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Constraining the symmetry energy at subsaturation densities using isotope binding energy difference and neutron skin thickness

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ABSTRACT

We show that the neutron skin thickness Δr_{np} of heavy nuclei is uniquely fixed by the symmetry energy density slope $L(\rho)$ at a subsaturation cross density $\rho_c \approx 0.11\ \text{fm}^{-3}$ rather than at saturation density ρ_0, while the binding energy difference ΔE between a heavy isotope pair is essentially determined by the magnitude of the symmetry energy $E_{\text{sym}}(\rho)$ at the same ρ_c. Furthermore, we find a value of $L(\rho_c)$ leads to a negative $E_{\text{sym}}(\rho_0)$–$L(\rho_0)$ correlation while a value of $E_{\text{sym}}(\rho_c)$ leads to a positive one. Using data on Δr_{np} of Sn isotopes and ΔE of a number of heavy isotope pairs, we obtain simultaneously $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV at 95% confidence level, whose extrapolation gives $E_{\text{sym}}(\rho_0) = 32.3 \pm 1.0$ MeV and $L(\rho_0) = 45.2 \pm 10.0$ MeV. The implication of these new constraints on the Δr_{np} of ^{208}Pb and the core–crust transition density in neutron stars is discussed.

1. Introduction

The determination of density dependence of the symmetry energy $E_{\text{sym}}(\rho)$, which characterizes the isospin dependent part of the equation of state (EOS) of asymmetric nuclear matter, is of fundamental importance due to its multifaceted roles in nuclear physics and astrophysics [1–4] as well as some issues of new physics beyond the standard model [5–7]. Due to the particularity of nuclear saturation density ρ_0 ($\sim 0.16\ \text{fm}^{-3}$), a lot of works have been devoted to constraining quantitatively the magnitude and density slope of the symmetry energy at ρ_0, i.e., $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$, by analyzing terrestrial nuclear experiments and astrophysical observations. Although significant progress has been made during the last decade, large uncertainties on $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ still exist (see, e.g., Refs. [2–4,8–11]). For example, while the value of $E_{\text{sym}}(\rho_0)$ is determined to be around 30 ± 4 MeV, the extracted $L(\rho_0)$ varies drastically from about 20 to 115 MeV, depending on the observables and analysis methods. To better understand the model dependence of the constraints and reduce the uncertainties is thus of critical importance and remains a big challenge in the community. In this Letter, we show the isotope binding energy difference and neutron skin thickness of heavy nuclei can be used to stringently constrain the subsaturation density behavior of the symmetry energy.

The neutron skin thickness $\Delta r_{np} = \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$ of heavy nuclei, i.e., the difference of the neutron and proton rms radii, has been shown to be a good probe of $E_{\text{sym}}(\rho)$ [12–23], and this provides a strong motivation for the Lead Radius Experiment (PREX) being performed at the Jefferson Laboratory to determine the $\langle r_n^2\rangle^{1/2}$ of ^{208}Pb to about 1% accuracy by measuring the parity-violating electroweak asymmetry in the elastic scattering of polarized electrons from ^{208}Pb [24–26]. Physically, the Δr_{np} depends on the pressure of neutron-rich matter in nuclei which will balance against the pressure due to nuclear surface tension [13]. Since the pressure of neutron-rich matter is essentially controlled by the density dependence of $E_{\text{sym}}(\rho)$ and the characteristic (average) density in finite nuclei is less than ρ_0 (see, e.g., Ref. [27]), one expects that the Δr_{np} should depend on the subsaturation density behaviors of the $E_{\text{sym}}(\rho)$ [2]. Brown and Typel [12] noted firstly that the Δr_{np} of heavy nuclei from model calculations is linearly correlated with the pressure of pure neutron matter at a subsaturation density of 0.1 fm^{-3}. The linear correlation of the Δr_{np} with both $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ has also been observed in mean-field calculations [12–21] using many existing nuclear effective interactions.

