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The chiral condensate in neutron matter

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ABSTRACT

We calculate the chiral condensate in neutron matter at zero temperature based on nuclear forces derived within chiral effective field theory. Two-, three- and four-nucleon interactions are included consistently to next-to-next-to-next-to-leading order (N^3LO) of the chiral expansion. We find that the interaction contributions lead to a modest increase of the condensate, thus impeding the restoration of chiral symmetry in dense matter and making a chiral phase transition in neutron-rich matter unlikely for densities that are not significantly higher than nuclear saturation density.

The understanding of the phase diagram of matter is a current frontier in nuclear physics. At high temperatures and vanishing net baryon density, the properties of strongly interacting matter have been studied in first-principle lattice QCD calculations. It is found that at a temperature of 154 ± 9 MeV matter exhibits a chiral and deconfinement crossover transition from the low-temperature hadronic phase, where chiral symmetry is spontaneously broken, to the chirally symmetric high-temperature phase, the quark–gluon plasma [1]. An order parameter for characterizing this transition is the chiral condensate [2–4].

Owing to the fermion sign problem, matter at non-zero baryon densities cannot be directly studied in lattice QCD. Therefore, there are no first-principle QCD results for the phase diagram in particular at low temperatures and high densities. These conditions are probed in neutron stars, which can reach several times nuclear saturation density in their interiors [5–8]. However, there has been much speculation on possible exotic phases that may appear in the center of neutron stars, such as Lee–Wick abnormal matter, pion- as well as kaon-condensed matter, hyperon matter and quark matter [5]. It has also been conjectured, see, e.g., Fig. 1 in Ref. [9], that in neutron matter the QCD phase transition may begin already below saturation density.

Recent observations of neutron stars with $2\,M_\odot$ masses [10,11] provide general constraints on the equation of state (EOS) of cold strongly interacting matter, and put into question whether exotic phases that tend to soften the EOS are realized in neutron stars. At densities $n \lesssim n_0$, where $n_0 = 0.16$ fm^{-3} denotes nuclear saturation density, the properties of nuclear systems have been studied systematically based on nuclear forces derived within chiral effective field theory (EFT) [12,13] and using renormalization group methods [14,15]. In this Letter, we use chiral EFT interactions to study the chiral condensate as a function of density in neutron matter, based on perturbative calculations around the first-order Hartree–Fock energy [16–18].

The chiral condensate can be obtained from the energy using the Hellman–Feynman theorem [3,19,20],

$$\langle\bar{q}q\rangle_n - \langle\bar{q}q\rangle_0 = n\frac{\partial}{\partial m_q}\left[\frac{E_{\text{free}}(m_q,k_{\text{F}})}{N} + \frac{E_{\text{int}}(m_q,k_{\text{F}})}{N}\right], \qquad (1)$$

where $\langle\bar{q}q\rangle_n$ and $\langle\bar{q}q\rangle_0$ are the chiral condensates at finite baryon density $n = k_{\text{F}}^3/(3\pi^2)$ (with Fermi momentum k_{F}) and in vacuum, respectively. Moreover, $E_{\text{free}}/N = m_N + 3k_{\text{F}}^2/(10m_N)$ is the energy per particle of a system consisting of N noninteracting degenerate neutrons in the nonrelativistic limit, E_{int} is the corresponding interaction energy, m_q denotes the average of the u and d quark masses, $\bar{q}q = \bar{u}u + \bar{d}d$, and m_N is the neutron mass.

The contribution from the nucleon mass to the chiral condensate is proportional to the pion–nucleon sigma term $\sigma_{\pi N}$, which accounts for the scalar quark density in the nucleon [3,21]:

$$\sigma_{\pi N} = \langle N|m_q\bar{q}q|N\rangle = m_q\frac{\partial m_N}{\partial m_q}. \qquad (2)$$

Here $|N\rangle$ represents the state of a nucleon at rest. The value of the pion–nucleon sigma term has been determined within different frameworks [21–25]. As a baseline we use the value

