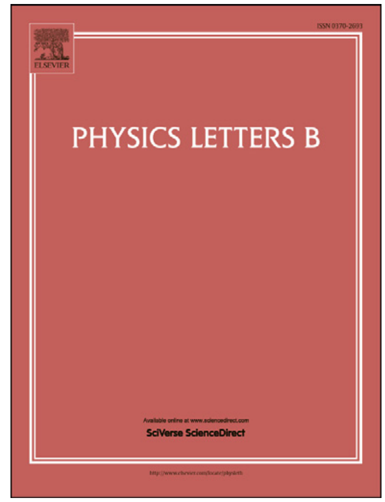
Accepted Manuscript



Thermodynamics of quasi-topological cosmology

M.H. Dehghani, A. Sheykhi, R. Dehghani

PII: S0370-2693(13)00447-4
DOI: 10.1016/j.physletb.2013.05.065
Reference: PLB 29449

To appear in: *Physics Letters B*

Received date: 30 January 2013
Revised date: 25 May 2013
Accepted date: 28 May 2013

Please cite this article in press as: M.H. Dehghani et al., Thermodynamics of quasi-topological cosmology, *Physics Letters B* (2013), http://dx.doi.org/10.1016/j.physletb.2013.05.065

Thermodynamics of Quasi-Topological Cosmology

M. H. Dehghani[1,2] , A. Sheykhi [1,2] and R. Dehghani [1]

In this paper, we study thermodynamical properties of the apparent horizon in a universe governed by quasi-topological gravity. Our aim is twofold. First, by using the variational method we derive the general form of Friedmann equation in quasi-topological gravity. Then, by applying the first law of thermodynamics on the apparent horizon, after using the entropy expression associated with the black hole horizon in quasi-topological gravity, and replacing the horizon radius, r_+, with the apparent horizon radius, $\tilde{r}_A$, we derive the corresponding Friedmann equation in quasi-topological gravity. We find that these two different approaches yield the same result which show the profound connection between the first law of thermodynamics and the gravitational field equations of quasi-topological gravity. We also study the validity of the generalized second law of thermodynamics in quasi-topological cosmology. We find that, with the assumption of the local equilibrium hypothesis, the generalized second law of thermodynamics is fulfilled for the universe enveloped by the apparent horizon for the late time cosmology.

I. INTRODUCTION

The most general Lagrangian which keeps the field equations of motion for the metric of second order, as the pure Einstein-Hilbert action, is Lovelock Lagrangian [1]. This Lagrangian is constructed from the dimensionally extended Euler densities and can be written as

$$\mathcal{L} = \sum_{p=0}^{m} \alpha_p \mathcal{L}_p, \tag{1}$$

where α_p and $\mathcal{L}_p$ are arbitrary constant and Euler density, respectively. In an $(n+1)$-dimensional spacetime $m = [n/2]$. $\mathcal{L}_0$ set to be one, and therefore α_0 plays the role of the cosmological constant. Because of the topological origin of the Lovelock terms, the second order (Gauss-Bonnet) term does not have any dynamical effect in four dimensions. Similarly, the cubic interaction only contributes to the equations of motion when the bulk dimension is seven or greater. In other words, although the equations of motion of pth order Lovelock gravity are second-order differential equations, the pth order Lovelock term has no contribution to the field equations in $2p$ and lower dimensions. Is it possible to construct a gravitational action with cubic curvature interactions or higher which has contribution in five dimension? The answer is positive and the corresponding theory is called "quasi-topological" gravity which was recently proposed in Refs. [2, 3] and [4] with cubic and quartic terms of Riemann tensor, respectively. This new gravitational theory provides a useful toy model to study a broader class of four (and higher) dimensional CFT's, involving three or more independent parameters [5]. Various aspects of pth- order quasi-topological terms which have at most second order derivatives of the metric in the field equations for spherically symmetric spacetimes in five and higher dimensions except $2p$ dimensions have been investigated [6–9].

Nowadays, it is a general belief that there is a profound connection between the gravitational field equations and the laws of thermodynamics. It was shown that the gravitational field equation of a static spherically symmetric spacetime in Einstein, Gauss-Bonnet and more general Lovelock gravity can be recast as the first law of thermodynamics [10]. The studies were also extended to other gravity theories such as $f(R)$ gravity [11] and scalar-tensor gravity [12]. In the cosmological setup, it was shown that the differential form of the Friedmann equation of Friedmann-Robertson-Walker (FRW) universe can be transformed to the first law of thermodynamics on the apparent horizon [13, 14]. In the context of brane cosmology, it was shown that the Friedmann equations on the brane can be expressed as $dE = TdS + WdV$ on the apparent horizon [15–17]. This procedure also leads to extract an expression for the entropy at the apparent horizon on the brane, which is useful in studying the thermodynamical properties of the black hole horizon on the brane [15–17].

Is the inverse procedure also possible? That is starting from the first law of thermodynamics to extract the general field equations of gravitational theory. Jacobson [18] was the first who disclosed that the Einstein field equation can be derived from the relation between the horizon area and entropy, together with the Clausius relation $\delta Q = T\delta S$. Also, in the cosmological setup, it was shown that the corresponding Friedmann equations of Einstein, Gauss-Bonnet and Lovelock gravity can be derived by applying the energy balance relation $-dE = TdS$ to the apparent horizon of a Friedmann- Robertson-Walker universe (FRW) with any spatial curvature [19]. Here, $-dE$ is actually just the heat flux δQ in [18] crossing the apparent horizon within an infinitesimal internal of time dt. In the framework of Horava-Lifshitz gravity, it was shown that the corresponding Friedmann equation cannot be derived by applying the first law of thermodynamics on the apparent horizon and using the entropy expression for static spherically symmetric