Recently, a remarkable negative correlation between $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ has been obtained by analyzing existing data on Δr_{np} of Sn isotopes [28], showing a striking contrast to other constraints that essentially give a positive $E_{\text{sym}}(\rho_0)$–$L(\rho_0)$ correlation (see, e.g., Refs. [9–11]). A negative correlation between $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ has also been observed for a fixed value of Δr_{np} in

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^{208}Pb [29]. It is thus of great interest to understand the physics behind this negative $E_{\rm sym}(\rho_0)$–$L(\rho_0)$ correlation. Using a recently developed correlation analysis method [28], we show here that the Δr_{np} of heavy nuclei is uniquely fixed by the $L(\rho)$ at a subsaturation cross density $\rho_c \approx 0.11$ fm^{-3}, which naturally leads to a negative $E_{\rm sym}(\rho_0)$–$L(\rho_0)$ correlation. Furthermore, we demonstrate that the binding energy difference between a heavy isotope pair is essentially determined by the magnitude of $E_{\rm sym}(\rho)$ at the same ρ_c. For the first time, we obtain simultaneously in the present Letter stringent constraints on both the magnitude and density slope of the $E_{\rm sym}(\rho)$ at $\rho_c \approx 0.11$ fm^{-3} by analyzing data of the isotope binding energy difference for heavy nuclei and Δr_{np} of Sn isotopes, which has important implications on the values of $E_{\rm sym}(\rho_0)$ and $L(\rho_0)$, the Δr_{np} of ^{208}Pb, and the core–crust transition density ρ_t of neutron stars.

2. Model and method

The EOS of asymmetric nuclear matter at baryon density ρ and isospin asymmetry $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$, given by its binding energy per nucleon, can be expanded to 2nd-order in δ as

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\delta^2 + O\left(\delta^4\right), \tag{1}$$

where $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

$$E_{\rm sym}(\rho) = \frac{1}{2!}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\bigg|_{\delta=0}. \tag{2}$$

Around a reference density ρ_r, the $E_{\rm sym}(\rho)$ can be characterized by using the value of $E_{\rm sym}(\rho_r)$ and the density slope parameter $L(\rho_r) = 3\rho_r \frac{\partial E_{\rm sym}(\rho)}{\partial\rho}|_{\rho=\rho_r}$, i.e.,

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_r) + L(\rho_r)\chi_r + O\left(\chi_r^2\right), \tag{3}$$

with $\chi_r = (\rho - \rho_r)/3\rho_r$.

In the present Letter, we use the Skyrme–Hartree–Fock (SHF) approach with the so-called standard form of Skyrme force (see, e.g., Ref. [30]) which includes 10 parameters, i.e., the 9 Skyrme force parameters σ, $t_0 - t_3$, $x_0 - x_3$, and the spin–orbit coupling constant W_0. This standard SHF approach has been shown to be very successful in describing the structure of finite nuclei, especially global properties such as binding energies and charge radii [30–32]. Instead of using directly the 9 Skyrme force parameters, we can express them explicitly in terms of 9 macroscopic quantities, i.e., ρ_0, $E_0(\rho_0)$, the incompressibility K_0, the isoscalar effective mass $m^*_{s,0}$, the isovector effective mass $m^*_{v,0}$, $E_{\rm sym}(\rho_r)$, $L(\rho_r)$, G_S, and G_V. The G_S and G_V are respectively the gradient and symmetry-gradient coefficients in the interaction part of the binding energies for finite nuclei defined as

$$E_{\rm grad} = G_S(\nabla\rho)^2/(2\rho) - G_V\left[\nabla(\rho_n - \rho_p)\right]^2/(2\rho). \tag{4}$$

Then, by varying individually these macroscopic quantities within their known ranges, we can examine more transparently the correlation of properties of finite nuclei with each individual macroscopic quantity. Recently, this correlation analysis method has been successfully applied to study the neutron skin [28] and giant monopole resonance of finite nuclei [33], the higher order bulk characteristic parameters of asymmetric nuclear matter [34], and the relationship between the nuclear matter symmetry energy and the symmetry energy coefficient in the mass formula [35], where the reference density ρ_r has been set to be ρ_0.

