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Fermions in a warped resolved conifold

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ABSTRACT

We investigated the localization of the spinorial field in a braneworld built as a warped product between a 3-brane and a 2-cycle of the resolved conifold. This scenario provides a geometric flow that controls the singularity at the origin and changes the properties of the fermion in this background. Furthermore, due the cylindrical symmetry according to the 3-brane and a smoothed warp factor, this geometry can be regarded as a near brane correction of the string-like branes. This geometry allows a normalizable and well-defined massless mode whose decay and value on the brane depend on the resolution parameter. For the Kaluza–Klein modes, resolution parameter also controls the height of the barrier of the volcano potential.

1. Introduction

The Randall–Sundrum model changed the way we understand the universe by allowing the space–time to have infinite extra dimensions [1,2]. In spite of the localization of the gravity on the 3-brane, the gauge and fermion fields are not trapped in this model [3]. One way to overcome this issue is to extend the RS model to higher dimensions [4].

In six dimensions, a static space–time with an infinite extra dimension and cylindrical symmetry is the so-called string-like model [4–12]. This model have the advantage of localize the massless mode of both fermions [5] and gauge [6] fields on the brane coupled with only the gravity. Furthermore, the correction to the gravitational potential is less than in RS model [7]. However, due the conical behavior near the brane, the string-like model has the problem of find non-zero induced field equations on the brane [13].

Another important property of the string-like model is the relationship between physics and geometry. Indeed, the geometry of the transverse manifold, as its deficit angle, is related to the mass-tension of the string-brane [4,7,10]. This effect motivated us to study how the fields on these models are affected by some geometrical flow in the extra dimensions.

We performed this task choosing as a parameter dependent transverse manifold a 2-cycle of the so-called resolved conifold. This smooth six-dimensional space whose parameter a controls the singularity on the tip of the cone is a special internal Calabi–Yau space of string-theory [14–21]. Thus, it is possible continuously to pass from a smooth to a singular manifold varying the parameter a. This geometrical resolution flow is also used in an extension of the AdS/CFT correspondence [17,20,22–24].

The study of the behavior of the fields on braneworlds with a resolved transverse conifold was addressed before in the literature. For the gravitational field, in a 10-dimensional space–time, the massless mode is located around the origin and the KK spectrum has an exponential decay [21]. In a six-dimensional set-up, we have shown that the scalar field has massless and massive modes trapped to the brane [25]. Moreover, the resolution flow changes the properties of the volcano potential for the KK modes, as the width of the well and the height of the barrier [25].

In this Letter we have used a different warp factor, firstly studied in [26], and that possess a Z_2 symmetry. This warp function satisfies the required regularity conditions, what renders this geometry as a smooth extension of the string-like scenario. This geometry represents a positive tension brane embedded in a space–time with negative cosmological constant [25]. Furthermore, for tiny values of a the components of the stress-energy tensor satisfy the weak and strong energy condition what extends the thin string-like model [7,11]. On the other hand, for $a \neq 0$, the 3-brane can be regarded as a brane embedded in a 4-brane with a compact extra dimension whose radius is the resolution parameter. This enable us to realize the RS1 model as a limit of the six-dimensional non-compact scenario.

Once studied the geometry we turned our attention to the behavior of a massless spinorial field minimally coupled in this scenario. For the massless mode, it turned out that this mode is

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normalizable provided there is a background gauge vector field, as done in [5]. Moreover, the new warp factor smooth out this mode at the origin while the resolution parameter controls the value of the gauge field on the brane.

Another improvement obtained is related to the conical behavior of the string-like models. Indeed, the conical geometry yields a divergence of the zero mode on the brane. On the other hand, if we consider a thin string-brane, taking into account only the exterior geometry, the metric does not satisfies the regularity conditions [7,11]. Also, it is possible to achieve a well-defined zero mode for others 6D conical geometries, but with compact transverse space [27]. The resolution parameter prevents this singular effect by smoothing out the cone at the origin.

For the KK modes, there is an attractive potential for only the left-handed fermion [5]. As for the scalar field, the depth of the well and the height of the barrier of the usual volcano potential depend on the resolution parameter [25]. Nevertheless, despite the lack of a potential well at the origin for the right-handed fermion, there is a potential well besides the brane.

This work is organized as follows. In Section 2 we built the warped product between a 3-brane and the 2-cycle of the conifold and studied the geometric properties of this scenario. In Section 3 we have studied the properties of the massless and KK spectrum of the fermionic field. Some conclusions and perspectives are outlined in Section 4.

2. Bulk geometry

Consider a six-dimensional warped bulk $\mathcal{M}_6$ of form $\mathcal{M}_6 = \mathcal{M}_4 \times \mathcal{M}_2$, where $\mathcal{M}_4$ is a 3-brane embedded in $\mathcal{M}_6$ and $\mathcal{M}_2$ is a two-dimensional transverse space.

The action for this model is defined as

$$S_g = \int_{\mathcal{M}_6} \left(\frac{1}{2\kappa_6} R - \Lambda + \mathcal{L}_m\right)\sqrt{-g}\, d^6x, \tag{1}$$

where $\kappa_6 = \frac{8\pi}{M_6^4}$, M_6^4 is the six-dimensional bulk Planck mass and $\mathcal{L}_m$ is the source matter Lagrangian.

