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Axially symmetric static scalar solitons and black holes with scalar hair

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ABSTRACT

We construct static, asymptotically flat black hole solutions with scalar hair. They evade the no-hair theorems by having a scalar potential which is not strictly positive. By including an azimuthal winding number in the scalar field ansatz, we find hairy black hole solutions which are static but axially symmetric only. These solutions possess a globally regular limit, describing scalar solitons. A branch of axially symmetric black holes is found to possess a positive specific heat.

1. Introduction

The energy conditions are an important ingredient of various significant results in general relativity [1]. Essentially, they imply that some linear combinations of the energy–momentum tensor of the matter fields should be positive, or at least non-negative. However, over the last decades, it has become increasingly obvious that these conditions can be violated, even at the classical level. Remarkably enough, the violation may occur also for the simplest case of a scalar field (see e.g. [2] for a discussion of these aspects).

Once we give up the energy conditions (and in particular the weak one), a number of results in the literature show that the asymptotically flat black holes may possess scalar hair,[1] which otherwise is forbidden by a number of well-known theorems [4]. Restricting to the simplest case of a minimally coupled scalar field with a scalar potential which is not strictly positive, this includes both analytical [5–9] and numerical [10,11] results.

Interestingly, in the limit of zero event horizon radius, some of these hairy black holes describe globally regular, particle-like objects, the so-called '*scalarons*' [10]. At the same time, a complex scalar field is known for long time to possess non-topological solitonic solutions [12], even in the absence of gravity. These are the Q-balls introduced by Coleman in [13]. Such configurations owe their existence to a harmonic time dependence of the scalar field and possess a positive energy density.

However, as argued below, the Q-balls can be reinterpreted as non-gravitating scalarons. The scalar field is static in this case and has a potential which takes negative values as well. As expected, the scalarons possess gravitating generalizations. However, different from the standard Q-ball case [14], their regular origin can be replaced with an event horizon. In this work we study such solutions for the simple case of a massive complex scalar field with a negative quartic self-interaction term in the potential. Apart from spherically symmetric configurations, we construct solitons and hairy black hole solutions which are static but axially symmetric only.

2. The model

Let us consider the action of a self-interacting complex scalar field Φ coupled to Einstein gravity in four spacetime dimensions,

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}g^{\mu\nu}\left(\Phi^*_{,\mu}\Phi_{,\nu}+\Phi^*_{,\nu}\Phi_{,\mu}\right)-U\right], \quad (1)$$

where R is the curvature scalar, G is Newton's constant and the asterisk denotes complex conjugation. Using the principle of variation, one finds the coupled Einstein–Klein–Gordon equations

$$E_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-8\pi G T_{\mu\nu}=0,$$
$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\Phi\right)=\frac{\partial U}{\partial|\Phi|^2}\Phi, \quad (2)$$

where $T_{\mu\nu}$ is the stress–energy tensor of the scalar field

$$T_{\mu\nu}=\left(\Phi^*_{,\mu}\Phi_{,\nu}+\Phi^*_{,\nu}\Phi_{,\mu}\right) - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\left(\Phi^*_{,\alpha}\Phi_{,\beta}+\Phi^*_{,\beta}\Phi_{,\alpha}\right)+U\right]. \quad (3)$$

In the above relations U denotes the scalar field potential, which, in order to retain the $U(1)$ symmetry of the whole Lagrangian,

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[1] One has to remark that the existence of black holes with scalar hair is perhaps the mildest consequence of giving up the energy conditions, see e.g. the discussion in [3].

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must be a function of $|\Phi|^2$. In what follows, we assume that U can be written as

$$U = \sum_{k\geqslant 1} c_k |\Phi|^{2k}, \tag{4}$$

the $k > 1$ terms taking effectively into account various interactions. Of interest here is the case of a potential which is not strictly positive definite. Then the polynomial $F(x) = \sum_{k\geqslant 1} c_k x^k$ is negative for some range of $x > 0$, which implies that at least one of coefficients c_k is smaller than zero. Since we assume[2] $\Phi \to 0$ asymptotically, the requirement to obtain a bound state imposes $c_1 = \mu^2 > 0$, with μ the scalar field mass.

3. Flat space solitons: Q-balls as scalarons

Let us start our discussion with the simple observation that when ignoring the gravity effects, a class of solutions of the model (1) is already known. We recall that in a flat spacetime background, the Klein–Gordon equation possesses non-topological soliton solutions–the so-called Q-balls, in which case the scalar has a harmonic time dependence, $\Phi = \phi(x)e^{-iwt}$ [13] (with $x^\mu = (x^a, t)$). As a result, the solutions possess a nonvanishing conserved Noether charge, $Q = 2w\int d^3x|\phi|^2$. Then, even though Φ depends on time, the energy–momentum tensor T_μ^ν is time independent and the effective action of this model reads

$$S_Q = -\int d^3x\, dt\, [\phi^*_{,a}\phi^{,a} - w^2|\phi|^2 + U_Q]. \tag{5}$$

The Q-balls have been extensively discussed in the literature (see the review work [12,15]) and they have found a variety of physically interesting applications.[3] If one assumes a potential of the form (4), then U_Q necessarily contains powers of $|\Phi|^2$ higher than two, the usual choice in the literature being $U_Q = \mu^2|\Phi|^2 - \lambda|\Phi|^4 + \nu|\Phi|^6$, with $\lambda > 0$, $\nu > 0$ and $\lambda^2 < 4\mu^2\nu$ for a positive potential.

