Physics Letters B 724 (2013) 301–305

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The Higgs mass and the emergence of new physics

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ARTICLE INFO

Article history:
Received 12 May 2013
Received in revised form 18 June 2013
Accepted 21 June 2013
Available online 27 June 2013
Editor: M. Cvetič

ABSTRACT

We investigate the physical implications of formulating the electroweak (EW) part of the Standard Model (SM) in terms of a superconnection involving the supergroup $SU(2/1)$. In particular, we relate the observed Higgs mass to new physics at around 4 TeV. The ultraviolet incompleteness of the superconnection approach points to its emergent nature. The new physics beyond the SM is associated with the emergent supergroup $SU(2/2)$, which is natural from the point of view of the Pati–Salam model. Given that the Pati–Salam group is robust in certain constructions of string vacua, these results suggest a deeper connection between low energy (4 TeV) and high energy (Planck scale) physics via the violation of decoupling in the Higgs sector.

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Introduction and overview

The Standard Model (SM) of particle physics is a phenomenally successful theory whose last building block has recently been detected [1,2]. In light of the apparent discovery of the Higgs boson, we address the connection between its mass and the structure of the electroweak (EW) sector of the SM, and argue that it points to some very exciting new physics at a rather low energy scale of 4 TeV.

A long time ago, Ne'eman [3] and Fairlie [4] independently discovered the relevance of a unique $SU(2/1)$ supergroup structure to SM physics. In this formalism, the even (bosonic) part of the $SU(2/1)$ algebra defines the $SU(2) \times U(1)$ gauge sectors of the SM, while the Higgs sector is identified as the odd (fermionic) part of the algebra. Although the model gives the correct quantum numbers of the SM, and it represents a more unified-hence more aesthetic-version of the SM, it suffers from the violation of the spin-statistic theorem, a common problem seen in the models using supergroups.[1]

In this work we adopt the superconnection approach of Ne'eman and Sternberg [5] who observed that the $SU_L(2) \times U_Y(1)$ gauge and Higgs bosons of the SM could be embedded into a unique $SU(2/1)$ superconnection, and the quarks and leptons into $SU(2/1)$ representations [6,7]. $SU(2/1)$ in this formalism is not imposed as a symmetry; it is rather only the structure group of the superconnection. Therefore, the $SU(2/1)$ structure can be interpreted as an emergent geometric pattern that involves the EW part of the SM, which avoids the problems with the ghosts.

The formalism fixes the ratio of the $SU_L(2) \times U_Y(1)$ gauge couplings, and thus the value of $\sin^2\theta_W$, and the quartic coupling of the Higgs. The value of $\sin^2\theta_W$ selects the scale $\Lambda \sim 4$ TeV[2] at which the superconnection relations can be imposed, and renormalization group (RG) running leads to a prediction of the Higgs mass. However, the claim of Refs. [6,7] that the predicted Higgs mass is around 130 GeV turns out to be incorrect.

In this Letter, we point out that the $SU(2/1)$ superconnection approach predicts the mass of the Higgs to be 170 GeV, which obviously disagrees with observation. Given the well-known issue with the ultraviolet incompleteness of the $SU(2/1)$ approach [6], which implies the emergent nature of this description, we should have no qualms in introducing new physics to fix the Higgs mass.

Here, we note a connection with the Spectral SM of Connes and collaborators [8,9] in which spacetime is extended to a product of a continuous four dimensional manifold by a finite discrete space with non-commutative geometry. The SM particle content and gauge structure are described by a unique geometry, where the Higgs appears as the connection in the extra discrete dimension [10]. Curiously, the original Higgs mass prediction of the Spectral SM was also 170 GeV [11], despite the fact that the boundary conditions imposed on the RG equations were quite different: in the Spectral SM, the usual $SO(10)$ relations among the gauge couplings are imposed at the GUT scale. In a recent paper

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[1] For example, there are anticommuting Lorentz scalars (the Higgs fields) which represent ghost-like degrees of freedom in the model.

[2] This scale is updated from the 5 TeV in Ref. [6] using more recent determinations of the gauge couplings. The difference does not play a noticeable role in the prediction of the Higgs mass.

