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# The five dimensional universal hypermultiplet and the cosmological constant problem

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ABSTRACT

We model the universe as a 3-brane embedded in five dimensional spacetime with $\mathcal{N}=2$ supersymmetry. The presence of the scalar fields of the universal hypermultiplet in the bulk results in a positive pressure effectively reducing the value of the cosmological constant and thereby providing a possible answer as to why the measured value of the cosmological constant is many orders of magnitude smaller than predicted from the vacuum energy. The solution allows for any number of parallel branes to exist and relates their cosmological constants (as well as matter densities and radiation pressures) to the value of the dilaton in the extra dimension. The results we find can be thought of as first order approximations, satisfying supersymmetry breaking and the Bogomol'nyi–Prasad–Sommerfield (BPS) conditions in the bulk only.

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## 1. Introduction

The 1998 discovery that the universe's expansion is accelerating [1,2] took the scientific community by storm. The term 'dark energy' was coined to account for the source of the mysterious negative pressure causing the acceleration. From the point of view of the Einstein gravitational field equations, this energy is modeled by the presence of a positive cosmological constant $\Lambda$. Although many different models explaining the origin of this dark energy exist, it is somewhat accepted today that the main contributor is almost certainly the energy of the vacuum as calculated from the standard model (see, for example, [3] and references within). There is a problem with this, however, as the vacuum's contribution seems to be many orders of magnitude *larger* than is actually observed, at least at the present epoch. The most recent estimates (e.g. [4]) put the difference to about 120 orders of magnitude; a catastrophic discrepancy that cannot simply be approximated away! Many solutions, as well as partial solutions, to this problem have been proposed. Of particular interest to us are models that embed our universe in a higher dimensional space, usually as a 'string-theoretic' 3-brane. Such models of 'brane cosmology' exist in abundance and range over various categories from non-supersymmetric solutions, to solutions that explain the acceleration with zero cosmological constant, and many others (see, for example, [5–15]).

In this Letter we consider five dimensional $\mathcal{N}=2$ supergravity with the scalar fields of the universal hypermultiplet (UH). Our universe is modeled as a 3-brane satisfying the conditions of homogeneous and isotropic expansion, i.e. the standard Robertson–Walker metric construction. The current work generalizes a static 3-brane solution that was found in [16] to one with time dependence. We assume the simple form of constant matter density and radiation pressure in the universe, as well as a possible cosmological constant in the bulk. As such our model does not chart the entire history of the universe, but rather just an effective snapshot of a specific epoch. We find that a generalization of our result to one with multiple brane solutions (parallel universes) is quite straightforward and is in fact almost demanded by the equations. The observed numerical values of the cosmological constant, matter and radiation pressure densities are tied in to the values of the UH fields in the bulk and as such change from brane to brane. Our universe's observed values are the way they are by virtue of our presence in this particular brane rather than another. Furthermore, we find that the values of these constants are controlled by a free parameter $B$ that is not determined by the model. But a lower bound may simply be placed on it by requiring that the fields vanish at bulk infinity. The entire result can be thought of as a first order approximation in the following sense: The brane's matter and radiation contents are not included in the theory's action and as such do not couple to the gravitini; so the supersymmetry variation equations are valid only in the bulk. This opens up a variety of questions and directions of possible future research, as will be subsequently discussed.

The Letter is organized as follows: In Section 2 we review the five dimensional supergravity theory formulated in the language

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of split complex numbers. In Section 3 we introduce our metric ansatz and calculate the components of the Einstein field equations. It is shown that two possible solutions exist. Only one is discussed in this Letter, the other is deferred to future work. The UH fields are found in Section 4. Finally, the modified Friedmann equations are derived and the full solution is summarized in Section 5.

