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Nonsingular electrovacuum solutions with dynamically generated cosmological constant

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ABSTRACT

We consider static spherically symmetric configurations in a Palatini extension of General Relativity including R^2 and Ricci-squared terms, which is known to replace the central singularity by a wormhole in the electrovacuum case. We modify the matter sector of the theory by adding to the usual Maxwell term a nonlinear electromagnetic extension which is known to implement a confinement mechanism in flat space. One feature of the resulting theory is that the nonlinear electric field leads to a dynamically generated cosmological constant. We show that with this matter source the solutions of the model are asymptotically de Sitter and possess a wormhole topology. We discuss in some detail the conditions that guarantee the absence of singularities and of traversable wormholes.

1. Introduction

In 1955 John Wheeler [1] pointed out that well-known solutions of the Einstein equations, such as Reissner–Nordström or Kerr–Newman, could be interpreted as topologically nontrivial objects connecting different regions of the space–time through a wormhole. This work, and subsequently that of Morris and Wheeler [2], has led to several physically interesting suggestions, such as the mass-without-mass and charge-without-charge mechanisms. In this view, the electromagnetic field is not originated by a point-like charge, but instead it arises as a consequence of a flux crossing the wormhole mouth, creating the illusion of a negatively charged object on one side, and a positively charged object on the other, even though no real sources generate the field. The mass of this object would correspond to the energy stored in the electric field.

After the developments on traversable wormholes of Morris and Thorne [3], a great deal of attention has been payed to construct wormholes from a variety of energy-momentum sources (see [4] for a review). The standard approach consists on proposing a physically interesting wormhole metric and then drive the Einstein equations back to find the matter source that generates that geometry. More recently, motivated by the observational evidence in favor of an accelerated expansion of the Universe, wormhole solutions including a cosmological constant have experienced renewed interest [5] (see also the review [6] and references therein).

In a number of previous works [7–9] the electrovacuum solutions of a simple Palatini extension of General Relativity (GR) containing R^2 and Ricci-squared terms in four space–time dimensions have been studied in detail by some of us. It has been found that the internal point-like singularity that typically arises in the Reissner–Nordström solution of GR is generically replaced by a region of finite nonzero area that represents the mouth of a wormhole. The existence of this wormhole is an effect of the modified gravitational dynamics since the electromagnetic field does not violate any of the classical energy conditions. The higher-order curvature terms characterizing the gravitational sector of this theory usually arise in approaches to quantum gravity [10] and in the quantization of fields in curved space–time [11]. The novelty of our approach lies on the fact that we relax the Levi-Civita condition on the metric and obtain the field equations following the Palatini approach, in which metric and connection are regarded as two physically independent entities (see [12] for a pedagogical discussion of these concepts). This implies that both metric and connection must be determined by solving their respective equations obtained through the application of the variational principle

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on the action. In this context one finds ghost-free, second-order field equations with Minkowski space–time as a stable vacuum solution [13]. These properties of the quadratic Palatini theory arise because of the existence of invariant volumes associated with symmetric connections [14], and are in sharp contrast with the usual metric formulation of quadratic gravity, in which the connection is imposed to be the Levi-Civita one *a priori*, resulting in fourth-order field equations generically affected by ghosts. It is worth noting that while in the metric formalism there exists a family of Lagrangians leading to second-order equations (Lovelock gravities [15]), the extra terms become topological invariants in a four-dimensional space–time, and the theory provides the same dynamics as GR. In contrast, the Palatini approach yields nontrivial modified dynamics in four dimensions as long as matter fields are present because they play an active role in the determination of the connection. In vacuum, our theory boils down to GR, which is a manifestation of the universality of Einstein's equations observed in Palatini theories [16] (see also [14]).

In this work we present wormhole-type solutions with a dynamically generated cosmological constant in the quadratic Palatini extension of GR studied in [7–9]. The matter sector of our theory, responsible for the dynamical generation of a cosmological constant [17] (see also [18]), will be described by a nonlinear theory of electrodynamics given by the Lagrangian density

$$\varphi(X)=X-g\sqrt{2X}, \tag{1}$$

where $X=-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the field strength tensor of the vector potential A_μ. In the absence of gravity, the square root term in (1) naturally arises as a spontaneous breakdown of the scale symmetry of the Maxwell Lagrangian X [19], being $g>0$ an integration constant responsible for this breakdown. Moreover, when coupled to charged fermions, the model (1) produces a confinement effective potential $V(r)=-\frac{q}{r}+\frac{g}{\sqrt{2}}r$, which is of the form of the well-known Cornell potential, which has been used for the effective description of heavy quark–antiquark strong interactions [20]. This implements 't Hooft's description of linear confinement phenomena [21] since the electromagnetic field energy becomes a linear function of the electric displacement field in the infrared region. Let us note that one could start with the non-abelian version of (1) and for static spherically symmetric solutions the non-abelian theory effectively boils down to the abelian one, as pointed out in [19] (see also [22]). Here we shall see that the coupling of this field to a quadratic Palatini model yields a dynamically generated cosmological constant for large distances and, in addition, the GR singularity is generically replaced by a wormhole. This work complements the results recently found in [7,8], extending them to the case of nonlinear electrodynamics with modifications in the infrared sector.

