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Evolution of light-like Wilson loops with a self-intersection in loop space

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ABSTRACT

Recently, we proposed a general evolution equation for single quadrilateral Wilson loops on the light-cone. In the present work, we study the energy evolution of a combination of two such loops that partially overlap or have a self-intersection. We show that, for a class of geometric variations, the evolution is consistent with our previous conjecture, and we are able to handle the intricacies associated with the self-intersections and overlaps. This way, a step forward is made towards the understanding of loop space, with the hope of studying more complicated structures appearing in phenomenological relevant objects, such as parton distributions.

1. Introduction

A reformulation of the Ambrose–Singer theorem in a gauge theory context [1] states that the holonomy variables U_Γ of the (gauge-)connection $\mathcal{A}_\mu = A_\mu^a t^a$:

$$U_{\Gamma_i} = \Phi(\Gamma_i) = \mathcal{P}\exp\left[ig \oint_{\Gamma_i} dz^\mu \mathcal{A}_\mu(z)\right], \tag{1}$$

where the Γ_i are loops, and where t^a are the generators of the Lie algebra of the gauge group in a certain representation (here the fundamental representation of $SU(N_c)$), contain the same information as the corresponding gauge theory. From this holonomy, which is some $N \times N$ matrix in the representation of the gauge group, a gauge invariant variable can be obtained by taking the trace.[1] This trace introduces, however, extra constraints on the loops: the so-called Mandelstam constraints [1–4], which assure that the product of traces over holonomies is again the trace over a holonomy. Gauge theory can then be represented in a loop space setting, where the observables are now built from the vacuum expectation values of products of traces over holonomies, referred to as Wilson loop variables [2,3,5–7]:

$$\mathcal{W}_n[\Gamma_1, \ldots, \Gamma_n] = \langle 0|\mathcal{T}\frac{1}{N_c}\operatorname{Tr}\Phi(\Gamma_1)\cdots\frac{1}{N_c}\operatorname{Tr}\Phi(\Gamma_n)|0\rangle. \tag{2}$$

In a previous paper [8], we calculated a quadrilateral Wilson loop on the light-cone to leading order, and introduced a new differential operator:

$$\frac{d}{d\ln\sigma} = s\frac{d}{ds} + t\frac{d}{dt}, \tag{3}$$

with s and t the Mandelstam variables (see Section 2). This operator was then used to derive an evolution equation for this class of Wilson loops, and inspired us to formulate a conjecture for a general evolution equation:

$$\lim_{\epsilon\to 0}\mu\frac{d}{d\mu}\left(\frac{d}{d\ln\sigma}\ln\mathcal{W}_1(\Gamma)\right) = -\sum_{cusps}\Gamma_{cusp}, \tag{4}$$

where ϵ is defined in the dimensional regularization procedure $D = 4 - 2\epsilon$. It was also shown that the evolution of the cusp and Π-shape configurations is consistent with this conjecture (see [8] for the details). It turns out that the operator, Eq. (4), is a special case of the Fréchet derivative [9,10], the details of which will be discussed elsewhere [11].

In this work, we consider some symmetrical combinations of two quadrilateral Wilson loops on the light-cone, for which we test conjecture (4). Put in a Wilson loop variable language: we are calculating $\mathcal{W}_1[\Gamma]$, with $\Gamma = \Gamma_1\Gamma_2$, where the product between the loops is defined in generalized loop space [12]. Important is that these Wilson loop configurations exhibit intricacies, associated with the self-intersection and overlap, that usually cause problems in a loop space approach. A close inspection of these intricacies indicate that one needs to be careful in counting cusps along the path. We show that the effective number of cusps can deviate from the number one would expect from a naive counting procedure.

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[1] U_Γ transforms under a gauge transformation as $U_\Gamma^g = g_x^{-1}U_\Gamma g_x$, where $x = \Gamma(0)$. The cyclicity of the trace assures that Tr U_Γ is gauge invariant.

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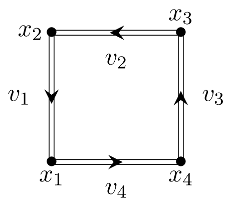

Fig. 1. Parametrization of the Wilson Loop in coordinate space.

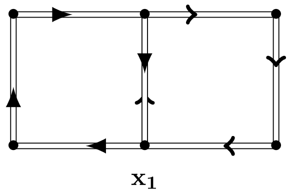

Fig. 2. Configuration 1 – overlap, equal orientation.

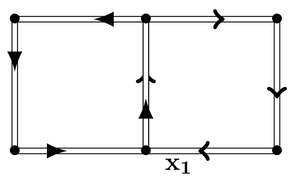

Fig. 3. Configuration 2 – overlap, opposite orientation.

Furthermore, the group structure of loop space is confirmed by our explicit calculations.

