Physics Letters B 727 (2013) 48–56

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Non-local massive gravity

Leonardo Modesto [a], Shinji Tsujikawa [b]

[a] Department of Physics & Center for Field Theory and Particle Physics, Fudan University, 200433 Shanghai, China
[b] Department of Physics, Faculty of Science, Tokyo University of Science, 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

ARTICLE INFO

Article history:
Received 26 July 2013
Received in revised form 2 October 2013
Accepted 15 October 2013
Available online 22 October 2013
Editor: M. Trodden

Keywords:
Perturbative quantum gravity
Non-local field theory

ABSTRACT

We present a general covariant action for massive gravity merging together a class of "non-polynomial" and super-renormalizable or finite theories of gravity with the non-local theory of gravity recently proposed by Jaccard, Maggiore and Mitsou (Phys. Rev. D 88 (2013) 044033). Our diffeomorphism invariant action gives rise to the equations of motion appearing in non-local massive gravity plus quadratic curvature terms. Not only the massive graviton propagator reduces smoothly to the massless one without a vDVZ discontinuity, but also our finite theory of gravity is unitary at tree level around the Minkowski background. We also show that, as long as the graviton mass m is much smaller the today's Hubble parameter H_0, a late-time cosmic acceleration can be realized without a dark energy component due to the growth of a scalar degree of freedom. In the presence of the cosmological constant Λ, the dominance of the non-local mass term leads to a kind of "degravitation" for Λ at the late cosmological epoch.

1. Introduction

The construction of a consistent theory of massive gravity has a long history, starting from the first attempts of Fierz and Pauli [1] in 1939. The Fierz–Pauli theory, which is a simple extension of General Relativity (GR) with a linear graviton mass term, is plagued by a problem of the so-called van Dam–Veltman–Zakharov (vDVZ) discontinuity [2]. This means that the linearized GR is not recovered in the limit that the graviton mass is sent to zero.

The problem of the vDVZ discontinuity can be alleviated in the non-linear version of the Fierz–Pauli theory [3]. The non-linear interactions lead to a well behaved continuous expansion of solutions within the so-called Vainshtein radius. However, the non-linearities that cure the vDVZ discontinuity problem give rise to the so-called Boulware–Deser (BD) ghost [4] with a vacuum instability.

A massive gravity theory free from the BD ghost was constructed by de Rham, Gabadadze and Tolley (dRGT) [5] as an extension of the Galileon gravity [6]. On the homogeneous and isotropic background, however, the self-accelerating solutions in the dRGT theory exhibit instabilities of scalar and vector perturbations [7]. The analysis based on non-linear cosmological perturbations shows that there is at least one ghost mode (among the five degrees of freedom) in the gravity sector [8]. Moreover it was shown in Ref. [9] that the constraint eliminating the BD ghost gives rise to an acausality problem. These problems can be alleviated by extending the original dRGT theory to include other degrees of freedom [10–12] (like quasidilatons) or by breaking the homogeneity [13] or isotropy [14,15] of the cosmological background.

Recently, Jaccard et al. [16] constructed a non-local theory of massive gravity by using a quadratic action of perturbations expanded around the Minkowski background. This action was originally introduced in Refs. [17,18] in the context of the degravitation idea of the cosmological constant. The resulting covariant non-linear theory of massive gravity not only frees from the vDVZ discontinuity but respects causality. Moreover, unlike the dRGT theory, it is not required to introduce an external reference metric.

Jaccard et al. [16] showed that, on the Minkowski background, there exists a scalar ghost in addition to the five degrees of freedom of a massive graviton, by decomposing a saturated propagator into spin-2, spin-1, and spin-0 components. For the graviton mass m of the order of the today's Hubble parameter H_0, the vacuum decay rate induced by the ghost was found to be very tiny even over cosmological time scales. The possibility of the degravitation of a vacuum energy was also suggested by introducing another mass scale μ much smaller than m.

In this Letter we propose a general covariant action principle which provides the equations of motion for the non-local massive gravity [16] with quadratic curvature terms. The action turns out to be a bridge between a class of super-renormalizable or finite theories of quantum gravity [19–25] and a diffeomorphism invariant theory for a massive graviton.

