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Coulomb gauge model for hidden charm tetraquarks

W. Xie [a], L.Q. Mo [a], Ping Wang [a,b], Stephen R. Cotanch [c,*]

[a] Institute of High Energy Physics, CAS, P.O. Box 918(4), Beijing 100049, China
[b] Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China
[c] Department of Physics, North Carolina State University, Raleigh, NC 27695-8202, USA

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ABSTRACT

The spectrum of tetraquark states with hidden charm is studied within an effective Coulomb gauge Hamiltonian approach. Of the four independent color schemes, two are investigated, the $(q\bar{c})_1(c\bar{q})_1$ singlet–singlet (molecule) and the $(qc)_3(\bar{q}\bar{c})_3$ triplet–triplet (diquark), for selected J^{PC} states using a variational method. The predicted masses of triplet–triplet tetraquarks are roughly a GeV heavier than the singlet–singlet states. There is also an interesting flavor dependence with $(q\bar{q})_1(c\bar{c}_1)$ states about half a GeV lighter than $(q\bar{c})_1(\bar{q}c)_1$. The lightest 1^{++} and 1^{--} predictions are in agreement with the observed $X(3872)$ and $Y(4008)$ masses suggesting they are molecules with $\omega J/\psi$ and ηh_c, rather than $D^*\bar{D}^*$ and $D\bar{D}$, type structure, respectively. Similarly, the lightest isovector 1^{++} molecule, having a $\rho J/\psi$ flavor composition, has mass near the recently observed charged $Z_c(3900)$ value. These flavor configurations are consistent with observed X, Y and Z_c decays to $\pi\pi J/\psi$.

1. Introduction

With the recent observed Higgs boson candidate at the LHC, the weak sector of the standard model may be approaching closure. However the strong interaction component is far from complete, especially hadronic structure where intense searches are under way for unconventional (non-$q\bar{q}$ or qqq) states. Indeed QCD allows for other color singlet combinations, such as glueballs, hybrid mesons, tetraquarks, pentaquarks and dibaryons. As early as 1977 $qq\bar{q}\bar{q}$ tetraquark states were proposed [1] and today there are now good candidates, the $\pi_1(1400)$ and $\pi_1(1600)$ [2,3], listed in the PDG meson summary table [4]. These states have unconventional quantum numbers $J^{PC} = 1^{-+}$ which are not possible for a $q\bar{q}$ system and have been reasonably described [5] as $(q\bar{q})_1(q\bar{q})_1$ color singlet–singlet states referred to as molecules. In the heavy quark sector no states with unconventional quantum numbers have yet to be confirmed. However, there are hidden charm states, some also listed in the PDG summary, which cannot be described as $c\bar{c}$ mesons. These X, Y and Z particles are charmonium-like but are not consistent with any $c\bar{c}$ spectrum predictions. The lightest is $X(3872)$ which was discovered by Belle in 2003 [6] and has a narrow peak near 3872 MeV in the $\pi^+\pi^- J/\psi$ invariant mass distribution from $B^- \to K^-\pi^+\pi^- J/\psi$ decay. This discovery was confirmed by BaBar in the same decay process [7]. Subsequently, additional X, Y and Z resonances were found, most recently [8] the first charged charmonium-like structure $Z_c(3900)$ which requires at least four quarks to have a non-zero electric charge. States with confirmed J^{PC} are listed in Table 1.

Theoretically, there have been many X, Y and Z investigations using a variety of methods, such as perturbative NRQCD [9,10], lattice QCD [11,12], effective field theory [13–15], QCD sum rule [16,17] and potential models [18–21], all reviewed in detail elsewhere [22,23]. One plausible conjecture [24–26] is that these states are tetraquarks containing light, $q = u$ or d, and charm, c, quarks in different color schemes. In this Letter we investigate this further and consider two color combinations, the somewhat conventional molecular $(q\bar{c})_1(c\bar{q})_1$ singlet–singlet [23] and the more exotic $(qc)_{\bar{3}}(\bar{q}\bar{c})_3$ triplet–triplet (diquark) [27]. We reserve the term exotic for the latter since it entails quarks in intermediate color states that are not singlets.

Our work is an extension of a previous light tetraquark study [5] utilizing the Coulomb gauge [CG] model, first implemented to predict a glueball spectrum [28,29] that was in good agreement with lattice QCD data. In this approach the exact QCD Hamiltonian in the Coulomb gauge is replaced with an effective field theoretical relativistic Hamiltonian. The bare parton (current quark and gluon) field operators are dressed by a Bardeen, Cooper and Schrieffer [BCS] rotation and variational ground state (vacuum) minimization. This generates a non-trivial vacuum in which chiral symmetry is dynamically broken and quark/gluon constituent masses and condensates emerge. The hadrons are represented as quasiparticle excitations using many-body techniques such as

* Corresponding author.
E-mail address: cotanch@ncsu.edu (S.R. Cotanch).

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Table 1
Properties of X, Y and Z mesons with established J^{PC}.

