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Elliptic flow from non-equilibrium initial condition with a saturation scale

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ABSTRACT

A current goal of relativistic heavy-ion collisions experiments is the search for a Color Glass Condensate (CGC) as the limiting state of QCD matter at very high density. In viscous hydrodynamics simulations, a standard Glauber initial condition leads to estimate $4\pi\eta/s \sim 1$, while employing the Kharzeev–Levin–Nardi (KLN) modeling of the glasma leads to at least a factor of 2 larger η/s. Within a kinetic theory approach based on a relativistic Boltzmann-like transport simulation, our main result is that the out-of-equilibrium initial distribution reduces the efficiency in building-up the elliptic flow. At RHIC energy we find the available data on v_2 are in agreement with a $4\pi\eta/s \sim 1$ also for KLN initial conditions. More generally, our study shows that the initial non-equilibrium in p-space can have a significant impact on the build-up of anisotropic flow.

Ultra-relativistic heavy-ion collisions (uRHICs) at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) create a hot and dense system of strongly interacting matter. In the last decade it has been reached a general consensus that such a state of matter is not of hadronic nature and there are several signatures that it is a strongly interacting quark–gluon plasma (QGP) [1–3]. A main discovery has been that the QGP has a very small shear viscosity to density entropy, η/s, which is more than one order of magnitude smaller than the one of water [4,5], and close to the lower bound of $1/4\pi$ conjectured for systems at infinite strong coupling [6]. A key observable to reach such a conclusion is the so-called elliptic flow, $v_2 = \langle\cos(2\varphi_p)\rangle = \langle(p_x^2 - p_y^2)/(p_x^2 + p_y^2)\rangle$, with φ_p being the azimuthal angle in the transverse plane and the average meant over the particle distribution. In fact, the expansion of the created matter generates a large anisotropy of the emitted particles that can be primarily measured by v_2. Its origin is the initial spatial eccentricity, $\epsilon_x = \langle x^2 - y^2\rangle/\langle x^2 + y^2\rangle$, of the overlap region in non-central collisions. The observed large v_2 is considered a signal of a very small η/s because it means that the system is very efficient in converting ϵ_x into an anisotropy in the momentum space v_2, a mechanism that would be strongly damped in a system highly viscous that dissipates and smooths anisotropies [7,9,11]. Quantitatively both viscous hydrodynamics [7–9,12,13,15], and transport Boltzmann-like approaches [16–20] agree in indicating an average η/s of the QGP lying in the range $4\pi\eta/s \sim 1$–3.

The uRHIC program offers the tantalizing opportunity to explore the existence of an exotic state, namely the Color Glass Condensate (CGC) [21,22], see [23,24] for reviews. Such a state of matter would be primarily generated by the very high density of the gluon parton distribution function at low x (parton momentum fraction), which triggers a saturation of the gluon distribution function at a p_T below the saturation scale, Q_s [25]. Even if at first sight surprisingly, the study of the shear viscosity η/s of the QGP and the search for the CGC are related. In fact, the main source of uncertainty for η/s comes from the unknown initial conditions of the created matter [8] and confirmed later by further works [10,12,26].

A simple geometrical description through the Glauber model [28] predicts an ϵ_x smaller at least 25–30% than the eccentricity of the CGC, for most of the centralities of the collisions, see for example results within the Kharzeev–Levin–Nardi (KLN) model [27,29,30], factorized KLN (fKLN) model [31], Monte Carlo KLN (MC-KLN) model [31,32] and dipole model [27,33]. The uncertainty in the initial condition translates into an uncertainty on η/s of at least a factor of two as estimated by means of several viscous hydrodynamical approaches [8,10,12,26]. More explicitly, the experimental data of $v_2(p_T)$ at the highest RHIC energy are in agreement with a fluid at $4\pi\eta/s \sim 1$ according to viscous hydrodynamics simulation,

