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First study of the negative binomial distribution applied to higher moments of net-charge and net-proton multiplicity distributions

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ABSTRACT

A study of the first four moments (mean, variance, skewness, and kurtosis) and their products ($\kappa\sigma^2$ and $S\sigma$) of the net-charge and net-proton distributions in Au + Au collisions at $\sqrt{s_{NN}} = 7.7$–200 GeV from HIJING simulations has been carried out. The skewness and kurtosis and the collision volume independent products $\kappa\sigma^2$ and $S\sigma$ have been proposed as sensitive probes for identifying the presence of a QCD critical point. A discrete probability distribution that effectively describes the separate positively and negatively charged particle (or proton and anti-proton) multiplicity distributions is the negative binomial (or binomial) distribution (NBD/BD). The NBD/BD has been used to characterize particle production in high-energy particle and nuclear physics. Their application to the higher moments of the net-charge and net-proton distributions is examined. Differences between $\kappa\sigma^2$ and a statistical Poisson assumption of a factor of four (for net-charge) and 40% (for net-protons) can be accounted for by the NBD/BD. This is the first application of the properties of the NBD/BD to describe the behavior of the higher moments of net-charge and net-proton distributions in nucleus–nucleus collisions.

1. Introduction

Colliding heavy ions is an essential experimental tool for exploring the nature of the deconfinement phase transition. The phase transition line between hadronic and partonic matter in the phase diagram of quantum chromodynamics (QCD) is frequently drawn as a function of baryon-chemical potential (μ_B) and temperature. Heavy ion collisions at a particular center-of-mass (CMS) energy essentially probe a singular value (small range) of μ_B. Baryon-chemical potential can be changed by adjusting the CMS collision energy. The Relativistic Heavy Ion Collider (RHIC) recently embarked on a program to study the QCD phase diagram by colliding Au-nuclei at CMS collision energies ($\sqrt{s_{NN}}$) of $\sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39, 62.4,$ and 200 GeV [1]. This 'energy scan' provides information at different values of μ_B from $\approx$ 20–400 MeV [1,2]. Lattice QCD (LQCD) and experimental data suggest that if there is a phase transition at high temperatures close to $\mu_B = 0$, it appears to be a smooth crossover [3]. Additional predictions from LQCD (dependent on the number of quark flavors and their masses) indicate the possible existence of a first order phase transition at larger values of μ_B. If this is the case, where the first order phase transition line and crossover phase transition region meet could be a critical point [4].

From previous studies of the thermodynamics of phase transitions, it has been seen that a system undergoing a phase transition that passes through a critical point can exhibit enhanced fluctuations (e.g. critical opalescence) [5,6]. Lattice QCD studies have shown that near the crossover phase transition the fourth-order quark number susceptibilities of conserved quantities (baryon quantum number, strangeness quantum number, and charge) change rapidly [7,8]. Because experiments cannot measure all produced baryons, strangeness, or charge, the higher order moments of the following proxy distributions are measured for each conserved quantity (baryon quantum number, strangeness quantum number, and charge): net-proton, net-kaon, and net-charge, respectively. In this case, the first through fourth-order moments correspond to the mean, variance, skewness, and kurtosis, respectively. There have been estimates that the skewness (S) and kurtosis (κ) should be sensitive to critical fluctuations generated by a phase transition trajectory that passes through (or close to) a critical point. The higher moments are proportional to powers of the correlation length (ξ) with the skewness (third moment) proportional to $\xi^{4.5}$ and the kurtosis (fourth moment) proportional to ξ^7 [9].

The moments of the (e.g. net-charge or net-proton) distributions contain trivial system volume dependencies. Constructing products and ratios of these moments can cancel the intrinsic volume dependence. Two examples of these volume independent products are $S\sigma$ and $\kappa\sigma^2$. These ratios of moments can also be expressed in terms of cumulants (C_n): $S\sigma = \frac{C_3}{C_2}$ and $\kappa\sigma^2 = \frac{C_4}{C_2}$.

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Quantitatively, their sensitivities to the correlation length (ξ) are $S\sigma \propto \xi^{2.5}$ and $\kappa\sigma^2 \propto \xi^5$ [10].

Several measurements of the higher moments of net-proton, net-kaon, and net-charge distributions have been carried out [11–13]. These measurements are frequently compared to a baseline that assumes Poisson statistics. Deviations from this Poisson baseline have been observed. However, the deviations from the Poisson baseline are not as large as those expected from strong enhancement due to the presence of a QCD critical point. One estimate of $\kappa\sigma^2$ for the net-proton distribution from a QCD-based model that includes a critical point is $\kappa\sigma^2 \approx 2.5, 35$, or 3700 for a critical point at $\sqrt{s_{NN}} = 200, 62.4,$ and 19.6 GeV, respectively [11]. Other predictions concluded that the sign of the skewness and/or kurtosis will change at a critical point [14,15]. This behavior has been observed from lattice QCD studies that have a critical point, whose location in energy and μ_B is dependent on parameters such as the critical temperature and the lattice spacing used [8].

