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Signature transition in Einstein–Cartan cosmology

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ABSTRACT

In the context of Einstein–Cartan theory of gravity, we consider a Friedmann–Lemaitre–Robertson–Walker (FLRW) cosmological model with Weyssenhoff perfect fluid. We focus attention on those classical solutions that admit a degenerate metric in which the scale factor has smooth behavior in the transition from a Euclidean to a Lorentzian domain. It is shown that the spin–spin contact interaction enables one to obtain such a signature changing solutions due to the Riemann–Cartan (U_4) structure of space–time.

1. Introduction

The study of cosmology has always been influenced by the choice of the matter field used to construct the energy–momentum tensor in Einstein field equations. The most widely used matter source has traditionally been the perfect fluid. However, the ubiquitous scalar field has also been playing an increasingly important role in more recent cosmological models as the matter source [1]. This of course is not surprising since it being a scalar field makes it somewhat easy to work with. One may also conceivably imagine a Universe filled with massless or massive spinor fields as the matter source. Such cosmological models have seldom been studied in the literature and, when they were, it was more often than not in the form of general formalisms [2]. In general then, it would be fair to say that cosmologies with spinor fields as the matter source are the least studied scenarios. In 1923 Élie Cartan introduced the relation between the intrinsic angular momentum of matter and the space–time torsion in the framework of a generalization of general relativity (GR) [3], nowadays known as Einstein–Cartan (EC) theory [4]. Indeed, the classical spin may be introduced in GR in two distinct ways. In the first approach, spin is considered as a dynamical quantity without changing the Riemannian structure of the space–time geometry [5]. The second method, which as we mentioned above was proposed by Cartan, is based on the generalization of space–time structure by assuming the metric and the non-symmetric affine connection as independent quantities. Since the first attempts of Cartan to bring spin into the curved space–time, many efforts have been made in this area and the corresponding results have been followed and developed by a number of works, see for instance [6–8]. The importance of the Cartan theory becomes more clear, if one tries to incorporate the spinor field into the torsion-free general theory of relativity. In this context one should apply the Cartan theory which possesses torsion as well as curvature [9]. In EC theory, torsion is not a dynamical quantity, instead it can be expressed completely in terms of the spin sources [6]. Consequently, in order to study the effects of torsion in U_4 geometry (it is usual to denote the Riemann–Cartan space–time as U_4 to distinguish it from the Riemannian space–time) one may consider the matter fields with intrinsic angular momentum. To do this, one of the usual ways is to consider a fluid with intrinsic spin density known as the Weyssenhoff exotic perfect fluid [10]. As in the case of other alternative theories of gravity, it is important to seek the cosmological solutions in the EC theory of gravity, i.e., in a theory in which the spin properties of matter and their influence on the geometrical structure of space–time are considered. This is done by some authors [11], who have investigated the effects of torsion and spinning matter in a cosmological setting and its possible role to remove the singularities, inflationary scenarios, explain the late time accelerated expansion of the Universe and so on.

A question of interest related to classical and quantum cosmological models is that of signature transition which has been attracting attention since the early 1980s. Traditionally, a feature in GR is that one usually fixes the signature of the space–time metric before trying to solve Einstein's field equations. However, there is no a priori reason for doing so and it is well known that the field equations do not demand this property, that is, if

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one relaxes this condition one may find solutions to the field equations which, when parameterized suitably, can either have Euclidean or Lorentzian signature. The notion of signature transition mainly started to appear in the works of Hartle and Hawking [12] where they argued that in quantum cosmology amplitudes for gravity should be expressed as the sum of all compact Riemannian manifolds whose boundaries are located at the signature changing hypersurface. This phenomenon has been studied at the classical and quantum cosmology level by other authors, see for example [13]. In what follows by a signature changing space–time we mean a manifold which contains both Euclidean and Lorentzian region. As it is shown in [14], in classical GR, a signature changing metric should be either degenerate or discontinuous, though Einstein's equations implicitly assume that the metric is non-degenerate and at least continuous.