State	Mass (MeV)	Γ (MeV)	J^{PC}	Decay	Production $B \to$
$X(3872)$	3871.4 ± 0.6	< 2.3	1^{++}	$\pi^+\pi^- J/\psi$	$K X(3872)$
			or 2^{-+}	$\gamma J/\psi$	$p\bar{p}$
$Z(3930)$	3929 ± 5	29 ± 10	2^{++}	$D\bar{D}$	$\gamma\gamma$
$Y(4008)$	4008^{+82}_{-49}	226^{+97}_{-80}	1^{--}	$\pi^+\pi^- J/\psi$	e^+e^-
$Y(4260)$	4260 ± 12	83 ± 22	1^{--}	$\pi^+\pi^- J/\psi$	e^+e^-
$Y(4360)$	4361 ± 13	74 ± 18	1^{--}	$\pi^+\pi^- \psi'$	e^+e^-
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$\pi^+\pi^- \psi'$	e^+e^-

Tamm–Dancoff and random phase approximations. This method was subsequently applied to mesons [30–32], hybrids [33–36] and light tetraquark states [5,37].

This Letter is organized into five sections. Section 2 specifies the CG model which is then applied to hidden charm tetraquark states in Section 3. Numerical results are presented and discussed in Section 4 followed by a summary, Section 5, detailing conclusions and future work.

2. The QCD Coulomb gauge model

The CG model provides a comprehensive, systematic approach to hadron structure. It is applicable to both quarks and gluons, light and heavy meson and baryon ground and excited states for any flavor and exotic systems involving different combinations of quarks and gluons in various color schemes. It also permits consistent Hamiltonian dynamical mixing between states of entirely different dressed partons while providing insight characteristic of a wave function picture. As further discussed below there are additional attractive theoretical features and it is significant to note that there are no free model parameters as the two dynamical constants, the string tension σ and Coulomb interaction α_s, are predetermined from the literature.

The exact QCD Hamiltonian in the Coulomb gauge [38] is

$$H_{\mathrm{QCD}} = H_q + H_g + H_{qg} + H_C, \tag{1}$$

$$H_q = \int d\mathbf{x}\, \Psi^\dagger(\mathbf{x})[-i\boldsymbol{\alpha}\cdot\nabla + \beta m]\Psi(\mathbf{x}), \tag{2}$$

$$H_g = \frac{1}{2}\int d\mathbf{x}\left[\mathcal{J}^{-1}\boldsymbol{\Pi}^a(\mathbf{x})\cdot\mathcal{J}\boldsymbol{\Pi}^a(\mathbf{x}) + \mathbf{B}^a(\mathbf{x})\cdot\mathbf{B}^a(\mathbf{x})\right], \tag{3}$$

$$H_{qg} = g\int d\mathbf{x}\,\mathbf{J}^a(\mathbf{x})\cdot\mathbf{A}^a(\mathbf{x}), \tag{4}$$

$$H_C = -\frac{g^2}{2}\int d\mathbf{x}\,d\mathbf{y}\,\rho^a(\mathbf{x})\mathcal{J}^{-1}K^{ab}(\mathbf{x},\mathbf{y})\mathcal{J}\rho^b(\mathbf{y}), \tag{5}$$

where g is the QCD coupling constant, Ψ is the quark field with current quark mass m, $A^a = (\mathbf{A}^a, A_0^a)$ are the gluon fields satisfying the Coulomb (transverse) gauge condition, $\nabla\cdot\mathbf{A}^a = 0$ ($a = 1,2,\ldots,8$), $\boldsymbol{\Pi}^a = -\mathbf{E}^a_{tr}$ are the conjugate momenta and

$$\mathbf{E}^a_{tr} = -\dot{\mathbf{A}}^a + g\left(1 - \nabla^{-2}\nabla\nabla\cdot\right)f^{abc}A_0^b\mathbf{A}^c, \tag{6}$$

$$\mathbf{B}^a = \nabla\times\mathbf{A}^a + \frac{1}{2}gf^{abc}\mathbf{A}^b\times\mathbf{A}^c \tag{7}$$

are the non-abelian chromodynamic fields. The color densities $\rho^a(\mathbf{x})$ and quark currents $\mathbf{J}^a(\mathbf{x})$ are

$$\rho^a(\mathbf{x}) = \Psi^\dagger(\mathbf{x})T^a\Psi(\mathbf{x}) + f^{abc}\mathbf{A}^b(\mathbf{x})\cdot\boldsymbol{\Pi}^c(\mathbf{x}), \tag{8}$$

$$\mathbf{J}^a(\mathbf{x}) = \Psi^\dagger(\mathbf{x})\boldsymbol{\alpha}T^a\Psi(\mathbf{x}), \tag{9}$$

