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Test of conformal gravity with astrophysical observations

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ABSTRACT

Since it can describe the rotation curves of galaxies without dark matter and can give rise to accelerated expansion, conformal gravity attracts much attention recently. As a theory of modified gravity, it is important to test conformal gravity with astrophysical observations. Here we constrain conformal gravity with SNIa and Hubble parameter data and investigate whether it suffers from an age problem with the age of APM 08279+5255. We find conformal gravity can accommodate the age of APM 08279+5255 at 3σ deviation, unlike most of dark energy models which suffer from an age problem.

1. Introduction

Many astronomical observations indicate that the Universe is undergoing late-time acceleration. An unknown energy component, dubbed as dark energy, is usually proposed to explain the accelerated expansion. The simplest and most attractive candidate is the cosmological constant model (ΛCDM). This model is consistent with most of current astronomical observations, but suffers from the cosmological constant problem [1], as well as age problem [2]. It is thus natural to pursue alternative possibilities to explain the mystery of the accelerated expansion. Over the past numerous dark energy models have been proposed, such as quintessence, phantom, k-essence, quintom, tachyon, etc. Rather than by introducing a dark energy, modified gravity, such as $f(R)$ theories (for reviews, see e.g. [3–6]), $f(T)$ theories (see e.g. [7,8]), and $f(R, R_{\mu\nu}R^{\mu\nu})$ [9–11] theories, are proposed as ways to obtain a late accelerated expansion by modifying the Lagrangian of general relativity. Conformal gravity (CG, following the original work by Weyl [12], for reviews, see [13–15]), as a special $f(R, R_{\mu\nu}R^{\mu\nu})$ theory, can give rise to accelerated expansion [16]. It was also claimed that CG can describe the rotation curves of galaxies without dark matter [17]. A static solution for a charged black hole in CG was presented in [18]. Perturbations in the cosmology associated with the conformal gravity theory were investigated in [19]. It had been shown that currently available SNIa and GRB samples were accommodated well by CG [21]. Although CG attracts much attention, it is also confronted with some challenges. It was argued in [22] that in the limit of weak fields and slow motions CG does not agree with the predictions of general relativity, and it is therefore ruled out by Solar System observations (recently, however, it was indicated in [23] that conformal gravity can potentially test well against all astrophysical observations to date). CG cannot describe the phenomenology of gravitational lensing [24] and of clusters of galaxies [25]. CG is not able to explain the properties of X-ray galaxy clusters without resorting to dark matter [26]. CG cannot pass the primordial nucleosynthesis test [27], however, this is an open problem, because all the possible mechanisms producing deuterium are still incomplete.

Besides the dark energy problem, the age problem is another important test for cosmological models. A spatially flat Friedmann–Robertson–Walker (FRW) universe dominated by matter (with age $T = 2/3H_0$), for example, is ruled out unless $h < 0.48$ ($h = H/100\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$) [28], compared with the 14 Gyr age of the Universe inferred from old globular clusters. Introducing dark energy cannot only explain the accelerated expansion, but also reconcile the age problem. However, the discovery of an old quasar APM 08279+5255 at $z = 3.91$ which was initially estimated to be around 2–3 Gyr [29] and re-evaluated to be 2.1 Gyr [30] has once again led to an age problem for cosmological models, such as ΛCDM [30,31], the creation of cold dark matter models [32], $\Lambda(t)$ model [33], the new agegraphic dark energy [34], parametrized variable Dark Energy Models [35,36], the $f(R) = \sqrt{R^2 - R_0^2}$ model [37], quintessence [38,39], holographic dark energy model [40,41], braneworld modes [42–45], and other models [46–48]. Most of these researches imposed a priori on the Hubble constant H_0, or on the matter density parameter Ω_{m0}, or on other parameters. To a certain extent, the age problem is dependent on the values of H_0 or Ω_{m0} one takes. In [2], the age problem in ΛCDM had been

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investigated in a consistent way, not any special values of $\Omega_{\rm m0}$ or H_0 have been taken with prejudice.

