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Two-loop renormalization factors of dimension-six proton decay operators in the supersymmetric standard models

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ABSTRACT

The renormalization factors of the dimension-six effective operators for proton decay are evaluated at two-loop level in the supersymmetric grand unified theories. For this purpose, we use the previous results in which the quantum corrections to the effective Kähler potential are evaluated at two-loop level. Numerical values for the factors are presented in the case of the minimal supersymmetric SU(5) grand unified model. We also derive a simple formula for the one-loop renormalization factors for any higher-dimensional operators in the Kähler potential, assuming that they are induced by the gauge interactions.

1. Introduction

Discovery of the Higgs boson [1,2] may suggest the existence of supersymmetry (SUSY). The supersymmetric theories may accommodate the hierarchical structure with the great desert naturally. Searches for rare processes, such as proton decay, would be one of methods to access the physics beyond the supersymmetric standard models (SUSY SMs). The processes are dictated with the effective higher-dimensional operators. When comparing the prediction with the observation precisely, we need to include the radiative corrections correctly.

The realization of the gauge coupling unification strongly motivates us to study the supersymmetric grand unified theories (SUSY GUTs) [3–6]. In the theories, proton decay is induced by the exchanges of the colored Higgs multiplets and the X gauge bosons, which yield the baryon and lepton number non-conserving interactions. They are expressed in terms of the dimension-five and -six effective operators, respectively. It is found that the former interactions in general give rise to dominant channels for proton decay, such as $p \to K^+\bar{\nu}$ [7,8]. However, the current experimental limits on the channel, $\tau(p \to K^+\bar{\nu}) > 3.3 \times 10^{33}$ yrs [9], are so severe that the contribution of the dimension-five operators is required to be suppressed by a certain mechanism; otherwise the model is excluded just as the case of the minimal SUSY SU(5) GUT unless the SUSY particles in the SUSY SM are much heavier than the weak scale [10,11]. A variety of such mechanisms have been proposed. For example, the Peccei–Quinn symmetry [12] would be exploited for the purpose. The R symmetry also plays a role in suppressing the dimension-five proton decay in the models with extra dimensions [13]. With such a suppression mechanism imposed, the dimension-six operators in turn become dominant. In this case, the main decay mode is the $p \to \pi^0 e^+$ channel; the present experimental limit on its lifetime is given by $\tau(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ yrs [14]. Since in the SUSY GUTs the GUT scale $M_{\rm GUT}$ is relatively high, *i.e.*, $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV, the predicted proton lifetime usually evades the experimental limit. However, the consequence might be altered if there exist extra particles in the intermediate scale. With such particles belonging to a representation of the grand unified group, the gauge coupling unification is still achieved, while its value at the unified scale turns out to be enhanced. Then, the proton lifetime is considerably reduced due to the large gauge coupling [15].

In order to study such possibilities based on the proton decay experiments, it is important to make a precise prediction for the decay rate. To that end, we need to determine the effects of the dimension-six operators, which are generated at the GUT scale, on the low-energy physics by using the renormalization group equations (RGEs). Indeed, there have been several literature in which the renormalization factors for the effective operators are evaluated. In Ref. [16], the long-distance QCD corrections are computed at two-loop level. For the short-distance factors, on the other hand, only the one-loop calculation is carried out in Ref. [17] in the SUSY SM.

In this Letter, therefore, we evaluate the renormalization factors of the dimension-six operators at two-loop level in the presence of the supersymmetry. In the calculation, we use the results for the two-loop corrections to the effective Kähler potential given in Ref. [18] Since in the SUSY GUTs, the most of the intermediate

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energy scales are supersymmetric, the short-distance renormalization factors are well approximated by those evaluated in purely SUSY theory. Thus, combined with the long-distance effects given in Ref. [16], our results offer a tool for making a prediction of the proton decay rate with accuracy of two-loop level.

We also derive a simple formula for the one-loop level renormalization factors of any higher-dimensional operators in the Kähler potential, when including only the gauge interaction contributions. It is applicable to other observables, such as the neutron–antineutron oscillation [19].

This Letter is organized as follows: in Section 2, we first write down the dimension-six effective operators in terms of the superfield notation. Our notations and conventions are also shown in the section. Then, in the subsequent section, we describe a way of calculating the renormalization factors of the operators by using the effective Kähler potential, and present the results for the computation. In Section 4, the comparison of the one- and two-loop renormalization factors is discussed in the minimal SUSY SU(5) GUT. Section 5 is devoted to conclusion and discussion.

2. Dimension-six effective operators

To begin with, we write the dimension-six effective operators for proton decay in a SUSY and gauge invariant manner with superspace notation:

$$\mathcal{O}^{(1)} = \int d^2\theta\, d^2\bar{\theta}\, \epsilon_{abc}\epsilon_{ij}\big(\overline{U}^\dagger\big)^a\big(\overline{D}^\dagger\big)^b e^{-\frac{2}{3}g_Y V_1}\big(e^{2g_3V_3}Q_i\big)^c L_j,$$
$$\mathcal{O}^{(2)} = \int d^2\theta\, d^2\bar{\theta}\, \epsilon_{abc}\epsilon_{ij}\overline{E}^\dagger e^{\frac{2}{3}g_Y V_1}\big(e^{-2g_3V_3}\overline{U}^\dagger\big)^a Q_i^b Q_j^c, \tag{1}$$

where all the chiral superfields correspond to left-handed fermions, and V_1 and V_3 are the $U(1)_Y$ and $SU(3)_C$ vector superfields with the gauge coupling constants g_Y and g_3, respectively. The subscripts i, j, are the $SU(2)_L$ indices, while a, b, c are the color indices. Furthermore, we omit the generation indices for simplicity.