0370-2693/$ – see front matter Published by Elsevier B.V.
http://dx.doi.org/10.1016/j.physletb.2013.06.044

[12] Chamseddine and Connes isolate a unique scalar degree of freedom that is responsible for the neutrino Majorana mass in their approach, which, when correctly coupled to the Higgs field, can reduce the mass of the Higgs boson to the observed value, 125–126 GeV.[3]

We argue that a similar 'fix' works for the superconnection formalism: one needs to introduce extra scalar degrees of freedom which modify the RG equations. We further point out that this can be accomplished by the embedding of $SU(2/1)$ into $SU(2/2)$, and thus, in effect, a left–right (LR) symmetric extension of the EW sector [13], which is also natural from the point of view of the Pati–Salam model [14]. The $SU(2/2)$ formalism, as in the $SU(2/1)$ case, selects the scale $\Lambda \sim 4$ TeV via the value of $\sin^2\theta_W$. Therefore, 4 TeV in this formalism is the *prediction* for the energy scale of *new physics*, which is the LR symmetric model in this case.

We also note the peculiarity of the Higgs sector, which due to the relation between the coupling and the mass, violates decoupling [15]. When interpreted from either the emergent superconnection or the non-commutative geometry viewpoint, this violation of decoupling offers an exciting connection between the SM and short distance physics, such as string theory, via the non-decoupling of the 4 TeV and the Planck scales.

In particular, the embedding of $SU(2/1)$ into $SU(2/2)$ would be interesting from the point of view of string vacua, where it has been observed that the Pati–Salam group appears rather ubiquitously in a large number of vacua [16]. Though we lack a fundamental understanding of this phenomenon, it is quite intriguing in our context as it would point to a new relationship between low energy (SM-like) and high energy physics (quantum-gravity-like) which is not seen in the standard effective field theory approach to particle physics.

The $SU(2/1)$ formalism and the Higgs mass

Here we summarize the superconnection approach to the SM based on the supergroup $SU(2/1)$ [6,7]. Obviously, this supergroup has as its bosonic subgroup the EW gauge group $SU(2)_L \times U(1)_Y$. What is highly non-trivial is that the embedding of $SU(2)_L \times U(1)_Y$ into $SU(2/1)$ also gives the correct quantum numbers for all the physical degrees of freedom. Furthermore, the Higgs sector comes out naturally as a counterpart of the gauge sector. These have natural analogs in the Spectral SM as well [8,9,12], as already emphasized in the conclusion to the review Ref. [6]. We concentrate on the superconnection formalism which should be understood as an emergent framework, because of the fundamental ultraviolet incompleteness of gauged supergroup theories.

We start by defining the supercurvature as $\mathcal{F} = \mathbf{d}\mathcal{J} + \mathcal{J}\cdot\mathcal{J}$ where $\mathcal{J}$ is the superconnection, which is of the form

$$\mathcal{J} = \begin{bmatrix} M & \phi \\ \bar{\phi} & N \end{bmatrix}. \tag{1}$$

Since we would like to embed $SU_L(2) \times U_Y(1)$ and the Higgs into $SU(2/1)$, M and N are respectively 2×2 and 1×1 g-even submatrices valued over one-forms, while ϕ and $\bar{\phi}$ are respectively 2×1 and 1×2 g-odd submatrices valued over zero-forms. The superconnection $\mathcal{J}$ is written as $\mathcal{J} = i\lambda_s^a J^a$, $a = 1, 2, \ldots, 8$. The generators λ_s^a are matrices with supertrace zero. Therefore, they are the usual $SU(3)$ λ-matrices except for λ_s^8 which is

$$\lambda_s^8 = \frac{1}{\sqrt{3}}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \tag{2}$$

To obtain the superconnection we need to make the identifications $J^i = W^i$ $(i = 1, 2, 3)$ and $J^8 = B$, where W^i and B are one-form fields corresponding to the $SU_L(2)$ and $U_Y(1)$ gauge bosons. The zero-form fields are identified as $J^4 \mp iJ^5 = \sqrt{2}\phi^{\pm}$, $J^6 - iJ^7 = \sqrt{2}\phi^0$, and $J^6 + iJ^7 = \sqrt{2}\phi^{0*}$.[4] Then, the superconnection is

$$\mathcal{J} = i\begin{bmatrix} \mathcal{W} - \frac{1}{\sqrt{3}}B\cdot\mathbf{I} & \sqrt{2}\Phi \\ \sqrt{2}\Phi^\dagger & -\frac{2}{\sqrt{3}}B \end{bmatrix}. \tag{3}$$