## 2. Five dimensional $\mathcal{N}=2$ supergravity

The dimensional reduction of $D=11$ supergravity theory (see [17] for a review) over a rigid Calabi–Yau 3-fold (constant Kähler and complex structure moduli) yields an ungauged $\mathcal{N}=2$ supersymmetric gravity theory in $D=5$ with a matter sector comprised of four scalar fields and their superpartners; collectively known as the universal hypermultiplet. These are: the dilaton $\sigma$ (volume modulus of the Calabi–Yau space), the universal axion $\varphi$, the pseudo-scalar axion $\chi$ and its complex conjugate $\bar{\chi}$ [18,19]. In [16], it was argued that another way to represent the theory is by employing split-complex numbers, as opposed to the more traditional complex representation. To do so, the axions are defined as follows

$$\chi = \chi_1 + j\chi_2$$
$$\bar{\chi} = \chi_1 - j\chi_2, \tag{1}$$

where $(\chi_1, \chi_2)$ are real functions and the 'imaginary' number $j$ is defined by $j^2 = +1$ but is *not* equal to $\pm 1$. In this representation, the bosonic action of the theory is:

$$S_5 = \int_5 \left[ R \star \mathbf{1} - \frac{1}{2} d\sigma \wedge \star d\sigma - e^{\sigma} d\chi \wedge \star d\bar{\chi} - \frac{1}{2} e^{2\sigma} \left( d\varphi + \frac{j}{2} f \right) \wedge \star \left( d\varphi - \frac{j}{2} \bar{f} \right) \right], \tag{2}$$

where $\star$ is the $D=5$ Hodge duality operator and we have defined

$$f = (\chi\, d\bar{\chi} - \bar{\chi}\, d\chi)$$
$$\bar{f} = -f, \tag{3}$$

for brevity.[1] The variation of the action yields the following field equations for $\sigma$, $(\chi, \bar{\chi})$ and $\varphi$ respectively

$$(\Delta\sigma) \star \mathbf{1} - e^{\sigma} d\chi \wedge \star d\bar{\chi} - e^{2\sigma} \left( d\varphi + \frac{j}{2} f \right) \wedge \star \left( d\varphi - \frac{j}{2} \bar{f} \right) = 0 \tag{4}$$

$$d^{\dagger} \left[ e^{\sigma} d\chi + j e^{2\sigma} \chi \left( d\varphi + \frac{j}{2} f \right) \right] = 0 \tag{5}$$

$$d^{\dagger} \left[ e^{\sigma} d\bar{\chi} - j e^{2\sigma} \bar{\chi} \left( d\varphi - \frac{j}{2} \bar{f} \right) \right] = 0 \tag{6}$$

$$d^{\dagger} \left[ e^{2\sigma} \left( d\varphi + \frac{j}{2} f \right) \right] = 0, \tag{7}$$

where $d^{\dagger}$ is the adjoint exterior derivative and $\Delta$ is the Laplace–De Rahm operator. The full action is invariant under the following set of supersymmetry (SUSY) transformations of the gravitini $\psi$ and hyperini $\xi$ fermionic fields respectively $(M = 0, \ldots, 4)$:

$$\delta_\epsilon \psi^1 = D\epsilon_1 + \frac{j}{4} e^{\sigma} \left( d\varphi + \frac{j}{2} f \right) \epsilon_1 - \frac{j}{4} e^{\frac{\sigma}{2}} d\chi\, \epsilon_2 \tag{8}$$

$$\delta_\epsilon \psi^2 = D\epsilon_2 - \frac{j}{4} e^{\sigma} \left( d\varphi - \frac{j}{2} \bar{f} \right) \epsilon_2 + \frac{j}{4} e^{\frac{\sigma}{2}} d\bar{\chi}\, \epsilon_1 \tag{9}$$

$$\delta_\epsilon \xi_1 = \frac{1}{2} \left[ (\partial_M \sigma) - j e^{\sigma} \left( \partial_M \varphi + \frac{j}{2} f_M \right) \right] \Gamma^M \epsilon_1 + j \frac{e^{\frac{\sigma}{2}}}{\sqrt{2}} (\partial_M \chi) \Gamma^M \epsilon_2 \tag{10}$$

