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Constraining the neutron–proton effective mass splitting using empirical constraints on the density dependence of nuclear symmetry energy around normal density

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ABSTRACT

According to the Hugenholtz–Van Hove theorem, nuclear symmetry energy $E_{\mathrm{sym}}(\rho)$ and its slope $L(\rho)$ at an arbitrary density ρ are determined by the nucleon isovector (symmetry) potential $U_{\mathrm{sym}}(\rho,k)$ and its momentum dependence $\frac{\partial U_{\mathrm{sym}}}{\partial k}$. The latter determines uniquely the neutron–proton effective k-mass splitting $m^*_{n-p}(\rho,\delta)\equiv(m^*_n-m^*_p)/m$ in neutron-rich nucleonic matter of isospin asymmetry δ. Using currently available constraints on the $E_{\mathrm{sym}}(\rho_0)$ and $L(\rho_0)$ at normal density ρ_0 of nuclear matter from 28 recent analyses of various terrestrial nuclear laboratory experiments and astrophysical observations, we try to infer the corresponding neutron–proton effective k-mass splitting $m^*_{n-p}(\rho_0,\delta)$. While the mean values of the $m^*_{n-p}(\rho_0,\delta)$ obtained from most of the studies are remarkably consistent with each other and scatter very closely around an empirical value of $m^*_{n-p}(\rho_0,\delta)=0.27\cdot\delta$, it is currently not possible to scientifically state surely that the $m^*_{n-p}(\rho_0,\delta)$ is positive within the present knowledge of the uncertainties. Quantifying, better understanding and then further reducing the uncertainties using modern statistical and computational techniques in extracting the $E_{\mathrm{sym}}(\rho_0)$ and $L(\rho_0)$ from analyzing the experimental data are much needed.

1. Introduction

The ultimate goal of investigating properties of neutron-rich nucleonic matter through terrestrial nuclear laboratory experiments and astrophysical observations is to understand the underlying isospin dependence of strong interaction in nuclear medium [1]. The Equation of State (EOS) of neutron-rich nucleonic matter can be written within the parabolic approximation in terms of the binding energy per nucleon at density ρ as $E(\rho,\delta)=E(\rho,\delta=0)+E_{\mathrm{sym}}(\rho)\delta^2+\mathcal{O}(\delta^4)$ where $\delta\equiv(\rho_n-\rho_p)/(\rho_p+\rho_n)$ is the neutron–proton asymmetry and $E_{\mathrm{sym}}(\rho)$ is the density-dependent nuclear symmetry energy. The latter has important applications in many areas of both nuclear physics, see, e.g., Refs. [2–8] and astrophysics, see, e.g., Refs. [9–11]. However, the density dependence of nuclear symmetry energy has been among the most uncertain properties of neutron-rich nucleonic matter. Predictions using various many-body theories and interactions diverge quite broadly especially at abnormal densities. It is thus exciting to see that significant progress has been made recently in constraining the $E_{\mathrm{sym}}(\rho)$ around ρ_0, see, e.g., Ref. [12] based on model analyses of experimental and/or observational data. In particular, as listed in Table 1 and also shown in Fig. 1 at least 28 studies have extracted the slope $L(\rho_0)\equiv[3\rho(\partial E_{\mathrm{sym}}/\partial\rho)]_{\rho_0}$ and $E_{\mathrm{sym}}(\rho_0)$ at ρ_0 [13–43]. It is thus interesting to ask timely what we can learn about the isospin dependence of in-medium nuclear interaction from the extracted constrains on $L(\rho_0)$ and $E_{\mathrm{sym}}(\rho_0)$. Here we study this question at the mean-field level by using a formulism developed earlier in Refs. [29,44,45] based on the Hugenholtz–Van Hove (HVH) theorem [46]. Specifically, we try to infer both the magnitude of the symmetry potential $U_{\mathrm{sym}}(\rho_0,k_F)$ and the neutron–proton effective k-mass splitting $m^*_{n-p}(\rho_0,\delta)$ corresponding to each of the 28 constraints on $E_{\mathrm{sym}}(\rho_0)$ and $L(\rho_0)$ at ρ_0. The consistency of the extracted values for $U_{\mathrm{sym}}(\rho_0,k_F)$ and $m^*_{n-p}(\rho_0,\delta)$ from various constraints is then examined. It is found that while the mean values of the $U_{\mathrm{sym}}(\rho_0,k_F)$ and $m^*_{n-p}(\rho_0,\delta)$ from different studies are consistent with each other and most of them scatter closely around $U_{\mathrm{sym}}(\rho_0,k_F)=29$ MeV and $m^*_{n-p}(\rho_0,\delta)=0.27\cdot\delta$, respectively, the individual uncertainties from many analyses are still too large. Quantifying, better understanding and reducing the uncertainties in extracting the symmetry energy from model analyses of the experimental data are much needed in order to use reliably the extracted mean values of the $U_{\mathrm{sym}}(\rho_0,k_F)$ and $m^*_{n-p}(\rho_0,\delta)$ in solving many important problems in both nuclear physics and astrophysics.