As a theory of modified gravity, it is important to test conformal gravity with astrophysical observations. Here we aim to test CG with observational data including the age of APM 08279+5255. Following [2], we obtain directly observational constraints on parameters from SNIa and $H(z)$ data in the framework of the CG, then investigate whether it suffers from an age problem in the parameter space allowed by these observations.

The structure of this Letter is as follows. In Section 2, we review the cosmology in CG. In Section 3, we consider constraints on the parameters of the cosmology in CG from SNIa and $H(z)$ data, and use the best-fit values to discuss the "age problem". Conclusions and discussions are given in Section 4.

2. Cosmology in conformal gravity

The action of CG with matter is given by

$$\mathcal{I} = -\alpha_{\rm g} \int C_{\mu\nu\kappa\lambda} C^{\mu\nu\kappa\lambda} \sqrt{-g}\, {\rm d}^4x + \mathcal{I}_{\rm m}, \tag{1}$$

where $\alpha_{\rm g}$ is a dimensionless coupling constant and $C_{\mu\nu\kappa\lambda}$ the Weyl tensor. This action is invariant under local conformal transformations: $g_{\mu\nu} \to e^{2\alpha(x)} g_{\mu\nu}$. This symmetry forbids the presence of any $\Lambda\sqrt{-g}\,{\rm d}^4x$ term in the action, so CG does not suffer from the cosmological constant problem. Secondly, since $\alpha_{\rm g}$ is dimensionless, unlike general relativity CG is renormalizable [15]. Thirdly, though the equations of motion are fourth-order, CG is a ghost-free theory [49]. However, CG is also confronted with some challenges as discussed in the previous section. The matter action can be of the form [27,50]

$$\mathcal{I}_{\rm m} = -\int \sqrt{-g}\, {\rm d}^4x \times \left[\frac{1}{2} S^{;\mu} S_{;\mu} + \lambda S^4 - \frac{1}{12} S^2 R + i\bar{\psi}\gamma^\mu D_\mu \psi - \zeta S \bar{\psi}\psi\right], \tag{2}$$

where scalar field $S(x)$ is introduced to spontaneously break the conformal symmetry and renders the particles massive, ψ is a fermion field representing all matter field, $D_\mu = \partial_\mu + \Gamma_\mu$ is the covariant derivative with Γ_μ the fermion spin connection, λ and ζ are dimensionless coupling constants, γ^μ are the general relativistic Dirac matrices. λS^4 represents the negative minimum of the Ginzburg–Landau potential [16] with $\lambda < 0$. For action (1), variation with respect to the metric generates the field equations

$$4\alpha_{\rm g} W_{\mu\nu} = T_{\mu\nu}, \tag{3}$$

where

$$W_{\mu\nu} = -\frac{1}{6} g_{\mu\nu} R^{;\lambda}_{;\lambda} + \frac{2}{3} R_{;\mu;\nu} + R^{;\lambda}_{\mu\nu;\lambda} - R^{;\lambda}_{\lambda\nu;\mu} - R^{;\lambda}_{\lambda\mu;\nu} + \frac{2}{3} R R_{\mu\nu} - 2R_{\mu\lambda} R^\lambda_\nu + \frac{1}{2} g_{\mu\nu} R_{\lambda\kappa} R^{\lambda\kappa} - \frac{1}{6} g_{\mu\nu} R^2, \tag{4}$$

and the energy–momentum tensor of matter is

$$T^{\mu\nu} = i\bar{\psi}\gamma^\mu D^\nu \psi + \frac{2}{3} S^{;\mu} S^{;\nu} - \frac{1}{6} g^{\mu\nu} S^{;\kappa} S_{;\kappa} - \frac{1}{3} S S^{;\mu;\nu} + \frac{1}{3} g^{\mu\nu} S S^{;\kappa}_{;\kappa} - g^{\mu\nu}\lambda S^4 - \frac{1}{6} S^2 \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right). \tag{5}$$

By using local conformal invariance, the energy–momentum tensor can be written as

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kin} - \frac{1}{6} S_0^2 \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) - g^{\mu\nu}\lambda S_0^4 \tag{6}$$

where $T^{\mu\nu}_{\rm kin} = i\bar{\psi}\gamma^\mu D^\nu \psi$ and S_0 is a constant. According to Eq. (3) and $W^{\mu\nu} = 0$, we have

$$T^{\mu\nu}_{\rm kin} - g^{\mu\nu}\lambda S_0^4 = \frac{1}{6} S_0^2 \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right). \tag{7}$$

