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Upper energy limit of heavy baryon chiral perturbation theory in neutral pion photoproduction

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ABSTRACT

With the availability of the new neutral pion photoproduction from the proton data from the A2 and CB-TAPS Collaborations at Mainz it is mandatory to revisit Heavy Baryon Chiral Perturbation Theory (HBChPT) and address the extraction of the partial waves as well as other issues such as the value of the low-energy constants, the energy range where the calculation provides a good agreement with the data and the impact of unitarity. We find that, within the current experimental status, HBChPT with the fitted LECs gives a good agreement with the existing neutral pion photoproduction data up to ∼170 MeV and that imposing unitarity does not improve this picture. Above this energy the data call for further improvement in the theory such as the explicit inclusion of the $\Delta(1232)$. We also find that data and multipoles can be well described up to ∼185 MeV with Taylor expansions in the partial waves up to first order in pion energy.

1. Introduction

Chiral Perturbation Theory (ChPT) is an effective field theory (EFT) of Quantum Chromodynamics (QCD) in the low-energy domain where quarks and gluons are confined into hadrons and conventional perturbation theory cannot be directly applied. Due to the spontaneous breaking of chiral symmetry in QCD the π meson appears as a pseudoscalar Nambu–Goldstone boson [1] becoming the carrier of the nucleon–nucleon interaction. However, when fully relativistic spin-1/2 matter fields (i.e. nucleon) are introduced in the theory the exact one-to-one correspondence between the loop expansion and the expansion in small momenta and quark masses is spoiled [2]. This is due to the fact that the nucleon mass M does not vanish in the chiral limit. A consistent power counting scheme known as Heavy Baryon Chiral Perturbation Theory (HBChPT) [3] overcomes this difficulty considering the baryons as heavy (static) sources. For πN scattering and pion photoproduction HBChPT has been successful at describing experimental data in the near threshold region [3,4]. In this Letter we address the question of how well it works for the latest and most accurate $\vec{\gamma} p \to \pi^0 p$ data to date [5] and to provide an energy range where HBChPT agrees with the latest pion photoproduction data — the recently completed Mainz data for the differential cross sections $d\sigma/d\Omega$ and linear polarized photon asymmetries Σ for the $\vec{\gamma} p \to \pi^0 p$ reaction taken from threshold through the $\Delta(1232)$ region. This was performed with a tagged photon beam with energy bins of 2.4 MeV. We also determined the low-energy constants (LECs) to see if they are actually constant as the photon energy is increased. The quality of the HBChPT fits — χ^2 per degree of freedom (χ^2/dof) — are also compared to a simple *empirical* benchmark fit, a Taylor expansion of the partial waves. The data in [5] are more accurate than previous experiments and the first measurement of the energy dependence of Σ. This has allowed an extraction of the real parts of the four dominant multipoles for the first time — the S-wave E_{0+} and the three P-wave multipoles $P_{1,2,3}$ (E_{1+}, M_{1+}, M_{1-}). This is a much more significant test of the agreement of HBChPT with experiment. As the photon energy increases and the calculations gradually stop agreeing with experiment we have determined whether or not this is caused by one particular multipole. This information, in addition to the behavior of the low energy constants with photon energy provide clues about what improvements are needed to make the HBChPT calculations more accurate.

2. Theoretical framework

Due to the symmetry breaking, the S-wave amplitude for the $\gamma p \to \pi^0 p$ reaction is small in the threshold region — vanishing in the chiral limit [4]. Additionally, the P-wave amplitude is large and leads to the $\Delta(1232)$ resonance at intermediate energies [6]. Hence, for the $\gamma p \to \pi^0 p$ reaction the S- and P-wave contributions

