Physics Letters B 727 (2013) 211–217

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Scalar supersymmetry

A. Jourjine

FG CTP, Hofmann Str. 6-8, 01277 Dresden, Germany

ARTICLE INFO

Article history:
Received 12 August 2012
Received in revised form 8 October 2013
Accepted 15 October 2013
Available online 22 October 2013
Editor: A. Ringwald

Keywords:
Supersymmetry
Bi-spinor gauge theory
String theory

ABSTRACT

We describe a new realization of supersymmetry, called scalar supersymmetry, acting in spaces of differential forms (bi-spinors), where transformation parameters are Lorentz scalars instead of spinors. The realization is related but is not reducible to the standard supersymmetry. Explicit construction of chiral multiplets that do not require doubling of the spectrum of a gauge theory is presented. A bi-spinor s-supersymmetric string action is described.

1. Introduction

It is well known that the use of supersymmetry [1] to extend the Standard Model results in a number of attractive features of the models. Apart from curing the Higgs mass finetuning problem, supersymmetry leads to gauge coupling unification, provides candidates for Dark Matter, and sets the stage for gravity unification via superstrings. Unbroken supersymmetry requires that each observed particle has a superpartner with equal mass. Since the observed particle mass spectrum of the SM is not mass-degenerate, supersymmetry must be broken. Breaking supersymmetry is a non-trivial problem; it must be broken softly to preserve the desired cancellations of divergences, and presently there exist a number of phenomenologically viable supersymmetric extensions of the Standard Model [2,3].

One characteristic feature of supersymmetry of such extensions appears in the particle spectrum even if supersymmetry is broken. Namely, each Standard Model particle must have a superpartner with spin differing by one-half. This is because in the Standard Model the bosonic gauge fields are real and transform in the adjoint representation of the gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ but the fermionic spinor fields are complex and transform in the fundamental representations of G_{SM}. As a result, one cannot combine the observed bosons and fermions into multiplets without violating gauge symmetry. In addition, the left and the right fermions couple differently to $SU(2)_L$. To accommodate the difference one is forced to use chiral supermultiplets. These can be only constructed if one pads each fermion with a superpartner of differing spin.

Despite an intensive search, most recently at LHC, no superpartners of the particle of the Standard Model have been detected. One explanation for this failure could be that the predicted superpartners don't exist, which implies that a different realization of supersymmetry, the one that does not require doubling of the observed spectrum, must be used.

In this Letter we describe such a realization of supersymmetry. If implemented in a modification of the Standard Model, it would not require doubling of the Standard Model particle spectrum. We also show that the s-supersymmetry can be realized as symmetry of string action. With s-supersymmetry the observed gauge and fermion fields are allowed to mix through a supersymmetry transformation and no superpartners are needed. Parameters of s-supersymmetry are scalars instead of spinors, as is in the standard supersymmetry. It acts in spaces that are direct sums of spaces of commuting and anti-commuting differential forms and it requires the use of bi-spinor formalism [4,5] to represent fermions. (Fermion bi-spinor fields are described by objects that transform as products of Dirac spinors and their Dirac conjugates.)

Although bi-spinors are seldom used for model building, the notion of bi-spinor is as old as that of Dirac spinor. In their anti-symmetric tensor form bi-spinors were discovered in 1928 by Ivanenko and Landau [6], in the same year Dirac proposed his theory of electron [7]. In fact, Ivanenko and Landau constructed an alternative to Dirac's solution of the electron's giromagnetic ratio problem.[1] However, the Ivanenko–Landau solution was more complicated than Dirac's by the standards of the time and naturally the latter won over as a basic descriptor of quantum fermionic matter.

Although bi-spinors have not been popular in phenomenology, they have been much in use in lattice gauge theory and, in

[1] Bi-spinors are also referred to as Ivanenko–Landau–Kähler (ILK) [8] or Dirac–Kähler (DK) spinors.