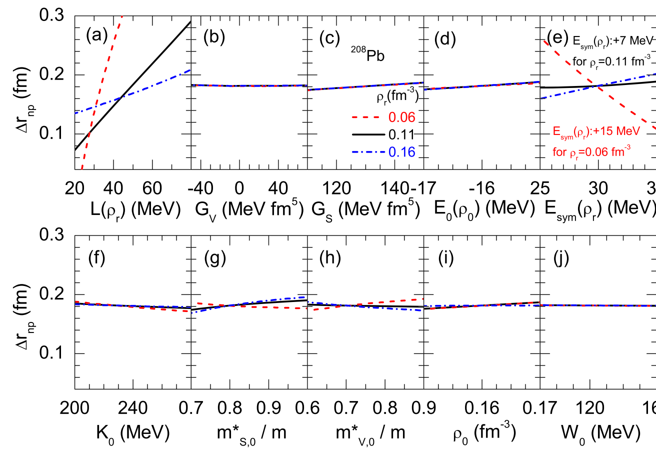

Fig. 1. (Color online.) The Δr_{np} of ^{208}Pb from SHF with MSL0 by varying individually $L(\rho_r)$ (a), G_V (b), G_S (c), $E_0(\rho_0)$ (d), $E_{\rm sym}(\rho_r)$ (e), K_0 (f), $m^*_{s,0}$ (g), $m^*_{v,0}$ (h), ρ_0 (i), and W_0 (j) for $\rho_r = 0.06$, 0.11, and 0.16 fm^{-3}. The $E_{\rm sym}(\rho_r)$ is shifted by adding 15 and 7 MeV for $\rho_r = 0.06$ and 0.11 fm^{-3}, respectively.

3. Results and discussions

To examine the correlation of the Δr_{np} of heavy nuclei with each macroscopic quantity, especially on $E_{\rm sym}(\rho_r)$ and $L(\rho_r)$, we show in Fig. 1 the Δr_{np} of ^{208}Pb from SHF with the Skyrme force MSL0 [28] by varying individually $L(\rho_r)$, G_V, G_S, $E_0(\rho_0)$, $E_{\rm sym}(\rho_r)$, K_0, $m^*_{s,0}$, $m^*_{v,0}$, ρ_0, and W_0 within their empirical uncertain ranges, namely, varying one quantity at a time while keeping all others at their default values in MSL0, for three values of ρ_r, i.e., $\rho_r = 0.06$, 0.11, and 0.16 fm^{-3}. It is seen from Fig. 1 that the Δr_{np} of ^{208}Pb exhibits a strong correlation with $L(\rho_r)$ and $E_{\rm sym}(\rho_r)$ while much weak correlation with other macroscopic quantities. In particular, while the Δr_{np} always increases with $L(\rho_r)$, it can increase or decrease with $E_{\rm sym}(\rho_r)$, depending on the value of ρ_r. Most interestingly, one can see that, for $\rho_r = \rho_c \approx 0.11$ fm^{-3}, the Δr_{np} becomes essentially independent of $E_{\rm sym}(\rho_r)$ and only sensitive to the value of $L(\rho_r)$. These features imply that the $L(\rho_c)$ is a unique quantity to determine the Δr_{np} of heavy nuclei, and the experimental data on Δr_{np} of heavy nuclei can put strong limit on the value of $L(\rho_c)$.

In order to determine the magnitude of the symmetry energy at ρ_c, i.e., $E_{\rm sym}(\rho_c)$, we propose here to use the difference of binding energy per nucleon between an isotope pair, denoted as ΔE. Before detailed quantitative calculations, it is instructive to estimate the ΔE from the well-known semiempirical nuclear mass formula in which the binding energy per nucleon for a nucleus with N neutrons and Z protons ($A = N + Z$) can be approximated by

$$E(N,Z) = a_{\rm vol} + a_{\rm surf}A^{-1/3} + a_{\rm sym}(A)\left(\frac{N-Z}{A}\right)^2 + a_{\rm Coul}\frac{Z(Z-1)}{A^{4/3}} + E_{\rm pair}. \tag{5}$$

For heavy spherical even–even nuclei, the ΔE can then be expressed approximately as

$$\begin{aligned}\Delta E &= E(N+\Delta N, Z) - E(N,Z) \\ &\approx a_{\rm sym}(A)\frac{4Z(N-Z)}{A^2}\times\frac{\Delta N}{A} \\ &\quad - a_{\rm Coul}\frac{4Z(Z-1)}{3A^{4/3}}\times\frac{\Delta N}{A} - \frac{a_{\rm surf}}{3A^{1/3}}\times\frac{\Delta N}{A},\end{aligned} \tag{6}$$

if we assume ΔN is significantly less than A and $N - Z$ and $a_{\rm sym}(A + \Delta N) \approx a_{\rm sym}(A)$. Since the Coulomb term is relatively well