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are comparable even very close to threshold [7] and even D waves have an important early contribution due to the weakness of the S wave [8]. The differential cross section and photon asymmetry can be written in terms of electromagnetic responses

$$\frac{d\sigma}{d\Omega}(s,\theta) = \frac{q}{k_\gamma} W_T(s,\theta) \tag{1}$$

$$\Sigma(s,\theta) \equiv \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel} = -\frac{W_S(s,\theta)}{W_T(s,\theta)} \sin^2\theta \tag{2}$$

where W_T and W_S are the electromagnetic responses, θ is the center of mass scattering angle, k_γ the center of mass photon energy, q the pion momentum in the center of mass, and s the squared invariant mass. The responses W_T and W_S are defined in term of the electromagnetic multipoles:

$$W_T = T_0(s) + T_1(s)\mathcal{P}_1(\theta) + T_2(s)\mathcal{P}_2(\theta) + \cdots \tag{3}$$

$$W_S = S_0(s) + S_1(s)\mathcal{P}_1(\theta) + \cdots \tag{4}$$

where $P_j(\theta)$ are the Legendre polynomials in terms of $\cos\theta$, the dots stand for negligible corrections, and

$$T_n(s) = \sum_{ij} \mathrm{Re}\{\mathcal{M}_i^*(s) T_n^{ij} \mathcal{M}_j(s)\} \tag{5}$$

$$S_n(s) = \sum_{ij} \mathrm{Re}\{\mathcal{M}_i^*(s) S_n^{ij} \mathcal{M}_j(s)\} \tag{6}$$

where $\mathcal{M}_j(s) = E_{0+}, E_{1+}, E_{2+}, E_{2-}, M_{1+}, M_{1-}, M_{2+}, M_{2-}$. The coefficients T_n^{ij} and S_n^{ij} can be found in Appendix A in [9].

The partial waves (electromagnetic multipoles) are not observables and have to be extracted from the experimental data within a theoretical framework (unless a complete experiment is possible [10]). In this Letter we employ three approaches to describe S and P waves that we present in forthcoming paragraphs: Section 2.1 HBChPT [11,12]; Section 2.2, Unitary HBChPT (U-HBChPT); and Section 2.3, Empirical. In all cases D waves are incorporated using the customary Born terms. Higher partial waves can be safely dismissed in this energy region [9]. The conventions employed in this Letter and further information on the structure of the observables in terms of the electromagnetic multipoles can be found in [9].

2.1. HBChPT

The explicit formulae for the S and P multipoles to one loop and up to $\mathcal{O}(q^4)$ can be found in [11,12]. Due to the order-by-order renormalization process six LECs appear: a_1 and a_2 associated with the E_{0+} counter-term:

$$E_{0+}^{ct} = e a_1 \omega m_{\pi^0}^2 + e a_2 \omega^3, \tag{7}$$

where ω is the pion energy in the center-of-mass; b_p associated with the $P_3 \equiv 2M_{1+} + M_{1-}$ multipole together with ξ_1 and ξ_2 associated with $P_1 \equiv 3E_{1+} + M_{1+} - M_{1-}$ and $P_2 \equiv 3E_{1+} - M_{1+} + M_{1-}$, respectively. The c_4 LEC associated with P_1, P_2, and P_3 has been taken from [13] where it was determined from pion–nucleon scattering inside the Mandelstam triangle. Some other parameters appear in the calculation, but these are fixed. The full list is: the pion–nucleon coupling constant $g_{\pi N} = 13.1$; the weak pion decay constant $f_\pi = 92.42$ MeV, together with the anomalous magnetic moments of the proton and neutron, the nucleon axial charge g_A (which we fix using the Goldberger–Trieman relation $g_A = g_{\pi N} f_\pi / M$); and the masses of the particles. The pair (a_1, a_2) LECs are highly correlated, $r(a_1, a_2) = -0.99$ [8,12], and it is more convenient to use the pair of LECs $(a_+ = a_1 + a_2,\ a_- = a_1 - a_2)$, where a_+ is the leading order for the counter-term close to threshold ($\omega \simeq m_{\pi^0}$) [8]. Henceforth, five LECs are fitted to the data under this approach: a_+, a_-, ξ_1, ξ_2, and b_p.