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assuming a standard Glauber initial condition. Assuming an initial fKLN or MC-KLN space distribution the comparison favors a fluid at $4\pi\eta/s \sim 2$. The reason is the larger initial ϵ_x of the fKLN, which leads to larger v_2 unless a large η/s is considered. However, in [10,26] it has been shown that viscous hydrodynamics fails to reproduce both v_2 and $v_3 = \langle\cos(3\varphi_p)\rangle$ if the same $4\pi\eta/s \sim 2$ is assumed. At variance a Glauber initial condition can account for both with the same $4\pi\eta/s = 1$. However the indirect effect on ϵ_x is not a unique and solid prediction of the CGC modelings; for example, the approach based on the solution of the Classical Yang–Mills (CYM) equations predicts a somewhat smaller initial eccentricity [34,36]. Very recently employing an *x-space* distribution inspired to the CYM approach in a viscous hydrodynamical approach [35] it has been show that not only v_2 but also higher harmonics can be correctly predicted with a $4\pi\eta/s \sim 1.5$ instead of ~ 2, which is in qualitative agreement with the fact that CYM tend to predict quite smaller ϵ_x with respect to fKLN. However our present studies focus on the effect of the initial non-equilibrium in p-space, an issue discarded in all previous studies including the recent ones [35,36].

In this Letter, we point out that the implementation of the melted CGC in hydrodynamics takes into account only the different space distribution with respect to a geometric Glauber model, discarding the key and more peculiar feature of the damping of the distribution for p_T below the Q_s saturation scale. We have found by means of kinetic theory that this has a pivotal role on the build-up of v_2.

We adopt the model which was firstly introduced by Kharzeev, Levin and Nardi [29] (KLN model) even if in the regime of over saturation in $A + A$ collisions some aspects are better caught by a CYM approach [48]. In particular, to prepare the initial conditions of our simulations we refer to the factorized KLN (fKLN) approach as introduced in [31,32]. This will allow for a direct comparison with viscous hydrodynamics results in which the coordinate space distribution function of gluons arising from the melted CGC is assumed to be

$$\frac{dN_g}{dy\,d^2\boldsymbol{x}_\perp} = \int d^2\boldsymbol{p}_T\, p_A(\boldsymbol{x}_\perp) p_B(\boldsymbol{x}_\perp) \Phi(\boldsymbol{p}_T, \boldsymbol{x}_\perp, y), \tag{1}$$

where Φ corresponds to the momentum space distribution in the k_T factorization hypothesis [37,38],

$$\Phi(\boldsymbol{p}_T, \boldsymbol{x}_\perp, y) = \frac{4\pi^2 N_c}{N_c^2 - 1}\frac{1}{p_T^2}\int^{p_T} d^2\boldsymbol{k}_T \alpha_S(Q^2)\phi_A(x_1, k_T^2; \boldsymbol{x}_\perp) \times \phi_B(x_2, (\boldsymbol{p}_T - \boldsymbol{k}_T)^2; \boldsymbol{x}_\perp). \tag{2}$$

Here $x_{1,2} = p_T \exp(\pm y)/\sqrt{s}$ and the ultraviolet cutoff $p_T = 3$ GeV/c assumed in the p_T integral in Eq. (1); α_S denotes the strong coupling constant, which is computed at the scale $Q^2 = \max(\boldsymbol{k}_T^2, (\boldsymbol{p}_T - \boldsymbol{k}_T)^2)$ according to the one-loop β function but frozen at $\alpha_s = 0.5$ in the infrared region as in [30,33,39]. In Eq. (1) $p_{A,B}$ denote the probability to find one nucleon at a given transverse coordinate, $p_A(\boldsymbol{x}_\perp) = 1 - (1 - \sigma_{in} T_A(\boldsymbol{x}_\perp)/A)^A$, where σ_{in} is the inelastic cross section and T_A corresponds to the usual thickness function of the Glauber model.