Consider a static and axisymmetric warped metric between a flat 3-brane $\mathcal{M}_4$ and the transverse manifold $\mathcal{M}_2$ given by [7,10,11,26]

$$ds_6^2 = W(r,c)\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dr^2 + \gamma(r,c,a)\, d\theta^2, \tag{2}$$

where $W \in C^\infty$ is the so-called warp factor. For the thin string-like models, the metric is given by [5–7,9–12]

$$W(r) = e^{-cr}, \qquad \gamma(r) = R_0^2 e^{-cr}, \tag{3}$$

where $c^2 = -\frac{2\kappa_6}{5}\Lambda$. The system in Eq. (3) describes the exterior geometry of the defect. It can be understood as a warped product between a 3-brane and a disc of radius R_0. Furthermore, the metric components in Eq. (3) do not satisfy the regularity conditions at the origin, namely,

$$W(0,c) = 1, \qquad W'(0,c) = 0, \tag{4}$$

where, the prime ($'$) stands for the derivative $\frac{d}{dr}$. In order to overcome this problem, in this work, we shall use a smoothed warp factor [26,28]

$$W(r,c) = e^{-(cr - \tanh cr)}. \tag{5}$$

The addition of the term $\tanh cr$ smoothes the warp factor near the origin, as shown in Fig. 1 (see also Fig. 2). Therefore, we can realize this warp function as a near brane correction to the thin string-like models [4–7].

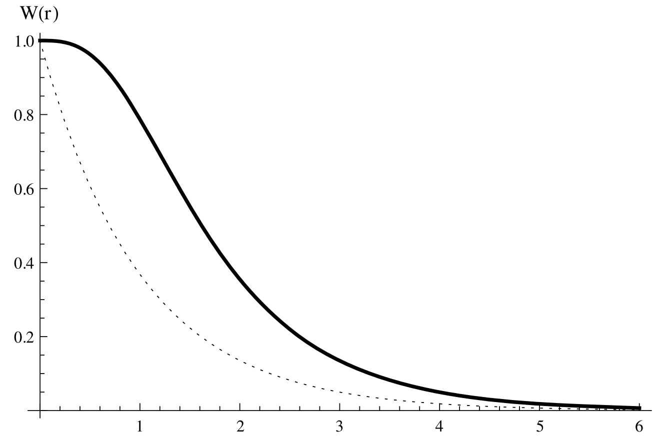

Fig. 1. Warp function for $c = 1$ (thick line). The thin string warp factor (dotted line) is defined only for the exterior of the string.

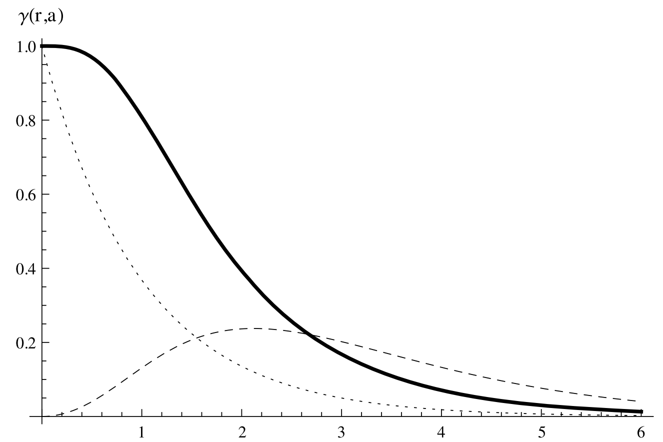

Fig. 2. Angular metric component for $c = 1$. For $a = 0$ (dashed line) there is a conical singularity and the thin string-like geometry is denoted by the dotted line.

Moreover, instead of use the disc, we have chosen a 2-section of the resolved conifold as the transverse manifold [14,17,18,21,25]

$$ds_2^2 = \left(\frac{u^2 + 6a^2}{u^2 + 9a^2}\right) du^2 + \frac{1}{6}\left(u^2 + 6a^2\right) d\theta^2. \tag{6}$$

Asymptotically, the resolved conifold has a conical shape. Near the origin the constant a, called the resolution parameter, controls the divergence of the conifold. This resolution flow provides a way to study the effects of a conical singularity has on the fields.

The coordinates u and r are related by

$$r_a(u) = \begin{cases} u, & a = 0, \\ -i\sqrt{6}aE(\operatorname{arcsinh}(\frac{i}{3a}u), \frac{3}{2}), & a \neq 0, \end{cases}$$

whose behavior is sketched in Fig. 3 (see also Fig. 4).

For the angular metric component, $\gamma : [0,\infty) \to [0,\infty)$, we have modified the string-like ansatz using as metric [7,10,26,28],

$$\begin{aligned} \gamma(r,c,a) &= W(r,c)\beta(r,a) \\ &= e^{-(cr - \tanh cr)}\left(\frac{u(r,a)^2 + 6a^2}{6}\right). \end{aligned} \tag{7}$$

The angular component (7) provides a resolved conical behavior to the transverse manifold. At the origin, the angular component satisfies $\gamma(0,c,a) = a^2$. Then, the geometrical flow of the resolved conifold yields a dimensional reduction $\mathcal{M}_6 \to \mathcal{M}_5$ at the origin. The string-like dimensional reduction $\mathcal{M}_6 \to \mathcal{M}_4$ is achieved only