2.2. U-HBChPT

From general principles such as time reversal invariance and unitarity the S wave can be written as the combination of a smooth part and a cusp part [9,14,15]

$$E_{0+} = e^{i\delta_0}[A_0 + i\beta q_+/m_{\pi^+}], \quad s > s_{thr}^{(\pi^+ n)}$$
$$E_{0+} = e^{i\delta_0}[A_0 - \beta|q_+|/m_{\pi^+}], \quad s < s_{thr}^{(\pi^+ n)} \tag{8}$$

where δ_0 is the $\pi^0 p$ phase shift (which is very small), $\sqrt{s}$ is the invariant mass, $\sqrt{s_{thr}^{(\pi^+ n)}}$ the invariant mass at the $\pi^+ n$ threshold, q_+ is the π^+ center-of-mass momentum, A_0 is E_{0+} in the absence of the charge exchange re-scattering (smooth part), and $\beta = \mathrm{Re}[E_{0+}(\gamma p \to \pi^+ n)] \times m_{\pi^+} a(\pi^+ n \to \pi^0 p)$ parameterizes the magnitude of the unitary cusp and can be calculated [14] on the basis of unitarity. Eq. (8) takes the static isospin breaking (mass differences) as well as πN scattering to all orders into account. In the electromagnetic sector it includes up to first order in the fine structure constant α. The π^+ center-of-mass momentum, q_{π^+}, is real above and imaginary below the π^+ threshold; this is a unitary cusp whose magnitude is parametrized by β which can be calculated [14] on the basis of unitarity and taking into account a theoretical evaluation of isospin breaking [16], obtaining $\beta = (3.35 \pm 0.08) \times 10^{-3}/m_{\pi^+}$ where $\mathrm{Re}\, E_{0+}(\gamma p \to \pi^+ n) = (28.06 \pm 0.27 \pm 0.45) \times 10^{-3}/m_{\pi^+}$ [17] and $a(\pi^+ n \to \pi^0 p) = (0.1195 \pm 0.0016)/m_{\pi^+}$ [18]. In HBChPT up to one loop and $\mathcal{O}(q^4)$, β is fixed by the imaginary part of E_{0+} — that is parameter-free — providing $\beta_{HBChPT} = 2.71 \times 10^{-3}/m_{\pi^+}$ which is far away from the unitary value. Because of the lack of unitarity of the S-wave amplitude [11] it is customary to substitute the S wave provided by HBChPT by a unitary prescription [9,11,12]. However, in this Letter instead of substituting the entire S wave for a prescription we prefer to substitute only the cusp part in E_{0+} from HBChPT by the cusp part of E_{0+} in Eq. (8), keeping the smooth part provided by HBChPT. In this way we keep the E_{0+} counter-term and both HBChPT and U-HBChPT approaches have the same LECs to fit to the data.

2.3. Empirical fit

The empirical fit is parameterization of the S and P waves with a minimal physics input: unitarity in the S wave through the β parameter and the angular momentum barrier. This is accomplished with a Taylor expansion in the pion energy in the center of mass ω up to first order on the smooth part of E_{0+} and P_i/q adding the cusp part in Eq. (8) to the S wave and keeping the imaginary part of the P waves equal to zero, in summary[1]

$$E_{0+} = E_{0+}^{(0)} + E_{0+}^{(1)} \frac{\omega - m_{\pi^0}}{m_{\pi^+}} + i\beta \frac{q_{\pi^+}}{m_{\pi^+}} \tag{9}$$

$$P_i/q = \frac{P_i^{(0)}}{m_{\pi^+}} + P_i^{(1)} \frac{\omega - m_{\pi^0}}{m_{\pi^+}^2}, \quad i = 1, 2, 3 \tag{10}$$

where $E_{0+}^{(0)}$, $E_{0+}^{(1)}$, $P_1^{(0)}$, $P_1^{(1)}$, $P_2^{(0)}$, $P_2^{(1)}$, $P_3^{(0)}$, and $P_3^{(1)}$ are free parameters that will be fitted to the experimental data. We note that

[1] The empirical parameterization in [5,9] expands on the photon energy in the laboratory frame E_γ while we prefer to expand in the pion energy in the center of mass frame ω in order to have direct comparison to HBChPT. Both approaches render equally good description of the observables and provide the same multipoles.