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Cosmologies of extended massive gravity

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ABSTRACT

We study the background cosmology of two extensions of dRGT massive gravity. The first is variable mass massive gravity, where the fixed graviton mass of dRGT is replaced by the expectation value of a scalar field. We ask whether self-inflation can be driven by the self-accelerated branch of this theory, and we find that, while such solutions can exist for a short period, they cannot be sustained for a cosmologically useful time. Furthermore, we demonstrate that there generally exist future curvature singularities of the "big brake" form in cosmological solutions to these theories. The second extension is the covariant coupling of galileons to massive gravity. We find that, as in pure dRGT gravity, flat FRW solutions do not exist. Open FRW solutions do exist – they consist of a branch of self-accelerating solutions that are identical to those of dRGT, and a new second branch of solutions which do not appear in dRGT.

1. Introduction and outline

An interacting theory of a massive graviton, free of the Boulware–Deser mode [1], has recently been discovered [2,3] (the dRGT theory, see [4] for a review), allowing for the possibility of addressing questions of interest in cosmology. Pure dRGT massive gravity admits self-accelerating solutions [5–11], in which the de Sitter Hubble factor is of order the mass of the graviton. Since having a light graviton is technically natural [13,12], such a solution is of great interest in the late-time universe to account for cosmic acceleration.

A natural question is whether a similar phenomenon might drive inflation in the early universe. To use the self-accelerating solution of massive gravity for inflation (i.e. "self-inflation"), the graviton mass would have to be of order the Hubble scale during inflation. Yet, we know that the current graviton mass cannot be much larger than the Hubble scale today [14].

Thus, for self-inflation to be possible, the graviton mass must change in time. One idea of how to realize this is to promote the graviton mass to a scalar field, Φ, which has its own dynamics and can roll [9,17]. The expectation value (VEV) of Φ then sets the mass of the graviton. We can imagine that at early times Φ has a large VEV, so that the graviton is very massive, and the universe self-inflates with a large Hubble constant. Then, at late times, Φ rolls to a smaller VEV, self-inflation ends and the graviton mass attains a small value consistent with present day measurements. This should be contrasted with the proposal of [17] in which the massive gravity terms are subdominant to the inflaton energy density.

In this Letter, we will see that, in practice, such an inflation-like implementation of massive gravity is difficult to achieve in this model. Pure dRGT theory has a constraint, stemming from the Bianchi identity, which forbids standard FRW evolution in the flat slicing [9] (the self-accelerating solutions are found in other slicings). There appears an analogous constraint in the variable mass theory, and this constraint, while it no longer forbids flat FRW solutions, implies that self-inflation cannot be sustained for a cosmologically relevant length of time. In addition, we show that non-inflationary cosmological solutions to this theory may exhibit future curvature singularities of the "big brake" type.

In the second half of this Letter (which can be read independently from the first), we consider the covariant galileon extension of massive gravity introduced in [15]. This theory has a new scalar degree of freedom π, which describes brane bending in an additional spatial dimension. (Unlike the variable mass theory, π here does not act to set the graviton mass, which is fixed. So we are not interested in self-inflation here, but are just exploring the basic cosmological equations.)

We derive the background cosmological equations for this theory, and find that the presence of the scalar leads to a more complicated constraint than in pure dRGT. We discuss the possible solutions in the case of zero and negative spatial curvature. We find that, as in pure dRGT theory, this constraint forbids flat FRW solutions. For an open FRW ansatz, however, solutions can exist and they come in two branches. The first branch consists of self-accelerating solutions that are identical to the self-accelerating solutions of pure dRGT theory. The second branch consists of novel solutions which are not found in pure massive gravity.

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2. Variable mass massive gravity

We start with variable mass massive gravity. This is dRGT theory in which the graviton mass squared is promoted to a scalar field Φ,

$$S = S_{\mathrm{EH}} + S_{\mathrm{mass}} + S_{\Phi}, \tag{1}$$

where

$$S_{\mathrm{EH}} = \frac{1}{2}M_{\mathrm{P}}^2 \int \mathrm{d}^4x \sqrt{-g}R, \tag{2}$$

$$S_{\mathrm{mass}} = M_{\mathrm{P}}^2 \int \mathrm{d}^4x \sqrt{-g}\Phi(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4), \tag{3}$$

$$S_{\Phi} = -\int \mathrm{d}^4x \sqrt{-g}\left[\frac{1}{2}g(\Phi)(\partial\Phi)^2 + V(\Phi)\right]. \tag{4}$$

Here α_3, α_4 are the two free parameters of dRGT theory. We have allowed for an arbitrary kinetic function $g(\Phi)$ and potential $V(\Phi)$, so that there is no loss of generality in the scalar sector. The mass term consists of the ghost-free combinations [3],

$$\begin{aligned}
\mathcal{L}_2 &= \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),\\
\mathcal{L}_3 &= \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),\\
\mathcal{L}_4 &= \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right),
\end{aligned} \tag{5}$$

where $K^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \sqrt{g^{\mu\sigma}\eta_{\sigma\nu}}$, $\eta_{\mu\nu}$ is the non-dynamical fiducial metric which we have taken to be Minkowski, and the square brackets are traces. To work in the gauge invariant formalism, we introduce four Stückelberg fields ϕ^a through the replacement $\eta_{\mu\nu} \to \partial_\mu\phi^a\partial_\mu\phi^b\eta_{ab}$.