The theory previously studied in Refs. [19–25] has an aim to provide a completion of the Einstein gravity through the introduction of a non-polynomial or semi-polynomial entire function (form

E-mail addresses: lmodesto@fudan.edu.cn (L. Modesto), shinji@rs.kagu.tus.ac.jp (S. Tsujikawa).

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.10.037

factor) without any pole in the action. In contrast, the non-local massive gravity studied in this Letter shows a pole in the classical action making it fully non-local. However, the Lagrangian for massive gravity can be selected out from the theories previously proposed [19–25] once the form factor has a particular infrared behavior. The non-local theory resulting from the covariant Lagrangian is found to be unitary at tree level on the Minkowski background. Moreover, the theory respects causality and smoothly reduces to the massless one without the vDVZ discontinuity.

We will also study the cosmology of non-local massive gravity on the flat Friedmann–Lemaître–Robertson–Walker (FLRW) background in the presence of radiation and non-relativistic matter.[1] Neglecting the contribution of quadratic curvature terms irrelevant to the cosmological dynamics much below the Planck scale, the dynamical equations of motion reduce to those derived in Ref. [16]. We show that, as long as the graviton mass m is much smaller than H_0, the today's cosmic acceleration can be realized without a dark energy component due to the growth of a scalar degree of freedom.

Our Letter is organized as follows. In Section 2 we show a non-local covariant Lagrangian which gives rise to the same equation of motion as that in non-local massive gravity with quadratic curvature terms. We also evaluate the propagator of the theory to study the tree-level unitarity. In Section 3 we study the cosmological implications of non-local massive gravity in detail to provide a minimal explanation to dark energy in terms of the graviton mass. We also discuss the degravitation of the cosmological constant induced by the non-local mass term. Conclusions and discussions are given in Section 4.

Throughout our Letter we use the metric signature $\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$. The notations of the Riemann tensor, the Ricci tensor, and Ricci scalar are $R^{\mu}_{\nu\rho\sigma} = -\partial_\sigma \Gamma^{\mu}_{\nu\rho} + \cdots$, $R_{\mu\nu} = R^{\sigma}_{\mu\nu\sigma}$ and $R = g^{\mu\nu} R_{\mu\nu}$, respectively.

2. Super-renormalizable non-local gravity

Let us start with the following general class of non-local actions in D dimension [19–25],

$$S = \int d^D x \sqrt{|g|} \Bigg[2\kappa^{-2} R + \bar{\lambda} + \overbrace{O\left(R^3\right) + \cdots + R^{\mathrm{N}+2}}^{\text{Finite number of terms}} + \sum_{n=0}^{\mathrm{N}} \left(a_n R (-\Box_M)^n R + b_n R_{\mu\nu} (-\Box_M)^n R^{\mu\nu} \right) + R h_0(-\Box_M) R + R_{\mu\nu} h_2(-\Box_M) R^{\mu\nu} \Bigg], \tag{1}$$

where $\kappa = \sqrt{32\pi G}$ (G is gravitational constant), $|g|$ is the determinant of a metric tensor $g_{\mu\nu}$, $\Box$ is the d'Alembertian operator with $\Box_M = \Box/M^2$, and M is an ultraviolet mass scale. The first two lines of the action consist of a finite number of operators multiplied by coupling constants subject to renormalization at quantum level. The functions $h_2(z)$ and $h_0(z)$, where $z \equiv -\Box_M$, are not renormalized and defined as follows

$$h_2(z) = \frac{V(z)^{-1} - 1 - \frac{\kappa^2 M^2}{2} z \sum_{n=0}^{\mathrm{N}} \tilde{b}_n z^n}{\frac{\kappa^2 M^2}{2} z},$$

$$h_0(z) = -\frac{V(z)^{-1} - 1 + \kappa^2 M^2 z \sum_{n=0}^{\mathrm{N}} \tilde{a}_n z^n}{\kappa^2 M^2 z}, \tag{2}$$

for general parameters $\tilde{a}_n$ and $\tilde{b}_n$, while

$$V(z)^{-1} := \frac{\Box + m^2}{\Box} e^{\mathrm{H}(z)}, \tag{3}$$

$$e^{\mathrm{H}(z)} = \left| p_{\gamma+\mathrm{N}+1}(z) \right| e^{\frac{1}{2}[\Gamma(0, p^2_{\gamma+\mathrm{N}+1}(z)) + \gamma_E]}. \tag{4}$$