2. Loops and parametrization

We combine two similar quadrilateral Wilson loops on the light-cone in two different configurations: with a partial overlap (Figs. 2 to 3), and with a self-intersection (Figs. 4 to 5). Each loop is parametrized by four vectors v_i on the light-cone (i.e. $v_i^2 = 0,\ \forall i$), as shown in Fig. 1. In the symmetric cases under consideration, both loops are equal in size and hence can be parametrized by these four vectors. We also introduce the Mandelstam variables[2]

$$s = (v_1 + v_2)^2 = 2v_1 v_2, \qquad t = (v_2 + v_3)^2 = 2v_2 v_3, \tag{5}$$

in order to simplify the notation of the results of the calculation. Each configuration has two different possible relative orientations for the constituting loops: one where the orientation is equal (Figs. 2 and 5) and one where the orientation is opposite (Figs. 3 and 4). These orientations define how the Wilson line – gluon vertices are ordered along the path, and also fix the color flow at the self-intersection or overlap. In the figures, the color flow along the loops is represented by the different arrow styles, and the point $\mathbf{x}_1$ represents the base-point of the considered loop space. Here we only consider color neutral objects, in other words: there are no gluons in the initial or final state.

3. Loop-group structure

Expanding Eq. (1) to second order

$$\Phi(\Gamma) = 1 + ig \oint_\Gamma dz^\mu \mathcal{A}_\mu(z) - \frac{g^2}{2!} \mathcal{P} \oint_\Gamma dz^\mu \, dz'^\nu \mathcal{A}_\mu(z) \mathcal{A}_\nu(z'), \tag{6}$$

[2] Note the signs!

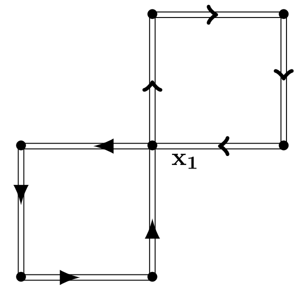

Fig. 4. Configuration 3 – self-intersection, opposite orientation.

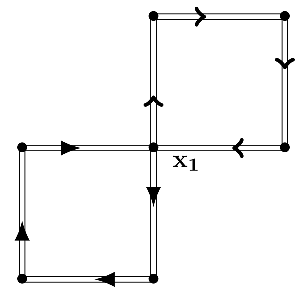

Fig. 5. Configuration 4 – self-intersection, equal orientation.

and taking traces and vacuum expectation values yields:

$$\mathcal{W}_1(\Gamma) = 1 - \frac{g^2}{2!} \mathrm{Tr}\big(t^a t^b\big) \langle 0 | \mathcal{T} \oint_\Gamma dz^\mu \, dz'^\nu A^a_\mu(z) A^b_\nu\big(z'\big) | 0 \rangle, \tag{7}$$

for the first order Wilson loop variable. The generators and gauge connections in Eq. (7) are ordered along the loop by the time-ordering operation $\mathcal{T}$, where the "time" is represented by the path parameter $t \in [0,1]$ such that $dz^\mu = \dot{z}^\mu \, dt$. Now, considering $\Gamma = \Gamma_1 \Gamma_2$, the group structure of generalized loop space [12–15] allows us to rewrite the integral in the second term of Eq. (7) as

$$\oint_{\Gamma_1 \Gamma_2} \mathcal{A}_\mu \mathcal{A}_\nu = \oint_{\Gamma_1} \mathcal{A}_\mu \mathcal{A}_\nu + \oint_{\Gamma_1} \mathcal{A}_\mu \oint_{\Gamma_2} \mathcal{A}_\nu + \oint_{\Gamma_2} \mathcal{A}_\mu \mathcal{A}_\nu, \tag{8}$$

where $\mathcal{A}_\mu$ and $\mathcal{A}_\nu$ are again ordered along the path[3] and the integral measures are suppressed. Eq. (8) makes it clear that there are three contributions: two coming from the loops considered independently, and one coming from the interference terms. Also, the group structure of generalized loop space[4] takes care of what happens when one changes the orientation of one of the two loops:

$$\begin{aligned}
&\oint_{\Gamma_1 \Gamma_2^{-1}} \mathcal{A}_\mu \mathcal{A}_\nu \\
&\quad = \int_{\Gamma_1} \mathcal{A}_\mu \mathcal{A}_\nu + \int_{\Gamma_1} \mathcal{A}_\mu \int_{\Gamma_2^{-1}} \mathcal{A}_\nu + \int_{\Gamma_2^{-1}} \mathcal{A}_\mu \mathcal{A}_\nu \\
&\quad = \int_{\Gamma_1} \mathcal{A}_\mu \mathcal{A}_\nu + (-1)^1 \int_{\Gamma_1} \mathcal{A}_\mu \int_{\Gamma_2} \mathcal{A}_\nu + (-1)^2 \int_{\Gamma_2} \mathcal{A}_\nu \mathcal{A}_\mu.
\end{aligned} \tag{9}$$

In the last term, the order of the algebra generators inside the trace needs to be reversed as well (i.e. $\mathrm{Tr}(t^a t^b) \to \mathrm{Tr}(t^b t^a)$). However, due to the cyclicity of the trace, at one-loop level both traces

[3] This means we do not need to consider the contribution $\int_{\Gamma_1} \mathcal{A}_\nu \int_{\Gamma_2} \mathcal{A}_\mu$.

[4] The group structure was confirmed by explicit calculation of the full diagrams.