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.10.036

particular, for building realizations of Dirac–Kähler twisting of the standard extended supersymmetry on the lattice [9–13]. Antisymmetric tensor form of bi-spinors also appears quite often in string theories in the form of p-forms, differential forms of fixed degree p. P-forms and their quantization have been studied both in supergravity and in string theory, including formulation of strings with two time parameters [14–18]. Theories of p-forms typically are restricted to commuting differential forms of a fixed degree. Here we will concentrate on the case where commuting and anticommuting inhomogeneous differential forms play equal role in the dynamics. For brevity we will concentrate on massless gauge fields and massless bi-spinors.

The Letter has three sections. In the following Section 2 we describe the needed basic ingredients of differential geometry. It can be skipped by readers familiar with the subject. Our results are contained in Section 3. Section 4 presents a brief summary.

2. Differential geometry, Z-basis, and spinbeins

Although our results also apply when background gravity is present, to emphasize applications to phenomenology we will work with four-dimensional Minkowski space–time M_4 with metric $g_{\mu\nu} = diag(1, -1, -1, -1)$. All of the mathematical constructs we will use generalize with minor modifications to an arbitrary (pseudo-) Euclidean space–time. We will use the following index conventions: capital Latin letters $A, B, \ldots$ are reserved for the fermion generations, lower case Latin letters $a, b, \ldots$ are for gauge group representations, lower case Greek letters $\alpha, \beta, \ldots$ for spinor indices, while $\mu, \nu, \ldots$ for Lorentz tensor indices.

The basic notions of differential geometry that we need are the standard operations with differential forms on a manifold [19,20], a basis in the space of differential forms, the Z-basis to define bi-spinors [21], and the spinbein decomposition of bi-spinors [22] to extract Dirac spinors from bi-spinors.

Given M_4 with coordinates x^μ, a differential form A in the coordinate basis (c-basis) is defined as a sum of homogeneous differential forms of degree p with values in the Lie algebra of the gauge group G

$$A(x) = \sum_{p=0}^{4} A_p(x), \qquad A_p(x) = A_{|\mu_1\cdots\mu_p|}(x)\, dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}, \tag{1}$$

where $\wedge$ is the exterior product and $|\mu_1 \cdots \mu_p|$ is a permutation of indices $\mu_1 \cdots \mu_p$ with increasing order. In bi-spinor formalism such differential forms play the role of the fields of the standard (quantum) field theory.

Additional basic differential-geometric constructs that we need are the main automorphism α, the main anti-automorphism β, and the contraction $(.,.)$ of a p-form A_p with a q-form B_q defined by

$$\alpha A_p = (-1)^p A_p, \qquad \beta A_p = (-1)^{p(p-1)/2} A_p,$$
$$(A_p, B_q) = \delta_{pq} tr(A_{\mu_1\cdots\mu_p})^+ B^{\mu_1\cdots\mu_p},$$

the exterior derivative d, $d^2 = 0$,

$$d: A_p \to A_{p+1}, \qquad dA_4 = 0,$$
$$dA_p = \partial_\nu A_{|\mu_1\cdots\mu_p|}\, dx^\nu \wedge dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}, \tag{2}$$

and the Hodge star operator $*$

$$*: A_p \to A_{4-p},$$
$$(*A)_{4-p} = A^{|\mu_1\cdots\mu_p|} \varepsilon_{\mu_1\cdots\mu_p|\nu_1\cdots\nu_{4-p}|}\, dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_{4-p}}, \tag{3}$$
$$* = (-1)^{p+1} = -\alpha, \tag{4}$$

where $\varepsilon^{\mu_1\cdots\mu_4}$ is the totally antisymmetric tensor of rank 4 with $\varepsilon^{0123} = 1$, $\varepsilon_{\mu_1\cdots\mu_4} = -\varepsilon^{\mu_1\cdots\mu_4}$. Very useful for us will also be operator $\star$, which we will call the chiral star operator, defined by

$$\star = -i * \alpha\beta = -i\beta*, \tag{5}$$
$$\star\star = 1. \tag{6}$$

From d and $*$ the covariant divergence operator δ is defined by

$$\delta: A_p \to A_{p-1}, \qquad \delta A_0 = 0, \tag{7}$$
$$\delta = *d*, \qquad \delta^2 = 0. \tag{8}$$

