Physics Letters B 725 (2013) 504–509

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Analytic structure of ϕ^4 theory using light-by-light sum rules

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ARTICLE INFO

Article history:
Received 8 April 2013
Received in revised form 22 July 2013
Accepted 29 July 2013
Available online 5 August 2013
Editor: J.-P. Blaizot

Keywords:
Sum rules
Light-by-light scattering
Analyticity
Scalar electrodynamics

ABSTRACT

We apply a sum rule for the forward light-by-light scattering process within the context of the ϕ^4 quantum field theory. As a consequence of the sum rule a stringent causality criterion is presented and the resulting constraints are studied within a particular resummation of graphs. Such resummation is demonstrated to be consistent with the sum rule to all orders of perturbation theory. We furthermore show the appearance of particular non-perturbative solutions within such approximation to be a necessary requirement of the sum rule. For a range of values of the coupling constant, these solutions manifest themselves as a physical bound state and a K-matrix pole. For another domain however, they appear as tachyon solutions, showing the inconsistency of the approximation in this region.

1. Introduction

Sum rules provide a powerful tool to study relativistic quantum field theories, and apply also outside the regime where perturbative expansions hold. As sum rules are consequences of such general principles as analyticity and unitarity, they allow to establish rigorous relations between physical observables, even when the underlying theory is non-perturbative in nature and cannot be solved exactly.

In recent works [1,2], we have derived three sum rules for the low-energy forward light-by-light scattering process. These $\gamma\gamma$-sum rules are non-perturbative in origin and demonstrate that the low and high-energy behaviors of the theory are related. We showed e.g. that a sum rule for the helicity-difference total cross-section of the photon–photon-fusion process, $\gamma\gamma \to X$, reveals in the hadron sector an intricate correlation between contributions of pseudoscalar and tensor mesons. In the charm quark sector, the $\gamma\gamma$ sum rules reveal an interplay between $c\bar{c}$ bound states, open charmed meson continuum states, as well as exotic resonance states [2]. Several experiments, primarily at e^+e^- collider facilities, are presently uncovering the rich spectroscopy of such systems, see e.g. [3,4] for some recent reviews.

Besides its relevance to hadron phenomenology, we can use the $\gamma\gamma$-sum rules in the same way in model field theories, where in the case of renormalizable models they can be applied perturbatively. When the conditions of applicability are fulfilled these sum rules were shown to hold in leading order calculations [2]. However, the realization of the causality constraints at higher orders as well as in the non-perturbative regime of quantum field theories is still an open issue.

Studies of causality constraints on the basis of different sum rules were carried out in the past in a number of different contexts. Especially the realization of the well-known Gerasimov–Drell–Hearn sum rule [5] within perturbative field theory was analyzed for spin-1/2 targets at the lowest nontrivial order [6] as well as at higher orders in QED [7]. In Refs. [8,9] consequences of the sum rules within asymptotically free theories were considered. In more recent years, they have also been discussed within the context of quantum gravity [10,11].

In the present work, we are using light-by-light scattering sum rule as a tool to study causality constraints within a model field theory, the ϕ^4 scalar theory. We consider a bubble-chain resummation and demonstrate it to be consistent with causality to all orders of perturbation theory. Furthermore, it is shown that the sum rule strictly defines the non-perturbative structure of the solutions which arise dynamically within this approximation. In a particular regime of the coupling constant the spectrum of solutions contains a dynamically generated bound state and a K-matrix pole. For another domain the solution possesses an unphysical pole

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http://dx.doi.org/10.1016/j.physletb.2013.07.058

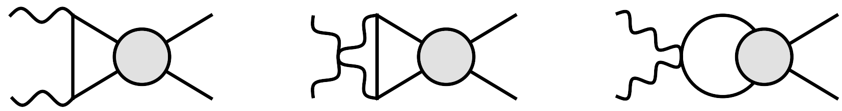

Fig. 1. The contribution to the $\gamma\gamma$-fusion process within the ϕ^4 field theory considered in this work. The solid lines denote the charged scalar fields.

with negative invariant mass being a direct sign of the inconsistency of the approximation.

The outline of this Letter is as follows. In Section 2, we compute the light-by-light scattering sum rule involving the helicity difference cross section for the $\gamma\gamma \to X$ process, within the ϕ^4 scalar field theory at one-loop level. In Section 3, we provide a calculation beyond the one-loop level in the "bubble-chain" approximation. In Section 4 we discuss how causality imposes constraints on the solutions for different values of the renormalized self-interaction coupling constant of the ϕ^4 theory. The summary and outlook are given in Section 5.

2. One loop

In this work, we will focus on a sum rule for the forward light-by-light scattering, which involves the helicity-difference cross-section for real photons [12,13,1] and reads as:

$$\int_{s_0}^{\infty} ds \, \frac{\Delta\sigma(s)}{s} = 0, \tag{1}$$

where $\Delta\sigma(s) = \sigma_2(s) - \sigma_0(s)$ is the total helicity-difference cross section of the two-photon fusion process $\gamma\gamma \to X$, with the Mandelstam variable $s = (q_1 + q_2)^2$, where q_1 and q_2 are the two photon 4-momenta, and s_0 is the lowest production threshold of the process.