The main ingredient to specify in Eq. (2) is the unintegrated gluon distribution function (uGDF) for partons coming from nucleus A, which is assumed to be:

$$\phi_A(x_1, k_T^2; \boldsymbol{x}_\perp) = \frac{\kappa\, Q_s^2}{\alpha_s(Q_s^2)}\left[\frac{\theta(Q_s - k_T)}{Q_s^2 + \Lambda^2} + \frac{\theta(k_T - Q_s)}{k_T^2 + \Lambda^2}\right] \tag{3}$$

where we see the peculiar feature of the CGC that is the saturation of the distribution for $p_T < Q_s$; a similar equation holds for partons belonging to nucleus B. Following [31] we take the saturation scale for the nucleus A as

$$Q_{s,A}^2(x, \boldsymbol{x}_\perp) = 2\ \text{GeV}^2\left(\frac{T_A(\boldsymbol{x}_\perp)}{1.53\, p_A(\boldsymbol{x}_\perp)}\right)\left(\frac{0.01}{x}\right)^\lambda, \tag{4}$$

with $\lambda = 0.28$, and similarly for nucleus B. This choice is the one adopted in fKLN or MC-KLN and in hydro simulations [12,31] to study the dependence of $v_2(p_T)$ on η/s. Using Eqs. (4) and (1) we find that $\langle Q_s\rangle \approx 1.4$ GeV where the average is understood in the transverse plane.

We employ transport theory as a base of a simulation code of the fireball expansion created in relativistic heavy-ion collision [19,20,40,41], therefore the time evolution of the gluons distribution function $f(\boldsymbol{x}, \boldsymbol{p}, t)$ evolves according to the Boltzmann equation:

$$p_\mu \partial^\mu f_1 = \int d\Gamma_2\, d\Gamma_{1'}\, d\Gamma_{2'} (f_{1'} f_{2'} - f_1 f_2) \times |\mathcal{M}|^2 \delta^4(p_1 + p_2 - p_{1'} - p_{2'}), \tag{5}$$

where $d^3\boldsymbol{p}_k = 2E_k(2\pi)^3 d\Gamma_k$ and $\mathcal{M}$ corresponds to the transition amplitude.

At variance with the standard use of transport theory, we have developed an approach that, instead of focusing on specific microscopic calculations or modelings for the scattering matrix, fixes the total cross section in order to have the wanted η/s. In Ref. [42] it has been shown in 1 + 1D such an approach is able to recover the Israel–Stewart viscous hydrodynamical evolution when η/s is sufficiently small. In 3 + 1D some of the authors have studied the analytical relation between η, temperature, cross section and density and as shown in [41,44], the Chapmann–Enskog approximation supplies such a relation with quite good approximation [43], in agreement with the results obtained using the Green Kubo formula. Therefore, we fix η/s and compute the pertinent total cross section by means of the relation

$$\sigma_{tot} = \frac{1}{15}\frac{\langle p\rangle}{\rho\, g(a)}\frac{1}{\eta/s}, \tag{6}$$

which is valid for a generic differential cross section $d\sigma/dt \sim \alpha_s^2/(t - m_D^2)^2$ as proved in [44]. In the above equation $a = T/m_D$, with m_D the screening mass regulating the angular dependence of the cross section, while

$$g(a) = \frac{1}{50}\int dy\, y^6\left[\left(y^2 + \frac{1}{3}\right)K_3(2y) - yK_2(2y)\right]h\left(\frac{a^2}{y^2}\right), \tag{7}$$

with K_n the Bessel function and h corresponding to the ratio of the transport and the total cross section. The maximum value of g, namely $g(m_D \to \infty) = h(m_D \to \infty) = 2/3$, is reached for isotropic cross section; a smaller value of $g(a)$ means that a higher σ_{tot} is needed to reproduce the same value of η/s. However, we notice that in the regime where viscous hydrodynamic applies (not too large η/s and p_T) the specific microscopic detail of the cross section is irrelevant and our approach is an effective way to employ transport theory to simulate a fluid at a given η/s. From the operative point of view, keeping η/s constant in our simulations is achieved by evaluating locally in space and time the strength of the cross section by means of Eq. (6), where both parton densities and temperature are computed locally in each cell. To realize a realistic freeze-out, when the local energy density reaches the cross-over region, the η/s increases linearly to match the estimated hadronic viscosity, as described in [20,41]; this affects in the same way all the cases considered in the following.

In the following, we will consider three different types of initial distribution function in the phase-space, two of which are the one employed till now for the investigation of the η/s, while the third one is the genuine novelty of the present study. Furthermore, we refer to Au + Au collision at $\sqrt{s} = 200$ AGeV and $b = 7.5$ fm. In this case, our result for initial eccentricity in the fKLN model is