In this Letter, we consider a smooth signature changing type of flat FLRW space–time in the framework of EC gravity with exotic Weyssenhof perfect fluid. For the case of a spatially flat Universe, field equations are then solved exactly for the scale factor as dynamical variable, giving rise to cosmological solutions with a degenerate metric, describing a continuous signature transition from a Euclidean domain to a Lorentzian space–time.

2. The model

In this section we start by briefly studying the EC gravity where the action is given by (we work in units where $c=1$ and consider the signature $(+,-,-,-)$ for the space–time metric)

$$\mathcal{S}=\int\sqrt{-g}\,d^4x\left[-\frac{1}{16\pi G}(\tilde{R}-2\Lambda)+\mathcal{L}_M\right], \tag{1}$$

where $\tilde{R}$ is the Ricci scalar constructed by the asymmetric connection $\tilde{\Gamma}^{\mu}{}_{\alpha\beta}$ and Λ is the cosmological constant. By using of the metricity condition [6]

$$\tilde{\nabla}_\alpha g_{\mu\nu}=0, \tag{2}$$

and also the definition of torsion

$$T^{\mu}{}_{\alpha\beta}:=\tilde{\Gamma}^{\mu}{}_{\alpha\beta}-\tilde{\Gamma}^{\mu}{}_{\beta\alpha}, \tag{3}$$

the connection $\tilde{\Gamma}^{\mu}{}_{\alpha\beta}$ can be expressed as

$$\tilde{\Gamma}^{\mu}{}_{\alpha\beta}=\Gamma^{\mu}{}_{\alpha\beta}+K^{\mu}{}_{\alpha\beta}, \tag{4}$$

where $\Gamma^{\mu}{}_{\alpha\beta}$ is the Levi-Civita connection (Christoffel symbol) and $K^{\mu}{}_{\alpha\beta}$ is the contorsion tensor related to the torsion $Q_{\alpha\beta}{}^{\mu}:=\tilde{\Gamma}_{[\alpha\beta]}{}^{\mu}$ via

$$K^{\mu}{}_{\alpha\beta}:=\frac{1}{2}\left(Q^{\mu}{}_{\alpha\beta}-Q_{\alpha}{}^{\mu}{}_{\beta}-Q_{\beta}{}^{\mu}{}_{\alpha}\right). \tag{5}$$

Also $\mathcal{L}_{\mathcal{M}}$ is essentially the Lagrangian density for matter field coupled to gravity. Our assumption is that instead of usual Big-Bang singularity in the early Universe, we have signature changing event. Therefore, we focus our attention on the early Universe epoch where the matter content of the model is of the form of fermionic matter, like quarks and leptons. The dynamical equations of motion can be obtained by performing the variation of the action with respect to the metric and contorsion [6], that is

$$\begin{cases} G^{\mu\nu}-\Lambda g^{\mu\nu}-\left(\nabla_\alpha+2Q_{\alpha\beta}{}^{\beta}\right)\left(T^{\mu\nu\alpha}-T^{\nu\alpha\mu}+T^{\alpha\mu\nu}\right)\\ \quad =8\pi G T^{\mu\nu},\\ T^{\mu\nu\alpha}=8\pi G\tau^{\mu\nu\alpha}, \end{cases} \tag{6}$$

where

$$T_{\mu\nu}{}^{\alpha}=Q_{\mu\nu}{}^{\alpha}+\delta^{\alpha}_{\mu}Q_{\nu\beta}{}^{\beta}-\delta^{\alpha}_{\nu}Q_{\mu\beta}{}^{\beta}, \tag{7}$$

and $G^{\mu\nu}$ and ∇_α are respectively the Einstein tensor and covariant derivative for the full nonsymmetric connection Γ. Also

$$\begin{cases} T^{\mu\nu}:=\dfrac{2}{\sqrt{-g}}\dfrac{\delta\mathcal{L}_M}{\delta g_{\mu\nu}},\\ \tau^{\mu\nu\alpha}:=\dfrac{1}{\sqrt{-g}}\dfrac{\delta\mathcal{L}_M}{\delta K_{\alpha\nu\mu}}, \end{cases} \tag{8}$$

