -i\,Tr\ln\Big\{ & i(S_0)^{-1}(x-y) \\ & - i\alpha g^2\bar{R}^{\mu\nu}(x-y)\gamma_\mu\sigma_i\left[(\bar{\psi}_y\gamma_\nu\sigma_i\psi_x) - i\gamma_5(\bar{\psi}_y i\gamma_5\gamma_\nu\sigma_i\psi_x)\right] \\ & + 2i\alpha g^2 R(x-y)\left[(\bar{\psi}_y\psi_x) + i\gamma_5\sigma_i(\bar{\psi}_y i\gamma_5\sigma_i\psi_x)\right]\Big\} - I_0, \end{aligned} \quad (3)$$

where Tr stands for traces of discrete internal quantum numbers indices and integration of spacetime coordinates/momentum and $I_0 = \int_x\left[\bar{\psi}\,(i\gamma\cdot\partial - m)\,\psi - \frac{g^2}{2}\int_y j^a_\mu(x)R^{\mu\nu}_{ab}(x-y)j^b_\nu(y)\right]$. In this expression the label $_1$ for the quark field was omitted because it is the only one remaining from here on. $(S_0)^{-1} = (S_0)^{-1}(x-y) \equiv (i\gamma\cdot\partial - m)$, with an implicit Dirac delta function, and where instead of m one could introduce an effective mass (m^*) which arise from the coupling to the scalar auxiliary variable s which produces the dynamical chiral symmetry breaking as discussed at length in Refs. [3,4,22,30,31]. The following kernels have also been defined from the Fierz transformation: $R = R(x-y) = 3R_T + R_L$ and $\bar{R}^{\mu\nu} = \bar{R}^{\mu\nu}(x-y) = g^{\mu\nu}(R_T + R_L) + 2\frac{\partial^\mu\partial^\nu}{\partial^2}(R_T - R_L)$ with implicit Dirac delta functions. By neglecting the derivative couplings, with a shorthand notation for which the non-local character of all the kernels is omitted, i.e $R = R(x-y)$, $\bar{R}^{\mu\nu} = \bar{R}^{\mu\nu}(x-y)$ and $S_0 = S_0(x-y)$, the quark determinant above can be rewritten [32] as:

$$\begin{aligned} I_d \equiv & -\frac{i}{2}Tr\ln\left[S^{-1}S^{\dagger -1}\right] = -\frac{i}{2}Tr\ln[\tilde{S}_0^{-1}] \\ & -\frac{i}{2}Tr\ln\Bigg[1 + \beta\tilde{S}_0\left(2R\bar{\psi}\psi - \bar{R}^{\mu\nu}\gamma_\mu\sigma_i\bar{\psi}\gamma_\nu\sigma_i\psi\right) \\ & + g^4\sum_{q,q'}\tilde{S}_0 a_{q,q'}(\Gamma_q\bar{\psi}\Gamma_q\psi)(\Gamma^\dagger_{q'}\bar{\psi}\Gamma_{q'}\psi)\Bigg], \end{aligned} \quad (4)$$

where $\beta = 2mg^2\alpha$ was defined for the quark mass (symmetry breaking term), $\tilde{S}_0 \equiv \tilde{S}_0(x-y) = -1/(\partial^2 + m^2)\delta(x-y)$ was factorized producing an irrelevant multiplicative constant in the generating functional, $a_{q,q'}$ are coefficients for each of the flavor channels, and crossed terms $(q, q' = s, p, v, a)$ with the corresponding operators Γ_q and kernels R_q. This expression still has a strong non-local character which is not written explicitly. This determinant will be expanded for small $\tilde{S}_0$, i.e. large quark (effective) mass by considering that m may be an effective (constituent) quark mass. A small coupling g^2 or weak quark field ψ_1 yields essentially the same results such that the final polynomial quark effective interactions are written in terms of effective coupling constants in the local limit of the resulting couplings. It can be noticed that all the chiral invariant interactions only appear from the contributions exclusively of the last term inside of the determinant. Therefore chiral invariant terms for this $SU(2)$ flavor will be $\mathcal{O}[(\bar{\psi}\psi)^2]^n$. All the interactions for which the second term contributes (proportional to the quark mass) will be not chiral invariant. One of the first order terms yields a contribution for the quark effective mass [22] of the form: $\Delta m^* = -i2\alpha g^2 m\; Tr\;\tilde{S}_0 R$.

3. SU(2) Quark effective interactions

The leading terms, by resolving the effective coupling constants in the longwavelength limit and the zero order derivative expansion, are:

$$\begin{aligned} \mathcal{L}_4 = & g_4\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\sigma_i i\gamma_5\psi)^2\right] \\ & - g_{v4}\left[(\bar{\psi}\sigma_i\gamma_\mu\psi)^2 + (\bar{\psi}\sigma_i\gamma_5\gamma_\mu\psi)^2\right] + \mathcal{L}_4^{sb} \end{aligned} \quad (5)$$

where $\mathcal{L}_4^{sb} = g_{4,sb}(\bar{\psi}\psi)^2 + g_{4,v,sb}(\bar{\psi}\sigma_i\gamma_\mu\psi)^2$ are symmetry breaking terms which emerge from the second order expansion although they are of the same order of magnitude as the first one, as it can be noted in the next expressions. These effective coupling constants were resolved as: