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Ehrenfest scheme for P-V criticality in the extended phase space of black holes

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ABSTRACT

In this Letter, the nature of phase transition at the critical point of P-V criticality in the extended phase space of RN-AdS black holes has been investigated. By treating the cosmological constant and its conjugate quantity as thermodynamic pressure and volume respectively, we introduce the original expressions of Ehrenfest equations directly into the black hole research instead of utilizing the analogy of Ehrenfest equations. We carry out an analytical check of Ehrenfest equations and prove that both Ehrenfest equations are satisfied. So the black hole undergoes a second order phase transition at the critical point. This result is consistent with the nature of liquid–gas phase transition at the critical point, hence deepening the understanding of the analogy of charged AdS black holes and liquid–gas systems.

1. Introduction

Since black holes were identified as thermodynamic objects with both temperature and entropy [1,2], black hole thermodynamics has been an exciting topic in theoretical physics. Phase transition, a fascinating phenomenon in the classical thermodynamics, has aroused the attention of researchers. The pioneer research of phase transitions of AdS black holes can be traced back to 1983, when Hawking and Page discovered that there exists a phase transition between the Schwarzschild AdS black hole and thermal AdS space [3]. From then on, phase transitions of black holes have been intensely investigated from different perspectives. It is believed that phase transitions of AdS black holes are of great importance because they may serve as a novel way to investigate phase transitions in the dual field theories [4].

Moreover it has been found that phase transitions of charged AdS black holes behave much like the liquid–gas systems. The observation of this phenomenon was first made by Chamblin et al. [5,6]. They investigated the first order phase transition in the Reissner–Nordstrom-AdS (RN-AdS) black hole and found that the critical behavior is analogous to the Van der Waals liquid–gas phase transition. Recently, there has been increasing attention of considering the variation of the cosmological constant in the first law of black hole thermodynamics [7–13]. Based on this idea, Kubizňák et al. [14] completed the analogy of charged AdS black holes with the liquid–gas systems by treating the cosmological constant as thermodynamic pressure and its conjugate quantity as thermodynamic volume. Their work has been elaborated subsequently [15–24] and P-V criticality in the extended phase space of black holes has led a new trend of phase transition research.

To investigate the phase transition of black holes, one may use techniques from classical thermodynamics or introduce a unique approach called thermodynamic geometry. The analogy of charged AdS black holes and liquid–gas systems has served as a good example of the former approach. Another excellent example may be the analogy of Ehrenfest equations. Recently, Banerjee et al. developed an innovative scheme based on Ehrenfest relations by considering the analogy ($V \leftrightarrow Q$, $P \leftrightarrow -\Phi$) between the thermodynamic state variables and black hole parameters [25–30]. To study phase transitions in black holes, they considered the black holes as grand-canonical ensembles and performed a detailed analysis of Ehrenfest relations using both analytical and graphical techniques.

In this Letter, we would like to elaborate the research on P-V criticality in the extended phase space of charged AdS black holes. As we know, Van der Waals liquid–gas system undergoes a first order phase transition. However, its phase transition is a second order one at the critical point. One may wonder whether charged AdS black holes behave alike at the critical point of P-V criticality. To further understand the analogy of charged AdS black holes and Van der Waals liquid–gas systems, we would investigate the nature of phase transition in the extended phase space of charged AdS black holes at the critical point. To achieve this goal, we would carry out an analytical check of Ehrenfest scheme. Instead of utilizing the analogy of Ehrenfest equations, we would endeavor to

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introduce the classical Ehrenfest equations directly into the black hole research. To our knowledge, we are the first to try such an approach.

The organization of this Letter is as follows. Firstly, a brief review of P-V criticality of charged AdS black holes will be carried out in Section 2. To investigate the nature of the phase transition at the critical point, we will introduce the classical Ehrenfest scheme and carry out an analytical check of both equations in Section 3. At the end, a brief conclusion will be drawn in Section 4.

2. A brief review of P-V criticality of charged AdS black holes in the extended phase space

In this section, we would like to briefly review the P-V criticality of charged AdS black holes in the extended phase space. Considering the spherical RN-AdS black hole, the metric is proposed as

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2^2, \tag{1}$$

where

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}. \tag{2}$$

The corresponding Hawking temperature, entropy and electric potential have been derived as

$$T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right), \tag{3}$$

$$S = \pi r_+^2, \tag{4}$$

$$\Phi = \frac{Q}{r_+}. \tag{5}$$

The key point of investigating P-V criticality is to treat the cosmological constant as thermodynamic pressure and its conjugate quantity as thermodynamic volume. In the paper pioneering the research of P-V criticality, Kubizňák et al. [14] identified the pressure and thermodynamic volume of RN-AdS black hole as