Before interpreting the results from the higher moments studies in terms of QCD critical point phenomena, the contribution from known physics must be quantified. It is well known that experimental multiplicity distributions are better described by the (negative) binomial distribution (NBD/BD) [16–22], than by a Poisson distribution. The NBD/BD and Poisson distributions are all discrete probability distributions. The Poisson distribution is a limiting case of the NBD/BD where the mean and the variance are equal. For a NBD/BD distribution, the variance is larger/smaller than the mean. There are several sources of correlation in particle production that can cause deviations from pure Poisson statistics: e.g. charge conservation, correlated production of particles from one source (e.g. strings or clusters), etc. Therefore, it is important to verify the experimental results compared to not only a Poisson distribution, but to other valid baselines that better describe the multiplicity distributions. The NBD/BD baseline is one of these cases.

2. Analysis

For this study, minimum bias Au + Au events at $\sqrt{s_{NN}} = 7.7$ (35 million events), 11.5 (12.2 million events), 19.6 (87.3 million events), 27 (62.4 million events), 39 (90.6 million events), 62.4 (109 million events) and 200 GeV (114 million events) were created with the default HIJING (v1.383) event generator [23]. Distributions of positive and negative particles and protons and anti-protons were produced. From each multiplicity distribution (positive and negative charge; proton and anti-proton) the mean value and the variance were extracted. Though the mean and variance are separately extracted from the positive and negative charged particle (or protons and anti-protons) multiplicity distributions, the higher moments of the net-charge and net-proton distributions are not automatically described by the knowledge of the mean and variance of the separate distributions. From this information, the higher order cumulants for each individual distribution could be calculated. From these, the combined cumulants of the net-charge and net-proton distributions were calculated. The additive properties of cumulants means that even order cumulants are added and odd order cumulants are subtracted (also for the NBD [24]) to calculate the required cumulants for the net-charge/proton distribution. A Poisson distribution can be fully described by the mean value μ and the cumulants $C_n = \mu$. For two Possion distributions with mean values μ_+ and μ_-, the odd cumulants of the net distribution (Skellam) are equal to $\mu_+ - \mu_-$ and the even cumulants are equal to $\mu_+ + \mu_-$. Ratios of the even cumulants (e.g C_4/C_2) are then equal to one, while C_3/C_2 is equal to $(\mu_+ - \mu_-)/(\mu_+ + \mu_-)$. A negative binomial distribution with mean μ and variance σ^2 ($\mu < \sigma^2$ for an NBD) is described by two parameters, p and n where,

$$p = \frac{\mu}{\sigma^2} \tag{1}$$

$$n = \frac{\mu p}{1 - p} \tag{2}$$

With knowledge of the mean and variance of an NBD, p, n, and the cumulants of the NBD can be calculated as,

$$C_1 = \frac{n(1-p)}{p} \tag{3}$$

$$C_2 = \frac{n(1-p)}{p^2} \tag{4}$$

$$C_3 = \frac{n(p-1)(p-2)}{p^3} \tag{5}$$

$$C_4 = \frac{n(1-p)(6-6p+p^2)}{p^4} \tag{6}$$

For two NBD distributions, the cumulants of each distribution are defined as either $C_{n,+}$ and $C_{n,-}$, where the odd cumulants of the net distribution are equal to $C_{n=odd} = C_{n,+} - C_{n,-}$ and the even cumulants are equal to $C_{n=even} = C_{n,+} + C_{n,-}$, from which the ratio of the cumulants can be calculated. A similar exercise can be carried out for the binomial distribution (where $\mu > \sigma^2$).

The centrality is defined using charged particle multiplicity in the region $0.5 < |\eta| < 1.0$. The measured charged particles and identified protons/anti-protons are restricted to the pseudorapidity range $|\eta| < 0.5$. For this measurement, this is required to prevent auto-correlations between the particles used for the measurement of the moments of the multiplicity distributions and those used for centrality determination. This method has been used in other analyses of particle correlations in pseudorapidity [25]. For net-charge studies, the transverse momentum range for particles in this analysis was $0.2 < p_T < 2.0$ GeV/c and for net-proton studies the range $p_T > 0.4$ and $p < 3.0$ GeV/c was used.

The moments are calculated in each multiplicity bin and weighted by the number of events in each bin to remove any centrality bin-width effects [26,25].

3. Results and discussion

Fig. 1 shows the multiplicity distribution for positively and negatively charged particles and the net-charge distribution (top panels) and for protons, anti-protons, and the net-proton distribution (bottom panels) from $\sqrt{s_{NN}} = 19.6$ GeV Au + Au collisions in a representative centrality bin (10–12.5%) from HIJING simulations. Two baseline distributions are calculated from the information in the histograms: the solid lines are a negative binomial distribution (NBD) and the dashed lines are a Poisson distribution. The only input to the Poisson curve is the mean of the multiplicity distribution, while for the NBD the mean and variance of the multiplicity distribution are required. For the positively charged particles, the χ^2/NDF for the Poisson is 650 and for the NBD is 0.92. For the protons, the χ^2/NDF for the Poisson distribution is 53, while for the NBD the χ^2/NDF is 1.2. Similar differences between the Poisson and NBD are observed for the negatively charged particle and anti-proton distribution. In both cases, the NBD is a better description of the data than the Poisson distribution. The same is true for negatively charged particles and anti-protons, which are treated separately.

While the positive and negative (or proton and anti-proton) multiplicity distributions are separately well described by the NBD/BD, the difference of two NBD/BD distributions to create the net-charge or net-proton distributions are not necessarily trivially described by an NBD/BD. This is seen clearly for the net-charge distribution (upper right panel) in Fig. 1(a), where the NBD describes