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Higgs inflation at NNLO after the boson discovery

Alberto Salvio

Departamento de Física Teórica, Universidad Autónoma de Madrid and Instituto de Física Teórica IFT-UAM/CSIC, Madrid, Spain

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ABSTRACT

We obtain the bound on the Higgs and top masses to have Higgs inflation (where the Higgs field is non-minimally coupled to gravity) at full next-to-next-to-leading order (NNLO). Comparing the result obtained with the experimental values of the relevant parameters we find some tension, which we quantify. Higgs inflation, however, is not excluded at the moment as the measured values of the Higgs and top masses are close enough to the bound once experimental and theoretical uncertainties are taken into account.

1. Introduction

The discovery of the Higgs boson [1,2] has allowed to fix the last Standard Model (SM) parameter, the Higgs mass. Making the strong but certainly economical assumption that the SM (appropriately extended to accommodate neutrino masses and dark matter) remains valid up to the Planck scale, it is now possible to obtain precise predictions in this vast energy range.

Ref. [3] argued that even the inflationary period of the Universe can be explained within the SM and the Higgs field and the inflaton can be identified if the term

$$\sqrt{-g}\xi H^\dagger H R, \tag{1}$$

with $\xi \gg 1$, is added to the Einstein–Hilbert plus SM Lagrangian $\mathcal{L}_{E-H} + \mathcal{L}_{SM}$, so that the total Lagrangian is

$$\mathcal{L}_{total} = \mathcal{L}_{E-H} + \mathcal{L}_{SM} + \sqrt{-g}\xi H^\dagger H R. \tag{2}$$

Here R is the Ricci scalar, H is the Higgs doublet and g is the determinant of the metric $g_{\mu\nu}$.

An inflaton with a non-minimal coupling of the form given in (1), and in particular Higgs inflation, is perfectly consistent with recent Planck results [4], which favor a simple single field inflation.

All this reinforces the interest in the possibility of Higgs inflation.

The non-minimal coupling in (1) can be eliminated by a redefinition of $g_{\mu\nu}$ (going to the so called Einstein frame), which leads to a non-polynomial Lagrangian for H. This redefinition shows that two regimes are present in the theory [5]: the small field one $|H| \ll M_P/\xi$, where the canonical SM is a good description, and the large field limit $|H| \gg M_P/\xi$, in which the physical Higgs mode decouples. Therefore, the latter limit corresponds to the chiral electroweak (EW) theory [6].

As we will review in Section 2, at the classical level this is a viable model of inflation if the non-minimal coupling ξ is chosen to match cosmic microwave background (CMB) observations. Quantum corrections may, however, render inflation impossible depending on the input parameters at the EW scale, in particular the Higgs and top pole masses M_h and M_t: if M_h is too small (or M_t is too large) the slope of the Higgs effective potential at large field values becomes negative preventing the field configuration to roll towards the EW vacuum.

In this Letter we improve on previous determinations [5] of the lower bound on the Higgs mass (or equivalently the upper bound on the top mass) to have Higgs inflation by using the following ingredients: (1) two loop effective potential in the inflationary regime including the effect of ξ and the leading SM couplings: the top Yukawa y_t, the strong gauge coupling g_3, the EW gauge couplings g_2 and g_1 and the quartic Higgs coupling λ; (2) three loop SM renormalization group equations (RGE) from the EW scale up to M_P/ξ for y_t, g_3, g_2, g_1 and λ including the effects of all these couplings; (3) two loop RGE for the same SM couplings and one loop RGE for ξ in the chiral EW theory; (4) recent precise determinations of these SM couplings at the top mass provided in [7], which are used as initial conditions for the RGE.[1]

A detailed description of these ingredients is provided in Section 3. In Section 4 we present our numerical results, including the

[1] See, however, Ref. [8] for a related possible issue if conformal invariance is required.

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determination of ξ and the lower bound on M_h (or M_t). Finally in Section 5 we conclude.

2. Classical analysis

Let us briefly review the model of [3] at the classical level. The part of the action in (2) that depends on the metric and the Higgs field *only* is

$$S_{gH} = \int d^4x \sqrt{-g}\left[\left(\frac{M_P^2}{2} + \xi H^\dagger H\right) R + |\partial H|^2 - V\right],$$

where $M_P \simeq 2.435 \times 10^{18}$ GeV is the reduced Planck mass, $V = \lambda(H^\dagger H - v^2/2)^2$ is the classical Higgs potential, and v is the EW Higgs vacuum expectation value.

