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Minimal see-saw model predicting best fit lepton mixing angles

Stephen F. King

School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, United Kingdom

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ABSTRACT

We discuss a minimal predictive see-saw model in which the right-handed neutrino mainly responsible for the atmospheric neutrino mass has couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(0, 1, 1)$ and the right-handed neutrino mainly responsible for the solar neutrino mass has couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(1, 4, 2)$, with a relative phase $\eta = -2\pi/5$. We show how these patterns of couplings could arise from an A_4 family symmetry model of leptons, together with Z_3 and Z_5 symmetries which fix $\eta = -2\pi/5$ up to a discrete phase choice. The PMNS matrix is then completely determined by one remaining parameter which is used to fix the neutrino mass ratio m_2/m_3. The model predicts the lepton mixing angles $\theta_{12} \approx 34^\circ, \theta_{23} \approx 41^\circ, \theta_{13} \approx 9.5^\circ$, which exactly coincide with the current best fit values for a normal neutrino mass hierarchy, together with the distinctive prediction for the CP violating oscillation phase $\delta \approx 106^\circ$.

1. Introduction

Daya Bay [1] and RENO [2] have measured a non-zero reactor angle $\theta_{13} \approx 0.15$ which excludes Tri-Bimaximal (TB) mixing [3]. Recent global fits also hint at deviations of the atmospheric and solar angles from their TB values (for a recent review see e.g. [4]). Such deviations may be expressed in terms of the deviation parameters (s, a and r) from TB mixing [5] (for a related parametrisation see [6]):

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s), \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a),$$
$$\sin\theta_{13} = \frac{r}{\sqrt{2}}. \tag{1}$$

With zero solar and atmospheric deviations from TB mixing, $s = a = 0$, and Cabibbo-like reactor mixing described by $r = \lambda$, with $\lambda = 0.225$ being the Wolfenstein parameter, one is led to Tri-Bimaximal Cabibbo (TBC) mixing [7]. However, as mentioned above, current global fits prefer non-zero solar and atmospheric TB deviation parameters,

$$s = -\lambda^2/2, \qquad a = -\lambda/3, \qquad r = \lambda, \tag{2}$$

corresponding to the angles,

$$\theta_{12} = 34.2^\circ, \qquad \theta_{23} = 40.8^\circ, \qquad \theta_{13} = 9.15^\circ. \tag{3}$$

These angles are close to the best fit values for all three global fits in the case of a normal neutrino mass ordering [4]. Assuming a normal neutrino mass *hierarchy* with $m_1 = 0$, one is led to [13],

$$\frac{m_2}{m_3} = \frac{3}{4}\lambda, \tag{4}$$

corresponding to $m_2/m_3 \approx 0.17$, close to the best fit value [4]. The deviation parameters in Eq. (2) have the feature that the atmospheric mixing angle is in the first octant and the solar mixing angle is somewhat less than its tri-maximal value, in agreement with the latest global fits for the case of a normal neutrino mass ordering. In particular it reproduces the best fit values of angles of all three global fits [4] to within one standard deviation.

There have been many attempts to describe the lepton mixing angles based on the type I see-saw model [8] combined with sequential dominance (SD) [9] in which the right-handed neutrinos contribute with sequential strength. Constrained sequential dominance (CSD) [10] involves the right-handed neutrino mainly responsible for the atmospheric neutrino mass having couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(0, 1, 1)$ and the right-handed neutrino mainly responsible for the solar neutrino mass having couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(1, 1, -1)$ and it led to TB mixing. CSD2 [11] was proposed to give a non-zero reactor angle and is based on the same atmospheric alignment but with right-handed neutrino mainly responsible for the solar neutrino mass having couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(1, 0, -2)$ or $(1, 2, 0)$ yielding a reactor angle $\theta_{13} \approx 6^\circ$ which unfortunately is too small, although the situation can be rescued by invoking charged lepton corrections [12]. The CSD3 model in [13] involves the right-handed neutrino mainly responsible for the solar neutrino mass having

E-mail address: king@soton.ac.uk.

