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Bifurcations in Boltzmann–Langevin one body dynamics for fermionic systems

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ABSTRACT

We investigate the occurrence of bifurcations in the dynamical trajectories depicting central nuclear collisions at Fermi energies. The quantitative description of the reaction dynamics is obtained within a new transport model, based on the solution of the Boltzmann–Langevin equation in three dimensions, with a broad applicability for dissipative fermionic dynamics.
Dilute systems formed in central collisions are shown to fluctuate between two energetically favourable mechanisms: reverting to a compact shape or rather disintegrating into several fragments. The latter result can be connected to the recent observation of bimodal distributions for quantities characterising fragmentation processes and may suggest new investigations.

1. Introduction

Phase transitions are general phenomena occurring in interacting many-body systems [1–4]. Over the past years, many efforts have been devoted to the identification of new features related to finite-size effects. As shown by recent thermodynamical analyses, first-order phase transitions in finite systems are characterised by negative specific heat and bimodal behaviour of the distribution of the order parameter [5,6]. The latter physically corresponds to the simultaneous presence of different classes of physical states for the same value of the system conditions that trigger the transition (like the temperature, for instance).

In particular, the appearance of phase transitions from the liquid to the vapour phases has been widely investigated in the context of the nuclear multifragmentation phenomenon [7,8,4,9]. Indeed, due to the analogies between the nuclear forces and the Van-der-Waals interaction, the nuclear matter equation of state (EOS) foresees such a possibility [10,11]. The theoretical findings cited above have stimulated corresponding thermodynamical analyses of the properties of the products issued from nuclear reactions at Fermi energies. Under suitable conditions, a bimodal character of experimental observables, such as the size of the heaviest cluster produced in each collision event [12], or the asymmetry between the charges of the two heaviest reaction products [13] has been revealed. Many investigations have also been focused on the complex nuclear many-body dynamics, to probe the reaction mechanisms governing the occurrence of phase transitions [14–17]. Within such a context, nuclear fragmentation studies at intermediate energies (above 50 MeV per nucleon) have recently pointed out that bimodality could have a dynamical origin, related to the fragment-formation mechanism [18], without necessarily requiring the reaching of thermodynamical equilibrium.

From a general point of view, interacting many-body systems may experience a very rich dynamics, ranging from mechanisms dominated by one-body (mean-field) effects to phenomena governed by strong fluctuations and correlations. In the regime of low-energy collective processes, nuclear dynamics presents a rather stable character; this is the domain where the fluctuation mechanism can be described in the small-amplitude limit, restricting to mean-field (quantum) fluctuations of collective observables [19,20]. This limit is exceeded when violent perturbations, like for instance dissipative heavy-ion collisions, bring the system beyond the one-body collective dynamics, with two-body nucleon collisions and correlations playing an important role. Along the compression–expansion path traced by the nuclear reaction, fluctuations introduce the anisotropy seeds from which 'nuclear droplets' can develop. More precisely, the system may access mechanically unstable regions of the EOS, called spinodal, where a density rise is related to a pressure fall; there, phase-space fluctuations are even amplified, leading to phase separation [17,8]. As soon as a mottling pattern stands out at low density, i.e. at the boundary of the phase separation, a bundle of bifurcations into a variety of different dynamical paths may set in.

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2. The Boltzmann–Langevin-One-Body model

The aim of this Letter is to further investigate the dynamical trajectory of disassembling nuclear systems, seeking for features associated with phase transitions. In particular, we will explore the possible occurrence of bifurcation patterns and bimodal behaviour in central heavy-ion reactions at beam energies around the multifragmentation threshold.

This study is undertaken in the framework of a new numerical implementation of the Boltzmann–Langevin (BL) equation, well suited to describe out-of-equilibrium processes, such as nuclear collisions: the Boltzmann–Langevin-One-Body (BLOB) model. The BL equation describes the time evolution of the semiclassical one-body distribution function $f(\mathbf{r},\mathbf{p},t)$ in response to the mean-field potential, incorporating the effect of fluctuations and correlations due to hard two-body scattering [21–24].

Hence the distribution function f evolves according to the action of the effective Hamiltonian $H[f]$, the average Boltzmann collision integral $\bar{I}[f]$, and the fluctuating term $\delta I[f]$ as:

$$\dot{f} = \partial_t f - \{H[f], f\} = \bar{I}[f] + \delta I[f]. \tag{1}$$

This form indicates that the residual interaction, represented by the right-hand side of Eq. (1), is expressed in terms of the one-body distribution function f.

Like in standard transport approaches [25], we sample the dynamics through the test-particle method, under the assumption of spatial and temporal locality of the two-body collisional process. N_{test} particles per nucleon are employed. The Boltzmann–Langevin theory describes fluctuations of f on a size scale of h^3, but it leaves the shape of such a phase-space volume arbitrary (see the discussion in Ref. [26]). This same arbitrarity characterises other molecular-dynamics or Boltzmann-like approaches. In this respect, we chose to follow the prescription of Bauer and Bertsch [27]: in order to solve the BL equation, they proposed to define nucleon wave packets by organising test particles in phase-space agglomerates of N_{test} elements. However, in Ref. [27] Pauli blocking was checked only for the centroids of the nucleon clouds: the effect of such approximation on the fermionic dynamics was analysed in Refs. [23,28], where it was concluded that an incomplete treatment of Pauli blocking affects the mechanism of fluctuation development. The above recommendation was taken into account in Ref. [26], in treating the schematic case of nuclear matter in a periodic box: such approach confirmed that an accurate treatment of the Pauli blocking is the key for correctly describing the fluctuation mechanism in full phase space. By further improving the above approach in a full model for heavy-ion collisions, we built a novel numerical procedure where nucleon–nucleon (N–N) correlations are implemented by accurately treating the Pauli-blocking factors of agglomerates of N_{test} elements of identical isospin. This arrangement, which simulates nucleon wave packets, is redefined at successive time steps and locally, for couples of colliding agglomerates. According to this rescaling and for elastic N–N collisions only, the average rate of change of the occupancy f_a around the phase-space location $(\mathbf{r}_a,\mathbf{p}_a)$ at a given time takes the form:

$$\dot{f}_a(\mathbf{r}_a,\mathbf{p}_a) = g\int \frac{\mathrm{d}\mathbf{p}_b}{h^3}\int \mathrm{d}\Omega\, W_{(\mathrm{AB}\leftrightarrow\mathrm{CD})}\, F_{(\mathrm{AB}\to\mathrm{CD})}, \tag{2}$$

where g denotes the degeneracy, and integrations are over momenta $\mathbf{p}_b$ and scattering angles Ω. The first integration argument is the symmetric transition rate from an intermediate state AB to a final configuration CD. To select the test particles defining the nucleon wave packets A and B, we adopt the following procedure: we consider a sphere, centred at the position $\mathbf{r}_a$, with radius equal to the scattering distance, associated with the free elastic N–N cross section at Fermi energies (taken equal to 50 mb); among all test particles inside the sphere, we pick up the N_{test} closest particles to the elements a and b in momentum space, respectively. The final state is represented by $c\in\mathrm{C}$ and $d\in\mathrm{D}$. The transition rate is obtained by averaging over all couples of test particles involved in the transition $\Sigma = (\mathrm{AB}\to\mathrm{CD})$, in terms of relative velocity and differential N–N cross section:

$$W_{(\mathrm{AB}\leftrightarrow\mathrm{CD})} = \left\langle |v_a - v_b| \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right\rangle_{\Sigma} = \left\langle W_{(\mathrm{ab}\leftrightarrow\mathrm{cd})}\right\rangle_{\Sigma}. \tag{3}$$

The second integration argument contains the product of occupancies of the entire agglomerates $f_{\mathrm{A}\ldots\mathrm{D}}$ and of the associated vacancies $\bar{f}_{\mathrm{A}\ldots\mathrm{D}}$:

$$F_{(\mathrm{AB}\to\mathrm{CD})} = \bar{f}_{\mathrm{A}}\bar{f}_{\mathrm{B}}f_{\mathrm{C}}f_{\mathrm{D}} - f_{\mathrm{A}}f_{\mathrm{B}}\bar{f}_{\mathrm{C}}\bar{f}_{\mathrm{D}} = \left\langle F_{(\mathrm{ab}\to\mathrm{cd})}\right\rangle_{\Sigma}. \tag{4}$$

Rewritten in terms of test-particles, the representation of Eq. (4) indicates that only the fraction of the packets which are really modified by the scattering can significantly contribute to the transition probability, while overlapping volumes contribute to the Pauli-blocking factors.

On this basis, full phase-space fluctuations are introduced in the equation of motion by moving simultaneously the test-particle agglomerates, in analogy with the extended-TDHF procedure of including perturbations in the Slater configuration [29]. The scattering is decided by confronting the probability $W\times F$ with a random number and scanning the entire phase space in search of collision configurations at successive time steps. Since all test particles belonging to the agglomerates A and B can be reconsidered as starting points of new collision processes, the scattering probability has to be suitably rescaled, dividing it by N_{test}^2. Once the sorting allows for a scattering to occur, modulation functions are applied to precisely adapt the density profile of final-states to the available vacancy profile $\bar{f}_{\mathrm{A}\ldots\mathrm{D}}$, with the requirement of imposing the most compact configuration compatible with the constraint of energy conservation [30]; the resulting occupation functions of the modulated final-state density profiles $(f_{\mathrm{A}\ldots\mathrm{D}})_{\mathrm{M}}$ should approach unity.

The extension of the wave packets makes necessary to pay special attention to scatterings close to the surface of the system, i.e. occurring across potential boundaries: confronting the shape of the wave packet to the shape of the surface, the blocking factors are increased in proportion to the spread of the nucleon packet outside of the boundary (similarly to what is done in some molecular-dynamics approaches [14]).

It occurs in some situations, for instance when low densities are attained, that the nuclear system is brought to explore regions of the phase diagram where it becomes unstable against density fluctuations, like the spinodal region. The action of the BL term results in agitating the density profile over several wave lengths. It can be proven for the proposed BL approach (through an analysis of the linear response in the mean field, see Refs. [31,17]) that the amplitude of the unstable modes grows according to the specific dispersion relation associated with the employed mean-field interaction.

3. Application to head-on heavy-ion collisions at Fermi energies

In the following, the BLOB model is applied to a highly constraining phenomenology: the low-energy threshold for multifragmentation in head-on heavy-ion collisions at Fermi energies. We will also refer to results of the so-called Stochastic Mean Field (SMF) model [24,17], that corresponds to an approximate