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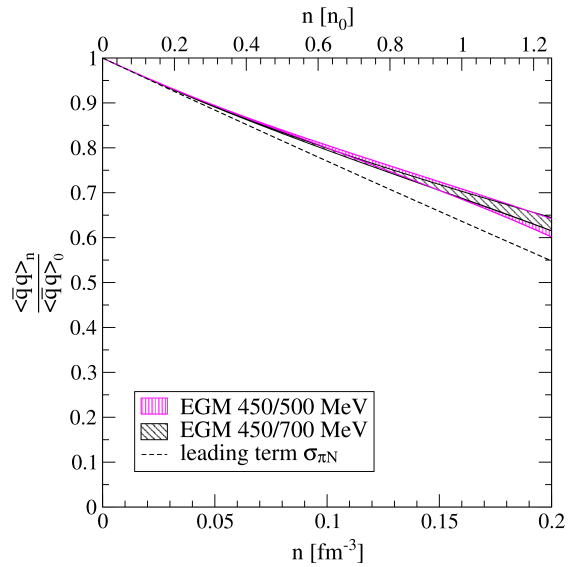

Fig. 1. (Color online.) Chiral condensate $\langle\bar{q}q\rangle_n/\langle\bar{q}q\rangle_0$ as a function of density in neutron matter. The dashed line is the leading pion–nucleon sigma-term contribution. The interaction contributions are obtained from the N^{3}LO neutron-matter calculation of Refs. [17,18], based on the EGM 450/500 MeV and 450/700 MeV N^{3}LO NN potentials plus 3N and 4N interactions to N^{3}LO, by varying the pion mass around the physical value. As in Refs. [17,18], the bands for each NN potential include uncertainties of the many-body calculation, of the c_i couplings of 3N forces, and those resulting from the 3N/4N cutoff variation.

$\sigma_{\pi N} \approx 50$ MeV [20]. The chiral condensate in neutron matter relative to the vacuum is then given by [3]

$$\frac{\langle\bar{q}q\rangle_n}{\langle\bar{q}q\rangle_0} = 1 - \frac{n}{f_\pi^2}\frac{\sigma_{\pi N}}{m_\pi^2}\left(1 - \frac{3k_F^2}{10m_N^2} + \cdots\right) - \frac{n}{f_\pi^2}\frac{\partial}{\partial m_\pi^2}\frac{E_{\text{int}}(m_\pi, k_F)}{N}, \tag{3}$$

where we have used the Gell-Mann–Oakes–Renner relation $m_q\langle\bar{q}q\rangle_0 = -f_\pi^2 m_\pi^2$. In our calculation we use for the pion mass the charge average $m_\pi = 138$ MeV and for the pion decay constant $f_\pi = 92.4$ MeV.

The leading $\sigma_{\pi N}$ contribution to the chiral condensate in Eq. (3), which is due to the mass term in E_{free}/N, is linear in density and is shown in Fig. 1 by the dashed line. By extrapolating this linear density dependence, one finds restoration of chiral symmetry at a density around (2.5–3)n_0 [3,19]. For the density range shown in Fig. 1, where chiral EFT interactions can be applied with confidence, the kinetic energy contribution to E_{free}/N gives only a 4% correction relative to the leading term. Relativistic corrections, indicated by the dots in Eq. (3), are negligible at these densities. The next term $9k_F^4/56m_N^4$ is a 0.3% correction. While the first correction to the chiral condensate in Eq. (3) is a consequence of the finite nucleon density, the long-range contributions from E_{int} can be attributed to the modification of the scalar pion density $\Delta n_\pi^s = n\,\partial(E_{\text{int}}/N)/\partial m_\pi$ due to the interactions between nucleons (cf. Ref. [26]).

The energy per particle of neutron matter has recently been calculated based on chiral EFT interactions to N^{3}LO, including two-nucleon (NN), three-nucleon (3N), and four-nucleon (4N) forces [17,18]. The pion-mass dependence of nuclear forces arises from two sources: First, due to the explicit m_π dependences in the long-range pion-exchange interactions, and second, implicitly, due to the quark-mass dependence of the pion–nucleon coupling g_A, the pion decay constant f_π, as well as the leading NN contact interactions C_S and C_T, and higher-order pion–nucleon and short-range NN and 3N contact interactions.