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Table 1
Lepton, Higgs and flavon superfields and how they transform under the symmetries relevant for the Yukawa sector of the model. The only non-trivial charged lepton charges are in the upper left of the table and the only non-trivial neutrino charges in the lower right of the table. Note that the only the lepton doublets L and A_4 symmetry, are common to both charged lepton and neutrino sectors and are given near the central column and row. The Standard Model gauge symmetries and $U(1)_R$ symmetry, under which all the leptons have a charge of unity while the Higgs and flavons have zero charge, are not shown in the table.

	θ	e^c	μ^c	τ^c	φ_e	φ_μ	φ_τ	H_1	L	H_2	ϕ_{atm}	ϕ_{sol}	N_{atm}	N_{sol}	ξ_{atm}	ξ_{sol}
Z_3^θ	ω	ω	ω^2	1	1	1	1	1	1	1	1	1	1	1	1	1
Z_3^e	1	ω^2	1	1	ω	1	1	1	1	1	1	1	1	1	1	1
Z_3^μ	1	1	ω^2	1	1	ω	1	1	1	1	1	1	1	1	1	1
Z_3^τ	1	1	1	ω^2	1	1	ω	1	1	1	1	1	1	1	1	1
A_4	**1**	**1**	**1**	**1**	**3**	**3**	**3**	**1**	**3**	**1**	**3**	**3**	**1**	**1**	**1**	**1**
Z_5^{atm}	1	1	1	1	1	1	1	1	1	1	ρ^3	1	ρ^2	1	ρ	1
Z_5^{sol}	1	1	1	1	1	1	1	1	1	1	1	ρ^3	1	ρ^2	1	ρ

couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(1,3,1)$ or $(1,1,3)$ with a relative phase $\mp\pi/3$ yielding a reactor angle $\theta_{13} \approx 8.5°$ close to the observed value. However CSD3 predicts approximate TBC mixing with an almost maximal atmospheric mixing angle disfavoured by the latest global fits, and so it may soon be challenged.

In this Letter we shall propose a model based on a new possibility called CSD4 which predicts the above best fit angles in Eq. (3) of the PMNS lepton mixing matrix and also makes predictions for the physical CP violating phases. Similar to all SD models, the CSD4 model involves effectively two right-handed neutrinos and a normal neutrino mass hierarchy, leading to $m_1 = 0$. As in CSD2 and CSD3, the CSD4 model only requires one input parameter, namely the ratio of neutrino masses which is selected to be $m_2/m_3 \approx 3\lambda/4$, which is a natural value that one would expect in such models. Also as in CSD2 and CSD3, once this value is chosen, the entire PMNS mixing matrix is then fixed by the theory (up to a discrete choice of phases) with no remaining free parameters. In the CSD4 model, the right-handed neutrino mainly responsible for the atmospheric neutrino mass has couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(0,1,1)$ and the right-handed neutrino mainly responsible for the solar neutrino mass has couplings to $(\nu_e, \nu_\mu, \nu_\tau)$ proportional to $(1,4,2)$, with a relative phase $\eta = -2\pi/5$. These couplings and phase relation were first discovered in [13] and shown to lead to lepton mixing angles in good agreement with the latest global fits, but no model has been proposed based on CSD4. The goal of this Letter is to show how CSD4 can arise from an A_4 family symmetry, together with additional discrete Z_5 and Z_3 symmetries, and to present the first model of leptons along these lines. This is necessary since it is far from clear whether alignments such as $(1,4,2)$ are possible to achieve within a realistic model. The CSD4 model presented here predicts the best fit PMNS angles in Eq. (3) with the distinctive prediction for the oscillation phase $\delta \approx 106°$.

2. A minimal predictive A_4 model of leptons

In this section we outline a supersymmetric (SUSY) A_4 model of leptons with CSD4 along the lines of the A_4 models of leptons discussed in [11,14]. The basic idea is that the three families of lepton doublets L form a triplet of A_4 while the right-handed charged leptons e^c, μ^c, τ^c, right-handed neutrinos N_{atm}, N_{sol} and the two Higgs doublets H_1, H_2 required by SUSY are all singlets of A_4. In addition the model employs an additional Z_3^θ family symmetry in order to account for the charged lepton mass hierarchy.