Considering a perfect fluid, $T^{\mu\nu}_{\rm kin} = (\rho + p)u^\mu u^\nu + p g^{\mu\nu}$, in the Friedmann–Robertson–Walker–Lemaître (FRWL) spacetime with the scale factor $a(t)$

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right], \tag{8}$$

where the spatial curvature constant $K = +1$, 0, and -1 corresponds to a closed, flat and open Universe, respectively, Eq. (7) takes the form

$$H^2 + \frac{K}{a^2} = -\frac{2\rho_{\rm m}}{S_0^2} - 2\lambda S_0^2, \tag{9}$$

where $H = \dot{a}/a$ and $\rho_{\rm m}$ represents energy density of matter which can be separated into a relativistic and a non-relativistic component: $\rho_{\rm m} = \rho_{\rm nr} + \rho_{\rm r} = \rho_{\rm nr0} a^3 + \rho_{\rm r0} a^4$. Taking substitutions $G = -3/(4\pi S_0^2)$ and $\Lambda = -6\lambda S_0^2$, Eq. (9) is identical to the standard Friedmann equation: $H^2 + K/a^2 = 8\pi G\rho/3 + \Lambda/3$. Both G and Λ, however, are negative and depend on the same parameter S_0^2 in the opposite way in CG. Constrained from the rotation curves of spiral galaxies, K must be negative: $K < 0$ [17].

If we define $\Theta_{\rm m} \equiv 2\rho_{\rm m}/(H^2 S_0^2) = \Theta_{\rm nr} + \Theta_{\rm r}$, $\Theta_\Lambda \equiv -2\lambda S_0^2/H^2$, and $\Theta_{\rm K} \equiv -K/(a^2 H^2)$ (because $K < 0$ and $\lambda < 0$, all these parameters are positive), Eq. (9) yields: $\Theta_\Lambda + \Theta_{\rm K} - \Theta_{\rm nr} - \Theta_{\rm r} = 1$. By taking the derivative of Eq. (9), we obtain

$$\frac{\ddot{a}}{a} = H^2\left(\Theta_\Lambda + \Theta_{\rm r} + \frac{\Theta_{\rm nr}}{2}\right), \tag{10}$$

which is always positive, whereas the deceleration parameter

$$q \equiv -\frac{\ddot{a}a}{\dot{a}} = -\Theta_\Lambda - \Theta_{\rm r} - \frac{\Theta_{\rm nr}}{2}, \tag{11}$$

is always negative. So the expansion of the universe in CG accelerates at all times, unlike the standard cosmology. With the present values of Θ parameters, Eq. (9) can be re-expressed as

$$H^2 = H_0^2\left(\Theta_{\Lambda 0} + \Theta_{\rm K0} a^{-2} - \Theta_{\rm nr0} a^{-3} - \Theta_{\rm r0} a^{-4}\right), \tag{12}$$

which is analogous to the standard Friedmann equation, but has negative signs in front of the matter parameters $\Theta_{\rm nr0}$ and $\Theta_{\rm r0}$. This equation implies that in CG a will reach a minimum value $a_{\rm min} > 0$, rather than the singularity $a = 0$. By adjusting parameters, we can obtain smaller and smaller $a_{\rm min}$ (lager and lager $z_{\rm max}$). Because there is no need to introduce dark matter in CG, terms $\Theta_{\rm nr0}$ and $\Theta_{\rm r0}$ can be neglect for large scale factor a, comparing with terms $\Theta_{\Lambda 0}$ and $\Theta_{\rm K0}$. The present age of the universe is found to be

$$H_0 t_0 = \frac{1}{\sqrt{-q_0}} {\rm atanh}(\sqrt{-q_0}). \tag{13}$$

3. Observational constraints on conformal gravity

In this section, we use the Union2.1 SNIa data and the observational Hubble parameter data to consider observational bounds on CG, and test it with the age of an old quasar by using the best-fit values constrained from SNIa and $H(t)$ data.