However, one can see from (5) that w^2 acts as an effective tachyonic contribution to the mass term, and thus it can be absorbed into μ^2. The scalar field is static in this case, $\Phi = \phi(x^a)$ and thus the Noether charge vanishes. Therefore all Q-ball solutions in a flat spacetime background can be interpreted as static scalar solitons, i.e. they become scalarons in a model with a shifted scalar field mass, for a new potential $U = U_Q - w^2|\phi|^2$. Note that although ϕ satisfies the same equation as before, the energy–momentum tensor and the total mass of the scalarons are different. Also, as implied by the Derick-type virial identity

$$\int d^3x\, [\phi^*_{,a}\phi^{,a} + 3U] = 0, \tag{6}$$

the redefined potential U is necessarily negative for some range of $|\phi|^2$ which is realised by the solutions. However, the scalarons' total mass is strictly positive,

$$M = \int d^3x\, [\phi^*_{,a}\phi^{,a} + U] = \frac{2}{3}\int d^3x\, \phi^*_{,a}\phi^{,a}. \tag{7}$$

Finally, we mention also that the mass of the scalarons is fixed by the mass M_Q and the Noether charge Q of the Q-balls, $M = M_Q - wQ$.

[2] This can always be realized *via* a redefinition of the scalar field.

[3] For example, the Q-ball solutions appear in supersymmetric generalizations of the standard model [16]. Also, they may be responsible for the generation of baryon number or may even be regarded as candidates for dark matter [17].

4. Spherically symmetric, gravitating solutions

However, the curved spacetime scalarons cannot be interpreted as boson stars and thus require a separate study. For example, following [14], one can show that, even in the absence of backreaction, one cannot add a black hole horizon inside a Q-ball[4] (this follows essentially because a Q-ball possesses an e^{-iwt} time dependence and $t \to \infty$ at the horizon of a black hole). However, this obstruction does not apply to scalarons, which possess finite energy, regular generalizations also for a static black hole background.

Let us start with a discussion of the spherically symmetric gravitating solutions of the model (1). These configurations are easier to study and some of their properties seem to be generic. A sufficiently general metric ansatz in this case reads

$$ds^2 = g_{rr}\,dr^2 + g_{\Omega\Omega}\,d\Omega_2^2 + g_{tt}\,dt^2, \tag{8}$$

(with $d\Omega_2^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$), and the scalar field is a function of r only, $\Phi = Z(r)$. One possible direction here is to choose a metric gauge with $-g_{tt} = 1/g_{rr} = V(r)$, $g_{\Omega\Omega} = P^2(r)$. Then the Einstein equations imply the relation $\frac{P''}{P} + 8\pi G Z'^2 = 0$ (where the prime denotes a derivative with respect to r). The approach taken in [5,6] (see also [7]) is to postulate an expression for the scalar field and to use this relation to derive P. In the next step, the remaining Einstein equations are used to reconstruct the scalar potential U and the metric function V compatible with Z and P. This approach has the advantage to lead to partially closed form solutions, but the resulting expressions are very complicated; also the potential cannot be written in the form (4).

In what follows we solve the field equations numerically for a given potential. In this case it is convenient to work in Schwarzschild-like coordinates with

$$g_{rr} = \frac{1}{N(r)}, \qquad g_{\Omega\Omega} = r^2,$$
$$g_{tt} = -N(r)\sigma^2(r), \quad \text{with } N(r) = 1 - \frac{2m(r)}{r}, \tag{9}$$

where $m(r)$ may be interpreted as the total mass–energy within the radius r; its derivative m' is proportional to the energy density $\rho = -T_t^t$. Then the field equations (2) reduce to

$$m' = 4\pi G r^2\big(NZ'^2 + U\big), \qquad \sigma' = 8\pi G r\sigma Z'^2,$$
$$Z'' + \left(\frac{\sigma'}{\sigma} + \frac{N'}{N} + \frac{2}{r}\right)Z' - \frac{1}{N}\frac{\partial U}{\partial Z^2}Z = 0. \tag{10}$$

For a generic U, it is possible to write an approximate form of the solutions close to the horizon (or at the origin) and also for large r. These asymptotics are connected by constructing numerically the solutions, which require to specify the expression for the scalar field potential.

The horizon of the black holes is located at $r = r_H > 0$, where the solutions have a power-series expansion

$$m(r) = \frac{r_H}{2} + m_1(r - r_H) + \cdots,$$
$$\sigma(r) = \sigma_0 + 8\pi G\sigma_0 r_H z_1^2(r - r_H) + \cdots,$$
$$Z(r) = z_0 + z_1(r - r_H) + \cdots, \tag{11}$$

in terms of two arbitrary parameters $Z(r_H) = z_0$ and $\sigma(r_H) = \sigma_0$ (with $m_1 = 4\pi G r_H^2 U(z_0)$, and $z_1 = \frac{r_H}{1-2m_1}\frac{\partial U}{\partial Z^2}|_{z_0} z_0$). One can write an approximate form of the solutions also for $r \to \infty$, with

[4] Note, however, the boson shells harbouring black holes in [18]. These solutions require a V-shaped scalar potential which is not of the form (4).