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi}\frac{1}{l^2}, \tag{6}$$

$$V = \frac{4}{3}\pi r_+^3. \tag{7}$$

It is worth noting that, with the newly introduced thermodynamic pressure and volume, the first law of black hole thermodynamics [14] and the corresponding Smarr relation [8] in the extended phase space take new forms as

$$dM = TdS + \Phi dQ + VdP, \tag{8}$$

$$M = 2(TS - VP) + \Phi Q. \tag{9}$$

The critical point was also obtained in Ref. [14] as

$$T_c = \frac{\sqrt{6}}{18\pi Q}, \qquad V_c = 8\sqrt{6}\pi Q^3, \qquad P_c = \frac{1}{96\pi Q^2}. \tag{10}$$

3. Analytical check of the classical Ehrenfest equations in the extended phase space

In classical thermodynamics, one can classify phase transitions as first order or higher order transitions by utilizing Clausius–Clapeyron–Ehrenfest equations. For a first order transition the Clausius–Clapeyron equation is satisfied while for a second order transition Ehrenfest equations are satisfied.

By considering the analogy ($V \leftrightarrow Q, P \leftrightarrow -\Phi$) between the thermodynamic state variables and black hole parameters, a beautiful analogy of Ehrenfest equations was proposed by Banerjee et al. [30] as

$$-\left(\frac{\partial \Phi}{\partial T}\right)_S = \frac{C_{\Phi_2} - C_{\Phi_1}}{TQ(\alpha_2 - \alpha_1)} = \frac{\Delta C_\Phi}{TQ\Delta\alpha}, \tag{11}$$

$$-\left(\frac{\partial \Phi}{\partial T}\right)_Q = \frac{\alpha_2 - \alpha_1}{\kappa_{T_2} - \kappa_{T_1}} = \frac{\Delta\alpha}{\Delta\kappa_T}, \tag{12}$$

where $\alpha = \frac{1}{Q}(\frac{\partial Q}{\partial T})_\Phi$ is the analog of volume expansion coefficient and $\kappa_T = \frac{1}{Q}(\frac{\partial Q}{\partial \Phi})_T$ is the analog of isothermal compressibility.

However, it is possible for us to introduce the classical Ehrenfest equations directly into the black hole research because P-V criticality provides us with a valuable circumstance to consider the specific heat of black holes at constant pressure. In this section, we would carry out an analytical check of the original Ehrenfest equations, whose expressions can be found in any textbook of classical thermodynamics as

$$\left(\frac{\partial P}{\partial T}\right)_S = \frac{C_{P_2} - C_{P_1}}{VT(\alpha_2 - \alpha_1)} = \frac{\Delta C_P}{VT\Delta\alpha}, \tag{13}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha_2 - \alpha_1}{\kappa_{T_2} - \kappa_{T_1}} = \frac{\Delta\alpha}{\Delta\kappa_T}, \tag{14}$$

where $\alpha = \frac{1}{V}(\frac{\partial V}{\partial T})_P$ is volume expansion coefficient and $\kappa_T = -\frac{1}{V}(\frac{\partial V}{\partial P})_T$ is isothermal compressibility coefficient.

Firstly, we would like to calculate the relevant quantities in (13)–(14) for P-V criticality in the extended phase space of RN-AdS black hole. Substituting Eqs. (4) and (6) into Eqs. (3), we obtain

$$T = \frac{1}{4\sqrt{\pi S}} \times \left(1 + 8PS - \frac{\pi Q^2}{S}\right). \tag{15}$$

Utilizing Eqs. (4), (7) and (15), the specific heat at constant pressure, volume expansion coefficient and isothermal compressibility coefficient can be derived as

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = \frac{2S(8PS^2 + S - \pi Q^2)}{8PS^2 - S + 3\pi Q^2}, \tag{16}$$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{12\sqrt{\pi}S^{3/2}}{8PS^2 - S + 3\pi Q^2}, \tag{17}$$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = \frac{24S^2}{8PS^2 - S + 3\pi Q^2}. \tag{18}$$

In the derivation of Eqs. (18), we have utilized the thermodynamic identity as

$$\left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P = -1. \tag{19}$$

It is quite interesting to note that C_P, α, κ_T share the same factor in their denominators, namely $8PS^2 - S + 3\pi Q^2$, which implies that α, κ_T may also diverge at the critical point just as the specific heat at constant pressure does. In fact, the condition that $8PS^2 - S + 3\pi Q^2 = 0$ can be easily examined by utilizing Eq. (10).

Now let's embark on checking the validity of Ehrenfest equations (13)–(14) at the critical point. From the definition of volume expansion coefficient α, we obtain

$$V\alpha = \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial V}{\partial S}\right)_P \left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial V}{\partial S}\right)_P \left(\frac{C_P}{T}\right), \tag{20}$$

then the R.H.S of Eq. (13) can be transformed into

$$\frac{\Delta C_P}{TV\Delta\alpha} = \left[\left(\frac{\partial S}{\partial V}\right)_P\right]_c. \tag{21}$$