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Magnetic moment of constituent fermions in ground states with spontaneously broken translational invariance

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ABSTRACT

Spontaneous symmetry breaking gives rise to a nonzero order parameter or a ground state expectation value (GEV) of the scalar field that generates energy gaps or constituent masses for the fermions via Yukawa interactions. There are several physical situations in which the order parameters or GEVs of the scalar field (and therefore constituent masses) can become space varying. This can change the definitions of several important physical operators. We investigate and rederive the generalized magnetic moment operator for 'constituent' fermions that arises from a space varying order parameter or GEV.

We especially consider the high baryon density π_0 condensed phase, in which chiral symmetry is spontaneously broken, with space varying expectation values of the σ and π_0 fields. This phase has a spin polarized Fermi sea as the ground state. We show that there is indeed generated a macroscopic magnetization in this phase, contrary to what one would have found, if one just used a primitive phenomenological magnetic moment formula for explicit/current fermion masses. This is important in the context of neutron stars, as such a high density state may be responsible for very high magnetic fields in the dense core of neutron stars.

1. Introduction

Spontaneous symmetry breaking gives rise to a nonzero order parameter that generates an energy gap, as in superconductivity. Similarly, spontaneous symmetry breaking gives rise to a nonzero order parameter or a vacuum expectation value (VEV) for the scalar field that generates constituent masses for the fermions. Constituent fermions or energy gaps arise in a variety of models from electron superconductivity, the Standard Model of the electroweak interactions via the Higgs VEV to the strong interactions (in the chiral symmetry limit) where the nucleon/quark masses are generated by the VEV of the scalar field.

Constituent masses are different from explicit or current masses. For example, constituent masses occur when chiral symmetry is spontaneously broken. In this case the exact chiral symmetric Hamiltonian remains invariant under a chiral symmetry transformation, whereas any current mass in the Hamiltonian explicitly breaks the chiral symmetry.

There are several situations in condensed matter physics in which space dependent order parameters or ground state expectation values (GEVs) occur, for example, in crystalline superconductors and charge density waves. Such space dependent GEVs or order parameters also occur for diquark superconductors in QCD [1,5,7] and in pion condensation [14,16,8,2,12,20]. Such states can occur when fermions from two asymmetric Fermi seas, with different Fermi energies or chemical potentials, pair together, for then, the pair carries a nonzero momentum. An example is the LOFF state, first explored by Larkin and Ovchinnikov [13], Fulde and Ferrell [9], and Takada and Izuyama [19] in the context of electron superconductivity in the presence of magnetic impurities. They found that in a small range of chemical potential difference in the two species, it is favorable to form a state in which the Cooper pairs have nonzero momentum. Such condensates spontaneously break translational and rotational invariance, leading to gaps which vary periodically in space.

Analogously, for the strong interactions in several physical situations the GEVs of the scalar (or and pseudoscalar) field or order parameters and therefore constituent masses can become space varying, further underlining the difference with current masses. Such space varying order parameters can change the field equations which in turn change the exact form of some operators which follow from the use of the field equations. In such a context

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the use of the usual phenomenological formulae for space constant order parameters or GEVs can be misleading and lead to apparent paradoxes as we show below. This is a matter of some import for such ground states both in condensed matter and the strong interactions. We shall illustrate this for an important operator, the magnetic moment operator, below.

One such case that we will consider is the case of a high baryon density ground state for strongly interacting nucleons, which has a π_0 condensate that is a (space varying) stationary wave [8,2,12, 20,18]. In this case when we calculate the magnetic moment of the nucleons in the presence of a π_0 condensate using the naive formula, we find a somewhat paradoxical result: that for a spin polarized neutron ground state, the magnetic moment vanishes when averaged over space [17] Many of the above authors have also used the naive formulae. However, when we compute the magnetic moment operator from first principles, from the chiral Lagrangian, we find it has the right space dependence, which cancels out with the space dependence of the ground state to give a magnetic moment that is proportional to the total spin. Furthermore, this reveals that the magnetization of this state goes up as the GEV, that determines the 'mass', comes down with increasing baryon density.

This is important in the context of neutron stars, as such a high density ground state may be responsible for very high magnetic fields in the dense core and could be the origin of magnetars [6, 11] — the stars with the largest magnetic fields in the universe.