We calculate the explicit m_π dependence of nuclear forces by varying the value of the pion mass in the pion-exchange NN, 3N, and 4N interactions. At the NN level, we use the N^{3}LO potentials of Epelbaum, Glöckle, and Meißner (EGM) [27,28] with cutoffs 450/500 and 450/700 MeV (and their N^{2}LO versions to study the order-by-order convergence). With these NN interactions neutron matter is perturbative at the densities considered here [17,18]. This result was recently validated using first Quantum Monte Carlo calculations with chiral EFT interactions [29]. We vary m_π by 0.5% in the corresponding potential routines. The derivative of the interaction energy with respect to m_π^2 in Eq. (3) is then computed numerically for different densities.

We estimate the impact of the quark-mass dependence of g_A and f_π using the results of Refs. [30,31]. In a perturbative calculation, the interaction energy per particle E_{int}/N is a polynomial in g_A and in $1/f_\pi$. Consider a term in this polynomial, in which g_A enters with the power α, $[E_{\text{int}}/N]_\alpha$. For the corresponding contribution to the chiral condensate, due to the pion-mass dependence of g_A, we thus have

$$\begin{aligned} &-\frac{n}{f_\pi^2}\frac{\partial}{\partial m_\pi^2}\left[\frac{E_{\text{int}}}{N}\right]_\alpha \\ &= -\frac{n}{f_\pi^2}\frac{\partial g_A}{\partial m_\pi^2}\frac{\alpha}{g_A}\left[\frac{E_{\text{int}}}{N}\right]_\alpha \\ &\approx -(4.4\text{–}5.7)\times 10^{-4}\ \text{MeV}^{-1}\,\alpha\left[\frac{E_{\text{int}}}{N}\right]_\alpha. \end{aligned} \tag{4}$$

Here, we have used the range for $\frac{\partial g_A}{\partial m_\pi^2}$ from Ref. [31]. Similarly, an interaction term, in which $1/f_\pi$ enters with the power β, $[E_{\text{int}}/N]_\beta$, leads to a contribution to the chiral condensate

$$\begin{aligned} &-\frac{n}{f_\pi^2}\frac{\partial}{\partial m_\pi^2}\left[\frac{E_{\text{int}}}{N}\right]_\beta \\ &= -\frac{n}{f_\pi^2}\frac{\partial f_\pi}{\partial m_\pi^2}\frac{\partial}{\partial f_\pi}\left[\frac{E_{\text{int}}}{N}\right]_\beta \\ &\approx (2.6\text{–}5.0)\times 10^{-4}\ \text{MeV}^{-1}\,\beta\left[\frac{E_{\text{int}}}{N}\right]_\beta, \end{aligned} \tag{5}$$

using $\frac{\partial f_\pi}{\partial m_\pi^2}$ from Ref. [31]. The uncertainty is larger in this case, because of the c_3 and c_4 uncertainties, which are taken as in the N^{3}LO calculations of Refs. [17,18].

For the leading-order one-pion-exchange NN interaction, which is proportional to g_A^2/f_π^2 and contributes ~ 10 MeV per particle at n_0, the terms (4) and (5) give a contribution to the chiral condensate ranging from -0.006 to $+0.001$. The leading N^{2}LO two-pion-exchange 3N forces also provide ~ 10 MeV per particle at n_0. These terms are proportional to g_A^2/f_π^4 and the corresponding contribution to the chiral condensate lies in the range -0.001 to $+0.011$. Combined, these corrections amount to at most a 25% increase of the uncertainty band in Fig. 1. We expect the contributions from the shorter-range interactions to start at a similar level. However, the extrapolation of their m_π dependence from lattice QCD results at heavier pion masses to the physical point is uncertain. This will be improved in the future once lattice QCD results for NN and 3N interactions for physical pion masses will become available. Because the estimated effects beyond the explicit m_π dependence are small compared to the band in Fig. 1, we do not include these contributions in the present Letter.