We define a scalar product $\langle A, B\rangle$ of differential forms A, B by linearity from

$$\langle A_p, B_q\rangle = \delta_{pq} \int tr\big[\alpha A_p^+ \wedge *B_q\big] = \delta_{pq} \int d^n x(\alpha A_p, B_q). \tag{9}$$

Note that $-\delta$ is the adjoint of d with respect to scalar product (9) and, therefore, $(d - \delta)$ is self-adjoint. For Euclidean space–time definition (9) must be modified by removing automorphism α.

We now introduce the Z-basis in the space of differential forms and establish the connection between antisymmetric tensors and bi-spinors. Given a set of Dirac γ-matrices, $\gamma^\mu = \{\gamma^\mu_{\alpha\beta}\}$, such that $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, the defining property of the Z-basis, $Z = \{Z_{\alpha\beta}\}$, is that operator $(d - \delta)$ takes the form of the Dirac operator [21]

$$(d - \delta)Z = Z\big(i\gamma^\mu \partial_\mu\big). \tag{10}$$

Z is an 4×4 matrix of differential forms.[2] Any differential form A can be represented in the Z-basis as

$$A = tr\big(Z\Psi(A)\big), \tag{11}$$

where $\Psi(A) = \{\Psi_{\alpha\beta}(A)\}$ are the coefficients of the representation and the trace is over the γ-matrix indices. Using (10) we obtain an explicit expression for Z [21]

$$Z = \sum_p \gamma_{\mu_p} \cdots \gamma_{\mu_1}\, dx^{|\mu_1} \wedge \cdots \wedge dx^{\mu_p|}. \tag{12}$$

Since differential forms do not depend on the basis in which they are defined, the coefficients $A_{\mu_1\cdots\mu_p}(A)$ of A in the c-basis and the coefficients $\Psi_{\alpha\beta}(A)$ of A in the Z-basis represent the same mathematical object. Also the transformation properties of the two sets of coefficients can be derived from basis independence of A: under Lorentz transformation $x \to \Lambda x$ the set $\{A_{\mu_1\cdots\mu_p}(A)\}$ transforms as a collection of antisymmetric tensors, while $\Psi_{\alpha\beta}(A)$ transforms as

$$\Psi(A) \to S(\Lambda)\Psi(A)S(\Lambda)^{-1}, \tag{13}$$

where $S(\Lambda)$ is the spinor representation of the Lorentz group. Transformation (13) is the transformation law for bi-spinors: by definition they transform as a product of a Dirac spinor and its Dirac conjugate. Thus, we can identify the space of all Ψ with the space of bi-spinors. Relations between the two sets of coefficients $\{A_{\mu_1\cdots\mu_p}(A)\}$ and $\Psi_{\alpha\beta}(A)$ are derived using (12) and the completeness relations for γ-matrices

$$tr\big(\big[\gamma^{|\mu_1} \cdots \gamma^{\mu_p|}\big]\big[\gamma^{|\nu_1} \cdots \gamma^{\nu_q|}\big]^+\big) = 4\delta^{pq}\delta^{\mu_1\nu_1} \cdots \delta^{\mu_q\nu_p}, \tag{14}$$
$$\sum_p \big[\gamma^{|\mu_1} \cdots \gamma^{\mu_p|}\big]^*_{\alpha\beta}\big[\gamma^{|\mu_1} \cdots \gamma^{\mu_p|}\big]_{\gamma\delta} = 4\delta_{\alpha\gamma}\delta_{\beta\delta}. \tag{15}$$

It is given by

[2] For notational convenience our definition of Z is the transposed of that in [21].