We will study the above sum rule in a particular model quantum field theory. We take one of the simplest examples: a self-interacting scalar field $\phi(x)$ with charge e and mass m as described by the following Lagrangian density,

$$\mathcal{L} = \left(D^\mu \phi\right)^* D_\mu \phi - m^2 \phi^* \phi - \frac{\lambda_0}{4} \left(\phi^* \phi\right)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \tag{2}$$

where λ_0 is the self-interaction coupling constant, while the covariant derivatives and electromagnetic field-strength tensor are given as usual by $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

We denote the helicity amplitudes for the process $\gamma\gamma \to \phi\phi^*$ by M_{++} and M_{+-}, where the subscripts indicate the photon helicities. Given these amplitudes, the cross section for total helicity-0 and 2 are found as:

$$\sigma_0(s) = \frac{\beta}{32\pi s} \int_{-1}^{1} d\cos\theta \left|M_{++}(s, \theta)\right|^2, \tag{3}$$

$$\sigma_2(s) = \frac{\beta}{32\pi s} \int_{-1}^{1} d\cos\theta \left|M_{+-}(s, \theta)\right|^2, \tag{4}$$

where θ is the angle of one of the members of the $\phi\phi^*$ pair w.r.t. the photon in the center-of-mass system, and where we introduced their relative velocity β as:

$$\beta = \sqrt{1 - \frac{4m^2}{s}}. \tag{5}$$

To leading order in $\tilde{\lambda}_0 \equiv \lambda_0/(4\pi)^2$ and in the fine-structure constant $\alpha \equiv e^2/4\pi$, the helicity amplitudes are found to be:

$$M_{++}(s, \theta) = 4\pi\alpha \left\{ \frac{2(1-\beta^2)}{1-\beta^2\cos^2\theta} + \tilde{\lambda}_0 2F(s) \right\}, \tag{6a}$$

$$M_{+-}(s, \theta) = 4\pi\alpha \frac{2\beta^2 \sin^2\theta}{1-\beta^2\cos^2\theta}, \tag{6b}$$

where the first term in M_{++} and the expression for M_{+-} correspond with the tree level amplitudes for $\gamma\gamma \to \phi\phi^*$ in scalar QED. The second term in M_{++} describes the one-loop production process corresponding with Fig. 1, where the gray blob in this case is given by the tree-level vertex, i.e. the four-particle pointlike coupling, proportional to λ_0. Furthermore in Eq. (6a), $F(s)$ denotes the (dimensionless) form factor describing the transition of photons to a scalar pair at one-loop order. The explicit expression for the one-loop production process is given by:

$$\begin{aligned} M_{++}^{(1\text{-loop})} &= -i4\pi\alpha\tilde{\lambda}_0 2\mu^{4-d} \varepsilon_\mu(q_1, \lambda_1 = +1)\varepsilon_\nu(q_2, \lambda_1 = +1) \int \frac{d^d l}{(2\pi)^d} \\ &\quad \times \left\{ \frac{(2l+q_1)_\mu (2l-q_2)_\nu}{((q_1+l)^2 - m^2)((q_2-l)^2 - m^2)(l^2 - m^2)} \right. \\ &\quad \left. - \frac{g_{\mu\nu}}{(l^2 - m^2)((q_1+q_2-l)^2 - m^2)} \right\}, \end{aligned} \tag{7}$$

where $\varepsilon^\mu(q, \lambda)$ denotes the photon polarization vector, with helicity $\lambda = \pm 1$. In Eq. (7), the first term describes the contribution of the first and the second diagrams and the second term corresponds to the contribution of the third graph in Fig. 1. Note that all three diagrams of Fig. 1 contribute only to M_{++}, i.e. helicity-0 amplitude, because we have only s-wave rescattering. Helicity-2 contributions would necessarily involve the d-waves and higher due to conservation of the angular momentum. For exactly the same reason we have no angular dependence of the loop contribution. The sum of the 3 diagrams is finite because it is proportional $q_1 q_2$, i.e., the two external momenta. This happens indeed due to current conservation. In this case the superficial divergence of the result is less 2 of the superficial divergence of each diagram. The explicit expression for $F(s)$ is given by:

$$F(s) = 1 + \frac{m^2}{s} \left[\ln \frac{1+\beta}{1-\beta} + i\pi \right]^2. \tag{8}$$

It is quite easy now to compute the cross sections for $\gamma\gamma \to \phi\phi^*$, the result for the helicity difference cross-section is:

$$\Delta\sigma(s) = \Delta\sigma^{(\text{tree})}(s) - \alpha^2 \tilde{\lambda}_0 \frac{4\pi}{s} \beta\xi \,\mathrm{Re}\, F(s), \tag{9}$$

where $\Delta\sigma^{(\text{tree})}$ is the tree-level cross section in scalar QED (cf., e.g., Appendix in [2]), and where we used the notation

$$\xi = \frac{(1-\beta^2)}{\beta} \ln \frac{1+\beta}{1-\beta}. \tag{10}$$

The tree-level cross section weighted with $1/s$ integrates to zero by itself. For the one-loop contribution, it was verified both analytically and numerically that

$$\int_{4m^2}^{\infty} ds \, \frac{\mathrm{Re}\, F(s)}{s^3} \ln \frac{1+\beta}{1-\beta} = 0. \tag{11}$$