Here, $\mathcal{W} = W^i\tau^i$ (where τ^i are the Pauli matrices) and $\mathbf{I}$ is a 2×2 unit matrix, and $\Phi = [\phi^+\ \phi^0]^{\mathrm{T}}$. To obtain the supercurvature $\mathcal{F}$, we recall the rule for supermatrix multiplication [5,7]

$$\begin{bmatrix} A & C \\ D & B \end{bmatrix}\cdot\begin{bmatrix} A' & C' \\ D' & B' \end{bmatrix} = \begin{bmatrix} A\wedge A' + (-1)^{|D'|}C\wedge D' & A\wedge C' + (-1)^{|B'|}C\wedge B' \\ (-1)^{|A'|}D\wedge A' + B\wedge D' & (-1)^{|C'|}D\wedge C' + B\wedge B' \end{bmatrix} \tag{4}$$

where $|A|$ denotes the Z_2 grading of the differential form A. Then, the supercurvature (after introducing the dimensionless coupling g, $\mathcal{J} \to g\mathcal{J}$) reads as

$$\mathcal{F} = ig\begin{bmatrix} F_W - \frac{1}{\sqrt{3}}F_B\cdot\mathbf{I} + 2ig\Phi\Phi^\dagger & \sqrt{2}D\Phi \\ \sqrt{2}(D\Phi)^\dagger & -\frac{2}{\sqrt{3}}F_B + 2ig\Phi^\dagger\Phi \end{bmatrix} \tag{5}$$

where $D\Phi = d\Phi + ig\mathcal{W}\Phi + ig\frac{1}{\sqrt{3}}B\Phi$, $F_B = dB$ and $F_W = (F_W)^k\tau^k = [dW^k + ig\epsilon^{ijk}W^i\wedge W^j]\tau^k$. The action reads as follows

$$\begin{aligned} \mathcal{S} &= \int \frac{-1}{4g^2}\mathrm{Tr}\big[\mathcal{F}\cdot\mathcal{F}^\star\big] \\ &= \int\bigg(\frac{1}{2}\big[-(F_W)^i\wedge\big(F_W^*\big)^i - F_B\wedge F_B^*\big] \\ &\quad + (D\Phi)^\dagger\wedge(D\Phi)^* - \lambda\big(\Phi^\dagger\Phi\big)\wedge\big(\Phi^\dagger\Phi\big)^*\bigg), \end{aligned} \tag{6}$$

where the $\star$ on $\mathcal{F}^\star$ denotes taking the Hermitian conjugate of the supermatrices and the Hodge dual (denoted as $*$) of the differential forms, and $\lambda \equiv 2g^2$. Note that we need to break $SU(2/1)$ explicitly in order to introduce the Higgs mass. In 4 dimensions we have the following explicit form of the Lagrangian (given the metric $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$):

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F^i_{W\mu\nu}F^{i\mu\nu}_W - \frac{1}{4}F_{B\mu\nu}F_B^{\mu\nu} \\ &\quad + (D_\mu\Phi)^\dagger\big(D^\mu\Phi\big) - \lambda\big(\Phi^\dagger\Phi\big)^2. \end{aligned} \tag{7}$$

Note that the explicit forms of the curvature strengths and the covariant derivatives have the standard forms: $F^i_{W\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + 2ig\epsilon^{jki}W^j_\mu W^k_\nu$, $F_{B\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $D_\mu\Phi = \partial_\mu\Phi + ig(\boldsymbol{\tau}\cdot\mathbf{W}_\mu)\Phi + ig'B_\mu\Phi$, with $g'/g = 1/\sqrt{3}$. To switch to the common SM convention we rescale g and g' as $g, g \to g/2, g'/2$ (which is the missing part in [7]) which also changes our constraint at the symmetry breaking energy to $\lambda = g^2/2$.[5] Now we address the prediction for the Higgs mass. In what follows we use the relation $M_H^2 = 8M_W^2(\lambda/g^2)$ and the RG equations for λ and top Yukawa coupling g_t which are

[3] Given the similarities between the outcomes of the Spectral Model of Connes and Chamseddine [12] and the superconnection formalism, there may be a relation between these models.

[4] Note that *, which we will use to denote the Hodge product later in the Letter, here denotes taking complex conjugate of a field.

[5] If we do not make these rescalings at this point then we need to make appropriate ones in Eq. (8).