The vacuum alignment that is required for the model is discussed in Appendix A. In Table 1 we have displayed the symmetries and superfields relevant for the Yukawa sector only. In Appendix A the transformation properties of the remaining superfields under $Z_3^l \times Z_5^{\nu_i}$ responsible for vacuum alignment is discussed and are consistent with the charges shown in Table 1, where we have written $\phi_{\text{atm}} \equiv \varphi_{\nu_3}$ and $\phi_{\text{sol}} \equiv \varphi_{\nu_4}$ and hence $Z_5^{\text{atm}} \equiv Z_5^{\nu_3}$ and $Z_5^{\text{sol}} \equiv Z_5^{\nu_4}$.

The charged lepton sector of the model employs the A_4 triplet flavons φ_e, φ_μ, φ_τ whose alignment is discussed in Appendix A. With the lepton symmetries in the upper left of Table 1 we may enforce the following charged lepton Yukawa superpotential at leading order

$$\mathcal{W}^e_{\text{Yuk}} \sim \frac{1}{\Lambda} H_1(\varphi_\tau \cdot L)\tau^c + \frac{1}{\Lambda^2}\theta H_1(\varphi_\mu \cdot L)\mu^c + \frac{1}{\Lambda^3}\theta^2 H_1(\varphi_e \cdot L)e^c, \tag{5}$$

which give the charged lepton Yukawa couplings after the flavons develop their vevs. Λ is a generic messenger mass scale, but in a renormalisable model the messengers scales may differ. The charged lepton symmetries include three lepton flavour symmetries $Z_3^{e,\mu,\tau}$ under which φ_e, φ_μ, φ_τ and e^c, μ^c, τ^c transform respectively as ω and ω^2, together with a lepton family symmetry Z_3^θ under which e^c, μ^c, τ^c transform as ω, ω^2, 1 respectively (where $\omega = e^{i2\pi/3}$) with the family symmetry breaking flavon θ transforming as ω and otherwise being a singlet under all other symmetries. H_1 and L and all other fields are singlets under $Z_3^{e,\mu,\tau}$ and Z_3^θ. With these charge assignments the higher order corrections are very suppressed.

The charged lepton Yukawa matrix is diagonal at leading order due to the alignment of the charged lepton-type flavons in Eq. (25) (where the driving fields responsible for the alignment in Eq. (24) absorb the charges under the newly introduced symmetries $Z_3^{e,\mu,\tau}$ and Z_3^θ) and has the form,

$$Y^e = \text{diag}(y_e, y_\mu, y_\tau) \sim \text{diag}\big(\epsilon^2, \epsilon, 1\big) y_\tau \tag{6}$$

where we choose $\epsilon \sim \langle\theta\rangle/\Lambda \sim \lambda^2$ in order to generate the correct order of magnitude charged lepton mass hierarchy, with precise charged lepton masses also dependent on order-one coefficients which we have suppressed here.

With the neutrino symmetries in the lower right part of Table 1 we may enforce the following leading order neutrino Yukawa superpotential

$$\mathcal{W}^\nu_{\text{Yuk}} \sim \frac{1}{\Lambda} H_2(\phi_{\text{atm}} \cdot L)N_{\text{atm}} + \frac{1}{\Lambda} H_2(\phi_{\text{sol}} \cdot L)N_{\text{sol}}. \tag{7}$$

Again the higher order corrections are completely negligible. The neutrino sector of the model exploits the A_4 triplet flavons $\phi_{\text{atm}} \equiv \varphi_{\nu_3}$, and $\phi_{\text{sol}} \equiv \varphi_{\nu_4}$ whose alignment is discussed in Appendix A.

As is typical in models of this kind [11,14], the RH neutrinos have no mass terms at the renormalisable level, but they become massive after some A_4 singlet flavons ξ_{atm} and ξ_{sol} develop their vevs due to the renormalisable superpotential,