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Lattice evidence for the family of decoupling solutions of Landau gauge Yang–Mills theory

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ABSTRACT

We show that the low-momentum behavior of the lattice Landau-gauge gluon and ghost propagators is sensitive to the lowest non-trivial eigenvalue (λ_1) of the Faddeev–Popov operator. If the gauge fixing favors Gribov copies with small λ_1 the ghost dressing function rises more rapidly towards zero momentum than on copies with large λ_1. This effect is seen for momenta below 1 GeV, and interestingly also for the gluon propagator at momenta below 0.2 GeV: For large λ_1 the gluon propagator levels out to a lower value at zero momentum than for small λ_1. For momenta above 1 GeV no dependence on Gribov copies is seen. Although our data is only for a single lattice size and spacing, a comparison to the corresponding (decoupling) solutions from the DSE/FRGE study of Fischer, Maas and Pawlowski (2009) [22] yields already a good qualitative agreement.

1. Introduction

Lattice calculations of the Landau-gauge gluon, ghost and quark propagators have attracted quite some interest during the last 15 years. Staunch supporters of pure lattice QCD (LQCD) may wonder about the enthusiasm with which such calculations have been performed and discussed in the past, in particular, as LQCD comes with the distinct advantage that one does not need to fix a gauge. This holds true, however, only as long as one is interested in gauge-invariant quantities. But besides LQCD there are also other (sometimes better suited) frameworks to tackle nonperturbative problems of QCD, and these require the exact knowledge of QCD's elementary two and three-point functions in Landau or other gauges.

Two continuum functional methods one has to mention here are the efforts to solve the infinite tower of Dyson–Schwinger equations (DSEs) of QCD or, likewise, the corresponding Functional Renormalization Group Equations (FRGEs) (see, e.g., the reviews [1–8] and references therein). Both these methods imply fixing a gauge (and often the Landau gauge is chosen for simplicity), but more importantly, these methods also require a truncation of the infinite system of equations to enable finding a numerical solution. These truncations are a potential source of error, which why corresponding (volume and continuum extrapolated) lattice results are so essential to render these truncations harmless or to even substitute parts of the DSE (or FRGE) solutions by (interpolated) nonperturbative data.

In what concerns the Landau-gauge gluon and ghost propagators, lattice results have helped much to improve truncations over the years. Currently, the continuum and lattice results overlap for a wide range of momenta, showing nice consistency among the so different approaches to QCD. Admittedly, the currently used truncations are still not perfect, as seen, for example, for the gluon propagator whose DSE solutions differ from the corresponding lattice or FRGE results in the intermediate momentum regime (i.e., for momenta 0.5–3 GeV), whereas FRGE and lattice results agree much better there (see, e.g., Fig. 2 in [9]). But this situation will certainly improve, as it did in the past (see, e.g., [10] for recent progress).

Another regime that remains to be fully settled yet is the low (infrared) momentum regime. About the infrared behavior of the gluon and ghost propagators in Landau gauge there has been much dissent in the community and it is difficult to assess on the lattice also. Currently, all lattice studies agree upon a gluon propagator and ghost dressing function which are (most likely) finite in the zero-momentum limit (see, e.g., [11–19]).[1] DSE and FRGE studies [20–23], on the other hand, assert that this infrared behavior is not unique, but depends on an additional (boundary) condition

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[1] Current lattice results for this regime are for finite lattice spacings and volumes only, and also the Gribov problem is only partially understood.

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on the ghost dressing function at zero momentum, $J(0)$. Explicitly, in Refs. [22,23], it is shown that for $J^{-1}(0) = 0$ one finds the so-called *scaling* behavior for the gluon and ghost propagators at low momentum, as it was first found in [24], while for finite $J(0)$, one finds a family of *decoupling* solutions for the DSEs and FRGEs, in qualitative agreement with DSE solutions proposed in the studies of Refs. [25–30], and with lattice results. For momenta above 1 GeV both types of solutions are practically indistinguishable. We interpret this ambiguity in the infrared as a remnant of the Gribov ambiguity of the Landau gauge condition, which is lifted by fixing $J^{-1}(0)$ to a constant.

In this Letter we will show that a part of this one-parameter family of decoupling solutions can be seen on the lattice, at least qualitatively and as far as it is possible on a finite and rather coarse lattice. Our approach also allows only for mild variations of the gluon and ghost propagators. Nonetheless, after outlining some technical details in the next section and a discussion about the distribution of the lowest non-trivial eigenvalue of the Faddeev–Popov (FP) operator, λ_1, on different Gribov copies (Section 3), we will demonstrate in Section 4 that the decoupling-like behavior of the lattice gluon and ghost propagators can be changed by a (yet simple-minded implementation of a) constraint on λ_1. Changes take place only in the low-momentum regime, but interestingly in a similar manner as one expects from the DSE/FRGE study [22], where a condition on $J(0)$ was used to change the low-momentum behavior.[2] Specifically, we show that on Gribov copies with small λ_1 the ghost dressing function at low momenta rises more rapidly towards zero momentum than on copies with large λ_1. Interestingly, a similar (though less pronounced) Gribov-copy effect is seen for the gluon propagator at low momentum. Qualitatively, our data thus resembles the change of the gluon and ghost dressing functions as expected from [22] for the corresponding decoupling solutions.[3]

Note that we still find Gribov copies by a maximization of the lattice Landau-gauge functional, but we are not interested in finding Gribov copies with large gauge-functional values, but on copies with comparably small (or large) λ_1, irrespective of the functional value. On Gribov copies with large gauge-functional values we see both propagators to rise less rapidly towards zero momentum, consistent with what was found in the past [11,31–34]

We should also mention here that similar effects were seen for the B-gauges by Maas [35]. For these gauges, one selects Gribov copies based on the ratio of the ghost dressing function at a small and a large lattice momentum on a particular copy. By construction the ghost dressing function in these gauges is then clearly enhanced or suppressed at low momenta. It remains to be seen if corresponding effects become clear also for the gluon propagator. The current data suggests, also this approach may reproduce a part of the family of decoupling solutions on the lattice [36,37].