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Table 1
Constrained values of $E_{sym}(\rho_0)$ and $L(\rho_0)$ from 28 analyses of terrestrial nuclear experiments and astrophysical observations.

Analysis	$E_{sym}(\rho_0)$	$L(\rho_0)$	Ref.
Thomas–Fermi model analysis of masses (Myers 1996)	32.65	50	[13]
Atomic masses (Liu 2010)	31.1 ± 1.7	66 ± 13	[14]
Liquid drop model analysis of atomic masses (Lattimer 2012)	$29.6 \pm 3.$	46.6 ± 37	[15]
FRDM analysis of atomic masses (Moller 2012)	32.5 ± 0.5	70 ± 15	[16]
Atomic masses and n-skin of Sn isotopes (Chen 2011)	30.5 ± 3	52.5 ± 20	[17]
Atomic masses and n-skin in an empirical approach (Agrawal 2012)	32.1	64 ± 5	[18]
IAS + n-skin (Danielewicz and Lee 2013)	31.95 ± 1.75	52.5 ± 17.5	[19]
SHF + n-skin (Chen 2010)	30.5 ± 5.5	41 ± 41	[20]
Droplet Model + n-skin (Centelles and Warda 2009)	31.5 ± 3.5	55 ± 25	[21,22]
IBUU04 analysis of isospin diffusion at 50 MeV/A (Chen and Li 2005)	31.6	86 ± 25	[23,24]
IQMD analysis of isospin diffusion at 50 MeV/A (Tsang 2009)	32.5 ± 2.5	77.5 ± 32.5	[25,26]
IQMD analysis of isospin diffusion at 35 MeV/A (Sun 2010)	30.1	52	[27]
Isoscaling analysis of fragments (Shetty 2007)	31.6	65	[28]
Global nucleon optical potential (Xu 2010)	31.3 ± 4.5	52.7 ± 22.5	[29]
Pygmy dipole resonances (Klimkiewicz 2007)	32 ± 1.8	43 ± 15	[30]
Pygmy dipole resonances (Carbone 2010)	32 ± 1.3	65 ± 16	[31]
AMD analysis of transverse flow (Kohley 2010)	30.5	65	[32]
α-decay energy (Dong 2013)	31.6 ± 2.2	61 ± 22	[33]
β-decay energy (Dong 2013)	32.3 ± 1.3	50 ± 15	[34]
Mass differences and n-skin (Zhang 2013)	32.3 ± 1.0	45.2 ± 10	[35]
Dipole polarizability of ^{208}Pb (Tamii 2013)	30.9 ± 1.5	46 ± 15	[36]
r-mode instability of neutron stars (Vidana 2012)	$30. \pm 5$	$\geqslant 50$	[37]
r-mode instability of neutron stars (Wen 2012)	32.5 ± 7.5	$\leqslant 65$	[38]
Mass-radius of neutron stars-analysis 1 (Steiner 2010)	31 ± 3	50 ± 10	[39]
Mass-radius of neutron stars-analysis 2 (Steiner 2012)	33 ± 1.6	46 ± 10	[40]
Torsional crust oscillation of neutron stars (Gearheart 2011)	32.5 ± 7.5	$\leqslant 50$	[41]
Torsional crust oscillation of neutron stars (Sotani 2012)	32.5 ± 7.5	115 ± 15	[42]
Binding energy of neutron stars (Newton 2009)	32.5 ± 7.5	$\leqslant 70$	[43]

2. Relationship between neutron–proton effective mass splitting and symmetry energy based on the Hugenholtz–Van Hove theorem