where $T^a = \frac{\lambda^a}{2}$ and f^{abc} are the $SU(3)$ color matrices and structure constants, respectively. The Faddeev–Popov determinant, $\mathcal{J} = \det(\mathcal{M})$, of the matrix $\mathcal{M} = \nabla\cdot\mathbf{D}$, with covariant derivative $\mathbf{D}^{ab} = \delta^{ab}\nabla - gf^{abc}\mathbf{A}^c$, is a measure of the gauge manifold curvature and the kernel in Eq. (5) is given by $K^{ab}(\mathbf{x},\mathbf{y}) = \langle\mathbf{x},a|\mathcal{M}^{-1}\nabla^2\mathcal{M}^{-1}|\mathbf{y},b\rangle$. The bare parton fields have the following normal mode expansions (bare quark spinors u, v, helicity, $\lambda = \pm 1$, and color vectors $\hat{\boldsymbol{\epsilon}}_{\mathcal{C}=1,2,3}$)

$$\Psi(\mathbf{x}) = \int\frac{d\mathbf{k}}{(2\pi)^3}\left[u_\lambda(\mathbf{k})b_{\lambda\mathcal{C}}(\mathbf{k}) + v_\lambda(-\mathbf{k})d^\dagger_{\lambda\mathcal{C}}(-\mathbf{k})\right]e^{i\mathbf{k}\cdot\mathbf{x}}\hat{\boldsymbol{\epsilon}}_{\mathcal{C}}, \tag{10}$$

$$\mathbf{A}^a(\mathbf{x}) = \int\frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{\sqrt{2k}}\left[\mathbf{a}^a(\mathbf{k}) + \mathbf{a}^{a\dagger}(-\mathbf{k})\right]e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{11}$$

$$\boldsymbol{\Pi}^a(\mathbf{x}) = -i\int\frac{d\mathbf{k}}{(2\pi)^3}\sqrt{\frac{k}{2}}\left[\mathbf{a}^a(\mathbf{k}) - \mathbf{a}^{a\dagger}(-\mathbf{k})\right]e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{12}$$

Our model's starting point is the Coulomb gauge QCD Hamiltonian. We then make the following substitutions: 1) replace the exact Coulomb kernel with a calculable confining potential; 2) use the lowest order, unit value for the Faddeev–Popov determinant. Therefore, H_C in the CG model Hamiltonian becomes

$$H_C^{\mathrm{CG}} = -\frac{1}{2}\int d\mathbf{x}\,d\mathbf{y}\,\rho^a(\mathbf{x})\hat{V}\left(|\mathbf{x}-\mathbf{y}|\right)\rho^a(\mathbf{y}). \tag{13}$$

The confining and leading canonical interaction is represented by a Cornell type potential $\hat{V}(r) = -\alpha_s/r + \sigma r$. Previous studies with this interaction were in good agreement with both lattice glueball masses [28] and the observed meson spectrum [31]. Performing a Fourier transform, the potential in momentum space is

$$V\left(|\mathbf{k}|\right) = -\frac{4\pi\alpha_s}{k^2} - \frac{8\pi\sigma}{k^4}. \tag{14}$$

Next, hadron states are expressed as BCS vacuum $|\Omega\rangle$ excitations involving dressed quark (antiquark) Fock operators $B^\dagger_{\lambda\mathcal{C}}$, $D_{\lambda\mathcal{C}}$ related to the bare operators $b_{\lambda\mathcal{C}}$, $d^\dagger_{\lambda\mathcal{C}}$ by the rotation

$$B_{\lambda\mathcal{C}}(\mathbf{k}) = \cos\frac{\theta(k)}{2}b_{\lambda\mathcal{C}}(\mathbf{k}) - \lambda\sin\frac{\theta(k)}{2}d^\dagger_{\lambda\mathcal{C}}(-\mathbf{k}), \tag{15}$$

$$D_{\lambda\mathcal{C}}(-\mathbf{k}) = \cos\frac{\theta(k)}{2}d_{\lambda\mathcal{C}}(-\mathbf{k}) + \lambda\sin\frac{\theta(k)}{2}b^\dagger_{\lambda\mathcal{C}}(\mathbf{k}), \tag{16}$$

where $\theta(k)$ is the BCS angle. Correspondingly, the dressed Dirac spinors are

$$\mathcal{U}_\lambda(\mathbf{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{1+\sin\phi(k)}\chi_\lambda \\ \sqrt{1-\sin\phi(k)}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\chi_\lambda\end{pmatrix}, \tag{17}$$

$$\mathcal{V}_\lambda(-\mathbf{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix}-\sqrt{1-\sin\phi(k)}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\chi_\lambda \\ \sqrt{1+\sin\phi(k)}\chi_\lambda\end{pmatrix}, \tag{18}$$

where $\phi(k)$ is the gap angle from the gap equation that minimizes the energy of the BCS vacuum [35] and is related to $\theta(k)$ by $\tan(\phi(k) - \theta(k)) = m/k$.

The four model parameters have all been predetermined [32]. The light and charm bare quark masses are 5 MeV and 1350 MeV, respectively, while the two dynamic constants are $\sigma = 0.18$ GeV^2