are the energy–momentum and the canonical spin-density tensors respectively. Now by using equations (6) and (7) one can obtain modified Einstein field equations

$$G^{\mu\nu}(\Gamma)=8\pi G\left(T^{\mu\nu}+\tau^{\mu\nu}\right), \tag{9}$$

where $G^{\mu\nu}(\Gamma)$ is the usual symmetric Einstein tensor and

$$\begin{aligned}\tau^{\alpha\beta}=\Big[&-4\tau^{\alpha\mu}{}_{[\nu}\tau^{\beta\nu}{}_{\mu]}-2\tau^{\alpha\mu\nu}\tau^{\beta}{}_{\mu\nu}+\tau^{\mu\nu\alpha}\tau_{\mu\nu}{}^{\beta}\\ &+\frac{1}{2}g^{\alpha\beta}\left(4\tau_{\lambda}{}^{\mu}{}_{[\nu}\tau^{\lambda\nu}{}_{\mu]}+\tau^{\mu\nu\lambda}\tau_{\mu\nu\lambda}\right)\Big]\end{aligned} \tag{10}$$

is the correction to the space–time curvature due to the spin [10]. If the spin vanishes then equation (9) reduces to the standard Einstein field equations. We assume that $\mathcal{L}_M$ describes a fluid of spinning particles in the early Universe minimally coupled to the metric and the torsion of the U_4 theory. For the spin fluid the canonical spin tensor is given by [10]

$$\tau^{\mu\nu\alpha}=\frac{1}{2}S^{\mu\nu}u^{\alpha}, \tag{11}$$

where $S^{\mu\nu}$ is the antisymmetric spin density and u^α is the 4-velocity of the fluid [15]. Then the energy–momentum tensor can be decomposed into the two parts: the usual perfect fluid $T_F{}^{\alpha\beta}$ and an intrinsic-spin part $T_S{}^{\alpha\beta}$, as

$$T^{\alpha\beta}=T_F{}^{\alpha\beta}+T_S{}^{\alpha\beta}, \tag{12}$$

so that we have explicitly for intrinsic-spin part

$$\begin{aligned}T_S{}^{\alpha\beta}=&\,u^{(\alpha}S^{\beta)\mu}u^{\nu}u_{\mu;\nu}+\left(u^{(\alpha}S^{\beta)\mu}\right)_{;\mu}+Q_{\mu\nu}{}^{(\alpha}u^{\beta)}S^{\nu\mu}\\ &-u^{\nu}S^{\mu(\beta}Q^{\alpha)}{}_{\mu\nu}-\omega^{\mu(\alpha}S^{\beta)}{}_{\mu}+u^{(\alpha}S^{\beta)\mu}\omega_{\mu\nu}u^{\nu},\end{aligned} \tag{13}$$

where ω is the angular velocity associated with the intrinsic spin and semicolon denotes covariant derivative with respect to Levi-Civita connection. If as usual interpretation of EC gravity we assume that $S_{\mu\nu}$ is associated with the quantum mechanical spin of microscopic particles [11], then for unpolarized spinning field we have $\langle S_{\mu\nu}\rangle=0$ and if we define

$$\sigma^2:=\frac{1}{2}\langle S_{\mu\nu}S^{\mu\nu}\rangle, \tag{14}$$

we get

$$\langle\tau^{\alpha\beta}\rangle=4\pi G\sigma^2u^{\alpha}u^{\beta}+2\pi G\sigma^2g^{\alpha\beta}, \tag{15}$$

and

$$\begin{cases}\langle T_F{}^{\alpha\beta}\rangle=(\rho+p)u^{\alpha}u^{\beta}-pg^{\alpha\beta},\\ \langle T_S{}^{\alpha\beta}\rangle=-8\pi G\sigma^2u^{\alpha}u^{\beta}.\end{cases} \tag{16}$$

Consequently the simplest EC generalization of standard gravity will be

$$G^{\alpha\beta}(\Gamma)=8\pi G\Theta^{\alpha\beta}, \tag{17}$$

where $\Theta^{\alpha\beta}$ describes the effective macroscopic limit of matter field

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