The non-minimal coupling (1) can be eliminated through the *conformal* transformation

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi H^\dagger H}{M_P^2}. \tag{3}$$

The original frame, where the Lagrangian has the form in (2), is called the Jordan frame, while the one where gravity is canonically normalized (obtained with the transformation above) is called the Einstein frame. In the unitary gauge, where the only scalar field is the radial mode $\phi \equiv \sqrt{2H^\dagger H}$, we have (after the conformal transformation)

$$S_{gH} = \int d^4x \sqrt{-\hat{g}}\left[\frac{M_P^2}{2}\hat{R} + K\frac{(\partial\phi)^2}{2} - \frac{V}{\Omega^4}\right], \tag{4}$$

where $K \equiv (\Omega^2 + 6\xi^2\phi^2/M_P^2)/\Omega^4$. The non-canonical Higgs kinetic term can be made canonical through the field redefinition $\phi = \phi(\chi)$ defined by

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}}. \tag{5}$$

Thus, χ feels a potential

$$U \equiv \frac{V}{\Omega^4} = \frac{\lambda(\phi(\chi)^2 - v^2)^2}{4(1 + \xi\phi(\chi)^2/M_P^2)^2}. \tag{6}$$

From (5) and (6) it follows [3] that U is exponentially flat when $\chi \gg M_P$, which is a key property to have inflation. Indeed, for such high field values the parameters

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{U}\frac{dU}{d\chi}\right)^2, \qquad \eta \equiv \frac{M_P^2}{U}\frac{d^2U}{d\chi^2},$$
$$\zeta^2 \equiv \frac{M_P^4}{U^2}\frac{d^3U}{d\chi^3}\frac{dU}{d\chi} \tag{7}$$

are guaranteed to be small. Therefore, the region in field configurations $\chi > M_P$ (or equivalently [3] $\phi > M_P/\sqrt{\xi}$) corresponds to inflation.

All the parameters of the model can be fixed through experiments and observations, including ξ [3,9], so that Higgs inflation is highly predictive and as such falsifiable. ξ can be fixed by requiring that the WMAP normalization of [10],

$$\frac{U}{\epsilon} = 24\pi^2\Delta_R^2 M_P^4 \simeq (0.02746 M_P)^4, \tag{8}$$

is reproduced for a field value $\phi = \phi_{WMAP}$ corresponding to an appropriate number of e-foldings [9]:

$$N = \int_{\phi_{\rm end}}^{\phi_{WMAP}} \frac{U}{M_P^2}\left(\frac{dU}{d\phi}\right)^{-1}\left(\frac{d\chi}{d\phi}\right)^2 d\phi \simeq 59, \tag{9}$$

where $\phi_{\rm end}$ is the field value at the end of inflation,

$$\epsilon(\phi_{\rm end}) \simeq 1. \tag{10}$$

This procedure leads to $\xi \simeq 4.7 \times 10^4\sqrt{\lambda}$, which is why ξ has to be much larger than one.

We can also extract the spectral index n_s, the tensor-to-scalar ratio r and the running spectral index $dn_s/d\ln k$:

$$n_s = 1 - 6\epsilon + 2\eta,$$
$$r = 16\epsilon,$$
$$\frac{dn_s}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\zeta^2. \tag{11}$$

These parameters are of interest as they are constrained by observations [4].

3. Quantum corrections

We now turn to the quantum corrections. We will use perturbation theory to compute them. It is important to keep in mind that perturbative unitarity[2] is violated above some high energy scale [11,12]. Once the background fields are taken into account, however, one can show [13] that such energy is parametrically higher than all relevant scales during the history of the Universe. Nevertheless some additional assumptions on the underlying ultraviolet complete theory are necessary (see [12–14]).

There are two options for the quantization of the classical theory defined before: one can either first perform the transformation in (3) and then quantize [3] (prescription I) or first quantize and then perform the conformal transformation (prescription II) [16]. The two options lead to different theories as they have different predictions [5]. We choose the first possibility because Ref. [5] found it to be the one leading to the weaker bound on M_t and such bound, as we will see, is already giving some tension with the experiments. We will make some more comments on prescription II at the end of Section 4, where we will check that it is indeed leading to a stronger bound even at full NNLO.

The procedure to compute quantum corrections has been introduced in [5]: we briefly summarize it in the following subsections giving both the order of approximation reached in [5] and our improvements.

3.1. Effective potential

The first element that we need is the (quantum) effective potential for χ, which is expanded in loops as

$$U_{\rm eff} = U + U_1 + U_2 + \cdots.$$

Here U is the classical contribution in Eq. (6) and U_1, U_2, ... are the one loop, two loop, ... contributions respectively. An observation that leads to useful simplifications is that we only need $U_{\rm eff}$ in the inflationary regime. Also, further simplifications can be achieved with a judicious gauge choice; we choose the Landau gauge.

[2] This unitarity problem can be solved by adding an extra real scalar field [15]. The extension of the present analysis to include such scalar is beyond the scope of this Letter.