Variable mass massive gravity was first considered in [9], and further studied in [17–20] (see also [16] for a more symmetric scalar extension of dRGT). dRGT gravity has been demonstrated to be ghost-free through a variety of different approaches [21–25], and the introduction of the scalar field does not introduce any new Boulware–Deser like ghost degrees of freedom into the system [17].

For cosmological applications we take a Friedmann–Robertson–Walker (FRW) ansatz for the metric, so that

$$\mathrm{d}s^2 = -N^2(t)\,\mathrm{d}t^2 + a^2(t)\Omega_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j, \tag{6}$$

where

$$\Omega_{ij} = \delta_{ij} + \frac{\kappa}{1-\kappa r^2}x^i x^j \tag{7}$$

is the line element for a maximally symmetric 3-space of curvature κ and $r^2 = x^2 + y^2 + z^2$. We also take the assumptions of homogeneity and isotropy for the scalar field,

$$\Phi = \Phi(t). \tag{8}$$

Consider first the case of flat Euclidean sections ($\kappa = 0$). We work in the gauge invariant formulation, and the Stückelberg degrees of freedom take the ansatz [9,10].

$$\phi^i = x^i, \qquad \phi^0 = f(t), \tag{9}$$

where $f(t)$, like $a(t)$, is a monotonically increasing function of t.

Inserting (6) and (9) into the action, we obtain the mini-superspace action

$$S_{\mathrm{EH}} = 3M_{\mathrm{P}}^2 \int \mathrm{d}t\left[-\frac{\dot{a}^2 a}{N}\right], \tag{10}$$

$$S_{\mathrm{mass}} = 3M_{\mathrm{P}}^2 \int \mathrm{d}t\,\Phi\left[NF(a) - \dot{f}G(a)\right], \tag{11}$$

$$S_{\Phi} = \int \mathrm{d}t\,a^3\left[\frac{1}{2}N^{-1}g(\Phi)\dot{\Phi}^2 - NV(\Phi)\right], \tag{12}$$

where

$$\begin{aligned}
F(a) &= a(a-1)(2a-1)\\
&\quad + \frac{\alpha_3}{3}(a-1)^2(4a-1) + \frac{\alpha_4}{3}(a-1)^3,
\end{aligned} \tag{13}$$

$$G(a) = a^2(a-1) + \alpha_3 a(a-1)^2 + \frac{\alpha_4}{3}(a-1)^3. \tag{14}$$

This mini-superspace action is invariant under time reparametrizations, under which f transforms like a scalar. Note that $a = 1$ corresponds to the arbitrary reference value of the scale factor which relates ϕ^i to x^i. Only the ratio of the scale factor relative to the reference value has physical meaning. Therefore we can take the reference value to be unity without loss of generality.

There are four equations of motion, obtained by varying with respect to F, N, Φ and a. As in GR, the Noether identity for time reparametrization invariance tells us that the acceleration equation obtained by varying with respect to a is a consequence of the other equations, so we may ignore it. After deriving the equations, we will fix the gauge $N = 1$ (this cannot be done directly in the action without losing equations).

Varying with respect to f we obtain the constraint pointed out in [18],

$$\Phi = \frac{\mathcal{C}}{G(a)}, \tag{15}$$

where $\mathcal{C}$ is an arbitrary integration constant. (Note that the analogous equation in the fixed mass theory implies that $a = \text{constant}$, so there are no evolving flat FRW solutions in that case [9].) Varying with respect to N, we obtain the Friedmann equation,

$$3M_{\mathrm{P}}^2\left[H^2 + \frac{\Phi F(a)}{a^3}\right] = \frac{1}{2}g(\Phi)\dot{\Phi}^2 + V, \tag{16}$$

and varying with respect to Φ we obtain the scalar field equation

$$\begin{aligned}
&g(\Phi)[\ddot{\Phi} + 3H\dot{\Phi}] + \frac{1}{2}g'(\Phi)\dot{\Phi}^2 + V'(\Phi)\\
&\quad = 3M_{\mathrm{P}}^2\left[\frac{F(a)}{a^3} - \dot{f}\frac{G(a)}{a^3}\right].
\end{aligned} \tag{17}$$

Rather than solving the coupled second-order Einstein-scalar equations of motion, one can instead reduce the system to a single first-order Friedmann equation. The relation (15) can be used to eliminate Φ and its first derivative from (16), which then becomes a first-order differential equation in a which determines the scale factor,

$$H^2 = \frac{V\left(\frac{\mathcal{C}}{G(a)}\right) - 3M_P^2\mathcal{C}\frac{F(a)}{a^3G(a)}}{3M_P^2 - \frac{1}{2}\mathcal{C}^2 g\left(\frac{\mathcal{C}}{G(a)}\right)\frac{G'(a)^2a^2}{G(a)^4}}. \tag{18}$$

Once we have solved for the scale factor, the scalar Φ is determined from (15) and the Stückelberg field f is determined by solving (17).[1]

[1] Note that in general the Stückelberg field cannot be chosen arbitrarily as in [18] but is non-trivially constrained by the choice of mass term, or in this case, kinetic function $g(\Phi)$.