The form factor $V(z)^{-1}$ in Eq. (3) is made of two parts: (i) a non-local operator $(\Box + m^2)/\Box$ which goes to the identity in the ultraviolet regime, and (ii) an entire function $e^{\mathrm{H}(z)}$ without zeros in all complex planes. Here, m is a mass scale associated with the graviton mass that we will discuss later when we calculate the two-point correlation function. $\mathrm{H}(z)$ is an entire function of the operator $z = -\Box_M$, and $p_{\gamma+\mathrm{N}+1}(z)$ is a real polynomial of degree $\gamma + \mathrm{N} + 1$ which vanishes in $z = 0$, while $\mathrm{N} = (D-4)/2$ and $\gamma > D/2$ is integer.[2] The exponential factor $e^{\mathrm{H}(z)}$ is crucial to make the theory super-renormalizable or finite at quantum level [19–25].

Let us expand on the behavior of $\mathrm{H}(z)$ for small values of z:

$$\mathrm{H}(z) = \sum_{n=1}^{\infty} \frac{p_{\gamma+\mathrm{N}+1}(z)^{2n}}{2n(-1)^{n-1} n!} = \frac{1}{2}\left[\gamma_E + \Gamma\left(0, p^2_{\gamma+\mathrm{N}+1}(z)\right) + \log\left(p^2_{\gamma+\mathrm{N}+1}(z)\right)\right],$$
$$\text{for } \mathrm{Re}\left(p^2_{\gamma+\mathrm{N}+1}(z)\right) > 0. \tag{6}$$

For the most simple choice $p_{\gamma+\mathrm{N}+1}(z) = z^{\gamma+\mathrm{N}+1}$, $\mathrm{H}(z)$ simplifies to

$$\mathrm{H}(z) = \frac{1}{2}\left[\gamma_E + \Gamma\left(0, z^{2\gamma+2\mathrm{N}+2}\right) + \log\left(z^{2\gamma+2\mathrm{N}+2}\right)\right],$$
$$\mathrm{Re}\left(z^{2\gamma+2\mathrm{N}+2}\right) > 0,$$
$$\mathrm{H}(z) = \frac{z^{2\gamma+2\mathrm{N}+2}}{2} - \frac{z^{4\gamma+4\mathrm{N}+4}}{8} + \cdots, \quad \text{for } z \approx 0. \tag{7}$$

In particular $\lim_{z\to 0} \mathrm{H}(z) = 0$. We will expand more about the limit of large z in Section 2.2, where we will explicitly show the power counting renormalizability of the theory.

2.1. Propagator

In this section we calculate the two point function of the gravitational fluctuation around the flat space–time. For this purpose we split the $g_{\mu\nu}$ into the flat Minkowski metric $\eta_{\mu\nu}$ and the fluctuation $h_{\mu\nu}$, as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \tag{8}$$

Writing the action (1) in the form $S = \int d^D x \mathcal{L}$, the Lagrangian $\mathcal{L}$ can be expanded to second order in the graviton fluctuation [33]

$$\mathcal{L}_{\mathrm{lin}} = -\frac{1}{2}\left[h^{\mu\nu} \Box h_{\mu\nu} + A_\nu^2 + (A_\nu - \phi_{,\nu})^2\right] + \frac{1}{4}\left[\frac{\kappa^2}{2} \Box h_{\mu\nu} \beta(\Box) \Box h^{\mu\nu} - \frac{\kappa^2}{2} A^{\mu}_{,\mu} \beta(\Box) A^{\nu}_{,\nu}\right.$$

[1] Note that cosmological consequences of non-local theory given by the Lagrangian $R f(\Box^{-1} R)$ have been studied in Refs. [26–32]. In this case the function $f(\Box^{-1} R)$ can be chosen only phenomenologically from the demand to realize the late-time cosmic acceleration and so on.

[2] $\gamma_E = 0.577216$ is Euler's constant, and

$$\Gamma(b, z) = \int_z^{\infty} t^{b-1} e^{-t} \, dt \tag{5}$$

is the incomplete gamma function [19].