2. Simulation details

Our study is based on 80 thermalized gauge field configurations, generated with the usual heatbath thermalization and Wilson's plaquette action for SU(2) lattice gauge theory. The lattice size is 56^4 and the coupling parameter $\beta = 2.3$. To reduce autocorrelations, configurations are separated by 2000 thermalization steps, each involving four over-relaxation and one heatbath step. For every configuration there are at least $N_{\text{copy}} = 210$ gauge-fixed (Gribov) copies, all fixed to lattice Landau gauge using an optimally-tuned over-relaxation algorithm for the gauge fixing that finds local maxima of the lattice Landau gauge functional

$$F_U[g] = \frac{1}{4V} \sum_x \sum_{\mu=1}^{4} \operatorname{Tr} g_x U_{x\mu} g^{\dagger}_{x+\hat{\mu}}. \tag{1}$$

Here $U \equiv \{U_{x\hat{\mu}}\}$ denotes the gauge configuration and $g \equiv \{g_x\}$ one of the many gauge transformation fields fixing U to Landau gauge. To ensure these Gribov copies are all distinct, the gauge-fixing algorithm always started from a random gauge transformation field. Interestingly, for all these 80×210 gauge-fixing attempts only a few Gribov copies were found twice.

For every single Gribov copy we determine the lowest three (non-trivial) eigenvalues $0 < \lambda_1 < \lambda_2 < \lambda_3$ of the Faddeev–Popov (FP) operator using PARPACK [38]. In what follows, we will use λ_1 to classify copies: The Gribov copy with lowest λ_1 (considered for each configuration separately) is labeled *lowest copy* (ℓc), while the copy with the highest λ_1 we call *highest copy* (hc). The first generated copy, irrespective of λ_1, gets the label *first copy* (fc). It represents an arbitrary (random) Gribov copy of a configuration. To compare with former lattice studies on the problem of Gribov copies we also reintroduce the label *best copy* (bc). It refers to that copy with the best (largest) gauge functional value $F_U[g]$ for a particular gauge configuration.

On those four sets of Gribov copies we calculate the SU(2) gluon and ghost propagators following standard recipes. That is, the gluon propagator is calculated for every lattice momentum using a fast Fourier transformation and the ghost propagator by using the plane-wave method for selected momenta. To accelerate the latter we use the preconditioned conjugate gradient algorithm of [11]. As a by-product of this calculation we also obtain the renormalization constant, $\widetilde{Z}_1$, of the ghost–ghost–gluon (gh–gl) vertex in Landau gauge for zero incoming gluon momentum. For more details on lattice Landau gauge and the calculation of the propagators and $\widetilde{Z}_1$ the reader may refer to Refs. [11,39–41] and references therein.

When quoting momenta in physical units we adopt the usual definition $ap_\mu(k_\mu) = 2\sin(\pi k_\mu / L_\mu)$ with $k_\mu \in (-L_\mu/2, L_\mu/2]$ and $L_\mu \equiv 56$, assume for the string tension $\sqrt{\sigma} = 440$ MeV and use $\sigma a^2 = 0.145$ for $\beta = 2.3$ from Ref. [42], where a denotes the lattice spacing.

3. Distribution of λ_1

Before comparing the propagator data for the different types of Gribov copies, it is instructive to look at the distribution of λ_1 on all copies first. In Fig. 1 we show this eigenvalue distribution (in lattice units) for $N_{\text{cp}} = 210$ Gribov copies. There, the big panel shows it separately for each of the 80 gauge configurations and the small panel (on top) for all configurations together as a histogram. One sees that for most of the copies λ_1 takes values between 0.5×10^{-3} and 1.9×10^{-3}, mostly between 1.5×10^{-3} and 1.7×10^{-3}, but for some configurations there are also copies with an exceptionally small value for λ_1, a value ($\lambda_1 < 10^{-4}$) far below the values found for the other copies. With our simple (brute-force) approach we are rather limited in finding more of these exceptional copies. The gauge-fixing and calculation of eigenvalues on a 56^4 lattice is computational quite demanding, and a more sophisticated gauge-fixing algorithm — one which would automatically select that Gribov copy with the smallest (or at least with small) λ_1 — does not exist. But it would be interesting to know if for each configuration a Gribov copy with such an exceptionally small λ_1 exists.

For a few configurations we generated more than 210 Gribov copies. These allow us now to have a closer look at the distribution

[2] Note that in [22] the ghost dressing function is denoted G.

[3] We thank C. Fischer for providing us access to their (decoupling) solutions including those for smaller $J(0)$ not shown in [22].