According to the well-known Lane potential [47] verified by various many-body theories and optical model analysis of nucleon–nucleus scattering data, the neutron/proton (n/p) single-particle potential $U_{n/p}(\rho,k,\delta)$ can be well approximated by

$$U_{n/p}(\rho,k,\delta) = U_0(\rho,k) \pm U_{sym}(\rho,k)\cdot\delta + \mathcal{O}(\delta^2), \quad (1)$$

where the $U_0(\rho,k)$ and $U_{sym}(\rho,k)$ are, respectively, the nucleon isoscalar and isovector (symmetry) potentials for nucleons with momentum k in asymmetric nuclear matter of isospin asymmetry δ at density ρ. Their momentum dependence is normally characterized by the nucleon effective k-mass

$$m^*_\tau/m = \left[1 + \frac{m}{\hbar^2 k_F}\frac{dU_\tau}{dk}\bigg|_{k_F}\right]^{-1} \quad (2)$$

where $\tau = n, p$ and 0 for neutrons, protons and nucleons, respectively, and $m = (m_n + m_p)/2$ is the average mass of nucleons in free-space. While the nucleon isoscalar potential and its momentum dependence, especially at ρ_0, have been relatively well determined, our knowledge about the isovector potential $U_{sym}(\rho,k)$ and its momentum dependence $\frac{\partial U_{sym}}{\partial k}$ even at normal density is still very poor. However, from the structure of rare isotopes and mechanism of heavy-ion reactions to the cooling of protoneutron stars, solutions to many interesting issues depend critically on the nucleon isovector potential and its momentum dependence.

Using the Brueckner theory [48] or the Hugenholtz–Van Hove (HVH) theorem [46], the $E_{sym}(\rho)$ and $L(\rho)$ can be expressed as [29,44,45,49]

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k_F^2}{2m_0^*} + \frac{1}{2}U_{sym}(\rho,k_F), \quad (3)$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k_F^2}{2m_0^*} + \frac{3}{2}U_{sym}(\rho,k_F) + \frac{\partial U_{sym}}{\partial k}\bigg|_{k_F} k_F, \quad (4)$$

where $k_F = (3\pi^2\rho/2)^{1/3}$ is the nucleon Fermi momentum. We emphasize that these relationships are general and independent of the many-body theory and/or interaction used to calculate the $U_{sym}(\rho,k)$ and m_0^*. In fact, all microscopic calculations of the nuclear EOS are required to satisfy the HVH theorem. It is also worth noting that adding the second-order symmetry potential $U_{sym,2}(\rho,k)\cdot\delta^2$ term to the Lane potential in Eq. (1) and considering the δ^2 terms consistently in applying the HVH theorem, while the expression for the $E_{sym}(\rho)$ remains the same as in Eq. (3), the expression for $L(\rho)$ has two additional terms due to the momentum dependence of the isoscalar effective mass m_0^* and the $U_{sym,2}(\rho,k_F)$, respectively [45]. However, at the saturation density ρ_0 these high-order terms were found completely negligible based on the optical model analyses of the latest and most complete neutron-nucleus scattering data base [50]. Thus, at least at ρ_0 Eqs. (3) and (4) are accurate decompositions of the symmetry energy and its density slope required by the HVH theorem. While it is not clear if all models satisfy the HVH theorem and the resulting equations (3) and (4), it is understandable that various observables may be sensitive to different components of the $E_{sym}(\rho)$ and $L(\rho)$ with different sensitivities, leading to the rather broad ranges of uncertainties and/or error bars in the results shown in Table 1 and Fig. 1. It is certainly an interesting task to find out for each observable whether/why it may only depend on the total values or some particular components of the $E_{sym}(\rho)$ and/or $L(\rho)$. We notice that not all models used in extracting the $E_{sym}(\rho_0)$ and $L(\rho_0)$ consider all the terms of the $E_{sym}(\rho)$and $L(\rho)$ in Eqs. (3) and (4). For instance, while most models consider the momentum dependence of the isoscalar potential albeit often use different values for the m_0^*, the momentum dependence of the isovector potential, i.e., the $\frac{\partial U_{sym}}{\partial k}$ term, has been frequently ignored so far. It may well be that some of the observables are not sensitive to this component of the $L(\rho)$ but still allow an accurate extraction of the $E_{sym}(\rho_0)$ and $L(\rho_0)$ within the framework of a given model used. In this work, we use the 28 sets of $E_{sym}(\rho_0)$ and $L(\rho_0)$ as quasi-data regardless how they were extracted from the model analyses of experimental data. Since the expressions for $E_{sym}(\rho)$ and $L(\rho)$ in Eqs. (3) and (4)