2. General formalism

We consider a family of Palatini theories defined as

$$S[g,\Gamma,\psi_m]=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R,Q)+S_m[g,\psi_m], \tag{2}$$

where $f(R,Q)$ represents the gravity Lagrangian, κ^2 is a constant with suitable dimensions (in GR, $\kappa^2\equiv 8\pi G$), $\Gamma\equiv\Gamma^\alpha_{\mu\nu}$ is the independent connection, $g_{\alpha\beta}$ is the space–time metric, $R=g^{\mu\nu}R_{\mu\nu}$, $Q=g^{\mu\alpha}g^{\nu\beta}R_{\mu\nu}R_{\alpha\beta}$, $R_{\mu\nu}=R^\rho{}_{\mu\rho\nu}$, $R^\alpha{}_{\beta\mu\nu}=\partial_\mu\Gamma^\alpha_{\nu\beta}-\partial_\nu\Gamma^\alpha_{\mu\beta}+\Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\beta}-\Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\beta}$ is the Riemann tensor, and S_m the matter action. For concreteness, in this work we shall focus on the quadratic Lagrangian

$$f(R,Q)=R+l_P^2\big(aR^2+bQ\big), \tag{3}$$

where $l_P\equiv\sqrt{\hbar G/c^3}$ is the Planck length, and a and b are dimensionless parameters. Performing independent variations of the action (2) with respect to metric and connection leads to

$$f_R R_{\mu\nu}-\frac{f}{2}g_{\mu\nu}+2f_Q R_{\mu\alpha}R^\alpha{}_\nu=\kappa^2 T_{\mu\nu}, \tag{4}$$

$$\nabla_\beta\big[\sqrt{-g}\big(f_R g^{\mu\nu}+2f_Q R^{\mu\nu}\big)\big]=0, \tag{5}$$

where $f_R\equiv\frac{df}{dR}$, $f_Q\equiv\frac{df}{dQ}$, and $T_{\mu\nu}$ is the energy–momentum tensor of the matter. For simplicity, we have set the torsion to zero, which guarantees that $R_{[\mu\nu]}=0$ (see [14] for details). To solve Eqs. (4) and (5) we introduce the matrix $\hat{P}$ (whose components are $P_\mu{}^\nu\equiv R_{\mu\alpha}g^{\alpha\nu}$), which allows us to express (4) as

$$2f_Q\hat{P}^2+f_R\hat{P}-\frac{f}{2}\hat{I}=\kappa^2\hat{T}, \tag{6}$$

where $\hat{T}$ is the matrix representation of $T_\mu{}^\nu$. Since in this notation we have $R=[\hat{P}]_\mu{}^\mu$ and $Q=[\hat{P}^2]_\mu{}^\mu$, we can see (6) as a nonlinear algebraic equation for $\hat{P}=\hat{P}(\hat{T})$. Bearing in mind the relation $\hat{P}=\hat{P}(\hat{T})$, the connection equation (5) can be written as

$$\nabla_\beta\big[\sqrt{-g}g^{\mu\alpha}\Sigma_\alpha{}^\nu\big]=0, \tag{7}$$

where we have introduced the object

$$\Sigma_\alpha{}^\nu=\big(f_R\delta^\nu_\alpha+2f_Q P_\alpha{}^\nu\big). \tag{8}$$

Note that since $\Sigma_\alpha{}^\nu$ and $g^{\mu\alpha}$ do not depend explicitly on $\Gamma^\alpha_{\mu\nu}$, the connection in (5) appears linearly and can be solved by algebraic means. This motivates the introduction of a symmetric rank-two tensor $h^{\mu\nu}$ satisfying

$$\nabla_\beta\big[\sqrt{-g}g^{\mu\alpha}\Sigma_\alpha{}^\nu\big]=\nabla_\beta\big[\sqrt{-h}h^{\mu\nu}\big]=0. \tag{9}$$

The existence of $h_{\mu\nu}$ implies that $\Gamma^\alpha_{\mu\nu}$ is the Levi-Civita connection of $h_{\mu\nu}$ [14]. Comparison of the terms within brackets in this equation leads to the following solution

$$h^{\mu\nu}=\frac{g^{\mu\alpha}\Sigma_\alpha{}^\nu}{\sqrt{\det\hat{\Sigma}}},\qquad h_{\mu\nu}=(\sqrt{\det\hat{\Sigma}})\Sigma_\mu{}^\alpha g_{\alpha\nu}. \tag{10}$$

Using the definition of $\Sigma_\mu{}^\nu$ and the relations (10), it is easy to see that (4) can be written as $P_\mu{}^\alpha\Sigma_\alpha{}^\nu=R_{\mu\alpha}h^{\alpha\nu}\sqrt{\det\hat{\Sigma}}=\frac{f}{2}\delta^\nu_\mu+T_\mu{}^\nu$, which allows to express the metric field equations using $h_{\mu\nu}$ in the compact form

$$R_\mu{}^\nu(h)=\frac{1}{\sqrt{\det\hat{\Sigma}}}\left(\frac{f}{2}\delta_\mu{}^\nu+\kappa^2T_\mu{}^\nu\right). \tag{11}$$

3. The equations and solutions for nonlinear electrodynamics

For the sake of generality, we shall consider as the matter action in (2) nonlinear electrodynamics defined as

$$S_m=\frac{1}{8\pi}\int d^4x\sqrt{-g}\varphi(X,Y), \tag{12}$$

where $\varphi(X,Y)$ is a given function of the two field invariants $X=-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ and $Y=-\frac{1}{2}F_{\mu\nu}{*F^{\mu\nu}}$, where ${*F^{\mu\nu}}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual of $F_{\mu\nu}$. The matter field equations, $\nabla_\mu(\sqrt{-g}(\varphi_X F^{\mu\nu}+\varphi_Y{*F^{\mu\nu}}))=0$, admit, for purely electrostatic configurations, $E(r)=F^{tr}(r)$, and assuming a line element of the form $ds^2=g_{tt}dt^2+g_{rr}dr^2+r^2d\Omega^2$, a first integral of the form

$$F^{tr}=\frac{q}{r^2\varphi_X\sqrt{-g_{tt}g_{rr}}}, \tag{13}$$