$$\delta_\epsilon \xi_2 = \frac{1}{2} \left[ (\partial_M \sigma) + j e^{\sigma} \left( \partial_M \varphi - \frac{j}{2} \bar{f}_M \right) \right] \Gamma^M \epsilon_2 - j \frac{e^{\frac{\sigma}{2}}}{\sqrt{2}} (\partial_M \bar{\chi}) \Gamma^M \epsilon_1, \tag{11}$$

where

$$D = d + \frac{1}{4} \omega^{\hat{M}\hat{N}} \Gamma_{\hat{M}\hat{N}}, \tag{12}$$

is the usual covariant derivative, the $\Gamma$'s are the $D=5$ Dirac matrices, $(\epsilon_1, \epsilon_2)$ are the $\mathcal{N}=2$ SUSY spinors, $\omega$ is the spin connection and the hatted indices are frame indices in a flat tangent space.

## 3. Spacetime background

From the point of view of $D=5$ SUGRA, our universe may be modeled by a 3-brane in a five dimensional bulk. This implies the embedding of the Robertson–Walker metric in five dimensional space as follows

$$ds_5^2 = e^{2C\sigma(y)} \left[ -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right) \right] + e^{2B\sigma(y)} b^2(t)\, dy^2 \tag{13}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the line element of the unit sphere $S^2$, the quantities $C$ and $B$ are constants to be determined, $a(t)$ is the usual scale factor and $b(t)$ is a bulk scale factor. The solution we seek should have $a(t) \sim e^{Ht}$, where the positive constant $H$ is the current value of the Hubble parameter, to account for the accelerating phase of the universe. Also, since current data [20] seems to imply that on a large scale our universe is essentially flat, we will then take the 'curvature' factor $k$ to be zero.

The matter content of this five dimensional space is comprised of the UH fields in the bulk, represented by the following stress tensor $(\mu, \nu = t, r, \theta, \phi)$:

$$T^{\text{bulk}}_{\mu\nu} = \frac{1}{4} g_{\mu\nu} (\partial_y \sigma)(\partial^y \sigma) + \frac{1}{2} g_{\mu\nu} e^{\sigma} (\partial_y \chi)(\partial^y \bar{\chi}) + \frac{1}{4} g_{\mu\nu} e^{2\sigma} \left( \partial_y \varphi + \frac{j}{2} f_y \right) \left( \partial^y \varphi - \frac{j}{2} \bar{f}^y \right)$$

$$T^{\text{bulk}}_{yy} = \frac{1}{4} g_{yy} (\partial_y \sigma)(\partial^y \sigma) - \frac{1}{2} (\partial_y \sigma)(\partial_y \sigma) + \frac{1}{2} e^{\sigma} g_{yy} (\partial_y \chi)(\partial^y \bar{\chi}) - e^{\sigma} (\partial_y \chi)(\partial_y \bar{\chi}) + \frac{1}{4} e^{2\sigma} g_{yy} \left( \partial_y \varphi + \frac{j}{2} f_y \right) \left( \partial^y \varphi - \frac{j}{2} \bar{f}^y \right) - \frac{1}{2} e^{2\sigma} \left( \partial_y \varphi + \frac{j}{2} f_y \right) \left( \partial_y \varphi - \frac{j}{2} \bar{f}_y \right), \tag{14}$$

in addition to the usual perfect fluid stress tensor on the brane:

$$\mathrm{T}^{\text{3brane}}_{\mu\nu} = \rho U_\mu U_\nu + P (g_{\mu\nu} + U_\mu U_\nu), \tag{15}$$

[1] It is noted that the action in this form suffers from the presence of high energy ghost-like terms. However, as we will see, these are exactly canceled in our solution and as such have no physical effect on our conclusions.