The relationship between bare and renormalized operators is written in the following form:

$$\mathcal{O}_B^{(I)} = Z^{(I)}\mathcal{O}^{(I)} \quad (I = 1, 2), \tag{2}$$

where the subscript B indicates the operator is bare. Then, the Wilson coefficients $C^{(I)}$ for the operators $\mathcal{O}^{(I)}$ obey the differential equations,

$$\mu\frac{d}{d\mu}C^{(I)}(\mu) = \gamma_{\mathcal{O}^{(I)}}C^{(I)}(\mu), \tag{3}$$

with $\gamma_{\mathcal{O}^{(I)}}$ the anomalous dimensions for the operators defined as

$$\gamma_{\mathcal{O}^{(I)}} \equiv \mu\frac{d}{d\mu}\ln Z^{(I)}. \tag{4}$$

The anomalous dimensions are obtained by analyzing the vertex functions (or the effective action) in which the operators are inserted. Since we now deal with the dimension-six operators which contain four chiral or anti-chiral superfields, it is sufficient to consider the four-point vertex functions which include the corresponding external superfields. Their renormalization group equations (RGEs) are given as

$$\left[\mu\frac{\partial}{\partial\mu} + \beta_\alpha\frac{\partial}{\partial g_\alpha} - \sum_i \gamma_i + \gamma_{\mathcal{O}^{(I)}}\right]\Gamma_{\mathcal{O}^{(I)}} = 0. \tag{5}$$

Here, $\Gamma_{\mathcal{O}^{(I)}}$ are the four-point vertex functions with an insertion of the operators $\mathcal{O}^{(I)}$. The gauge coupling constants and their beta functions are denoted by g_α and β_α, respectively, and the sum over each gauge group is implicit. Further, γ_i shows the anomalous dimension of each superfield contained in the operators. From now on, we often omit the superscript (I) for brevity.

3. Renormalization factors

In this section, we present the formulae for the renormalization factors. They are derived from the effective Kähler potential given in Ref. [18]. In the calculation, the dimensional reduction scheme ($\overline{\rm DR}$) [20] is employed for the regularization. We first obtain the one-loop results and confirm the results in Ref. [17] in the former subsection. Then, in the latter subsection, we evaluate the two-loop contribution.

3.1. One-loop

Let us first evaluate the vertex functions at one-loop level. For this purpose, we use the results in Ref. [18], where the effective Kähler potential for generic four-dimensional $N = 1$ SUSY theories is computed up to two-loop level. According to the results, the one-loop correction[1] to the Kähler potential is given as

$$\Delta K_1 = -\sum_\alpha \frac{1}{16\pi^2}\,\mathrm{Tr}\, M^2_{C(\alpha)}\left(2 - \ln\frac{M^2_{C(\alpha)}}{\bar{\mu}^2}\right), \tag{6}$$

where $\bar{\mu}^2 \equiv 4\pi e^{-\gamma}\mu^2$ defines the $\overline{\rm MS}$ renormalization scale, and the mass matrix $M^2_{C(\alpha)}$ is defined by

$$\big(M^2_{C(\alpha)}\big)_{AB} \equiv 2g_\alpha^2\bar{\phi}_a\big(T_A^{(\alpha)}\big)^a{}_b G^b{}_c\big(T_B^{(\alpha)}\big)^c{}_d\phi^d, \tag{7}$$

with ϕ the background for the chiral superfield Φ and $G^a{}_b$ the Kähler metric

$$G^a{}_b \equiv \frac{\partial^2}{\partial\bar{\phi}_a\partial\phi^b}K(\bar{\phi}, \phi). \tag{8}$$

In Eq. (6), Tr denotes the trace over the adjoint representation of a gauge group whose coupling constant is g_α and generators are given by $T_A^{(\alpha)}$. Moreover, in the following calculation, we only take the gauge interactions into account, *i.e.*, we neglect the superpotential.[2]

In order to obtain the renormalization factors for the higher-dimensional effective operators, we consider the Kähler potential

$$K = \bar{\phi}_a\phi^a + C\mathcal{O} + C\mathcal{O}^\dagger, \tag{9}$$

with C the Wilson coefficient of the operator $\mathcal{O}$. In this case, the Kähler metric reads

$$G^a{}_b = \delta^a_b + C\mathcal{O}^a{}_b + C\mathcal{O}^{\dagger a}{}_b, \tag{10}$$

with $\mathcal{O}^a{}_b \equiv \partial^2\mathcal{O}/\partial\bar{\phi}_a\partial\phi^b$. By substituting the above equations to Eq. (6), we have

$$\Delta K_1 = -\sum_\alpha \frac{g_\alpha^2}{16\pi^2}2\big(1 + \ln\bar{\mu}^2\big)\big[C_\alpha(a)\bar{\phi}_a\phi^a$$
$$+ \big\{C\big(\bar{\phi}T_A^{(\alpha)}\big)_a\mathcal{O}^a{}_b\big(T_A^{(\alpha)}\phi\big)^b + \text{h.c.}\big\}\big], \tag{11}$$

where $C_\alpha(i)$ are the quadratic Casimir group theory invariants for the superfield Φ_i, defined in terms of the Lie algebra generators T_A by $(T_A^{(\alpha)}T_A^{(\alpha)})^a_b = C_\alpha(i)\delta^a_b$. Further, we keep only the terms up to the first order with respect to the Wilson coefficient, C, and do

[1] This one-loop result is first derived in Ref. [21].

[2] Experimental constraints on the effective operators in Eq. (1) are particularly severe when the external lines of the operators are of the first and/or second generations. In such a case, the size of the Yukawa couplings are negligible.