2. Current and constituent masses

2.1. QED

Quantum electrodynamics is a theory with an explicit or current electron mass, m_e, which breaks chiral symmetry explicitly. The case of the electron magnetic moment operator is explicitly worked out in the text of Sakurai (Advanced Quantum Mechanics) via the Gordon decomposition (see [15, 3-3, p. 85]).

$$H_{mag} = -\frac{e}{2m_e}\frac{1}{2}F_{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi. \tag{1}$$

($\hbar = c = 1.$)

2.2. The chiral symmetric Gellmann–Levy sigma model

In a chiral symmetric theory, the mass of the nucleon/quark comes from the VEV of $\langle\sigma\rangle$, for example in the linear σ model of Gellmann and Levy [10]. In the case of exact chiral symmetry we have the following Lagrangian when we couple the Gellmann–Levy sigma model to the electromagnetic field

$$L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \sum\overline{\psi}\big(\not{D} + g_y(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\big)\psi$$
$$-\frac{1}{2}(\partial\mu\sigma)^2 - \frac{1}{2}(\partial\mu\vec{\pi})^2 - \frac{\lambda^2}{4}\big(\sigma^2 + \vec{\pi}^2 - (F)^2\big)^2. \tag{2}$$

In the limiting case of a small explicit pion mass being set to zero, the usual (uniform in space) symmetry breaking that follows on the minimization of the potentials above is $\langle\sigma\rangle = F = f_\pi$ and $\langle\vec{\pi}\rangle = 0$.

The masses of the scalar (PS) and fermions are given by

$$\langle\sigma\rangle^2 = F^2 \tag{3}$$

where F is the pion decay constant. It follows that

$$m_\sigma^2 = 2\lambda^2F^2, \qquad m = g\langle\sigma\rangle = gF. \tag{4}$$

The quark or nucleon mass, m, is a spontaneous mass that is generated from the spatially uniform VEV. In this case the usual magnetic moment formula above (1) works.

On the other hand when the VEVs of $\langle\sigma\rangle$ and $\langle\vec{\pi}\rangle$ depend on space coordinates, the above expression for the magnetic moment will not work as we will find in the following section.

3. The π_0 condensation: space dependent VEVs

3.1. The π_0 condensed ground state

Here we shall consider another realization of the expectation value of $\langle\sigma\rangle$ and $\langle\vec{\pi}\rangle$ corresponding to π_0 condensation. This phenomenon was first considered in the context of nuclear matter [8,2]. Such a phenomenon also occurs with our quark based chiral σ model and was considered at the mean field level by Kutschera, Broniowski and Kotlorz for the 2 flavor case [12] and by one of us for the 3 flavor case [18]. Working in the chiral limit they found that the pion condensed state has lower energy than the uniform symmetry breaking state we have just considered for all density. This is expected, as the ansatz for the pion condensed phase is more general.

The expectation values now carry a particular space dependence

$$\langle\sigma\rangle = F\cos(\vec{q}.\vec{r}), \tag{5}$$

$$\langle\pi_3\rangle = F\sin(\vec{q}.\vec{r}), \tag{6}$$

$$\langle\pi_1\rangle = 0, \tag{7}$$

$$\langle\pi_2\rangle = 0. \tag{8}$$

Note that the relation, $\langle\sigma^2\rangle + \langle\vec{\pi}^2\rangle = F^2$, is preserved under this pattern of symmetry breaking. Also, when $|\vec{q}|$ goes to zero, we recover the usual space uniform phase above.

The Dirac Equation in this background is solved in [8,12] by the artifact of writing the wave function, ψ, in terms of a chirally rotated wave function, $\chi(k)$,

$$\psi = \exp\big(-i(\tau_3/2)\gamma_5\vec{q}\cdot\vec{r}\big)\cdot\chi(k) \tag{9}$$

where $\chi(k)$ are momentum eigenfunctions.

The Hamiltonian reduces to

$$H\chi(k) = \left(\vec{\alpha}.\vec{k} - \frac{1}{2}\vec{q}.\vec{\alpha}\gamma_5\tau_3 + \beta m\right)\chi(k) = E(k)\chi(k) \tag{10}$$

where $m = g\sqrt{\langle\sigma\rangle^2 + \langle\vec{\pi}\rangle^2} = gF$

The second term arises from the condensate and has been written in terms of the relativistic spin operator, $\vec{\alpha}\gamma_5$. It is evident that if spin is parallel to $\vec{q}$ and $\tau_3 = +1$ (proton/up quark) this term is negative and if $\tau_3 = -1$ (neutron/down quark) it is positive. For spin antiparallel to $\vec{q}$ the signs of this term for $\tau_3 = +1$ and -1 are reversed.

The spectrum for the Hamiltonian is the quasi particle spectrum and can be found to be [8,12]

$$E_{(-)}(k) = \sqrt{m^2 + k^2 + \frac{1}{4}q^2 - \sqrt{m^2q^2 + (\vec{q}.\vec{k})^2}}, \tag{11}$$

$$E_{(+)}(k) = \sqrt{m^2 + k^2 + \frac{1}{4}q^2 + \sqrt{m^2q^2 + (\vec{q}.\vec{k})^2}}. \tag{12}$$

The lower energy eigenvalue $E_{(-)}$ has spin along $\vec{q}$ for $\tau_3 = 1$, or has spin opposite to $\vec{q}$ for $\tau_3 = -1$. The higher energy eigenvalue $E_{(+)}$ has spin along $\vec{q}$ for $\tau_3 = -1$, or has spin opposite to $\vec{q}$ for $\tau_3 = +1$.