Physics Letters B 726 (2013) 431–435

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Classical solutions of higher-derivative theories

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ARTICLE INFO

Article history:
Received 23 January 2013
Received in revised form 13 August 2013
Accepted 13 August 2013
Available online 22 August 2013
Editor: L. Alvarez-Gaumé

ABSTRACT

We present exact classical solutions of the higher-derivative theory that describes the dynamics of the position modulus of a probe brane within a five-dimensional bulk. The solutions can be interpreted as static or time-dependent throats connecting two parallel branes. In the nonrelativistic limit the brane action is reduced to that of the Galileon theory. We derive exact solutions for the Galileon, which reproduce correctly the shape of the throats at large distances, but fail to do so for their central part. We also determine the parameter range for which the Vainshtein mechanism is reproduced within the brane theory.

1. Introduction

Exact classical solutions of field theories are interesting as they can describe configurations that differ substantially from the usual perturbative vacuum. The purpose of this Letter is to present a class of exact solutions of certain higher-derivative scalar theories in $3+1$ dimensions. The theories have a geometric origin, as they describe hypersurfaces, which we term branes, embedded in a higher-dimensional spacetime. From the point of view of the brane observer the action involves only derivative terms of a particular form for one scalar degree of freedom.

We obtain our solutions by generalizing known static and time-dependent ones for the Dirac–Born–Infeld (DBI) theory with vanishing gauge fields. This theory can be viewed as the effective description of a $(3+1)$-dimensional brane embedded in a Minkowski bulk spacetime with one additional spatial dimension. A known static solution is the catenoidal configuration obtained by joining two branches with opposite first derivatives at the point where they display square-root singularities [1,2]. The result is a smooth surface that looks like a throat or wormhole connecting two asymptotically parallel branes. A similar time-dependent solution describes two branes connected by a throat whose radius evolves with time. The throat shrinks to a minimal radius and subsequently re-expands [3,4]. Alternatively, the square-root singularity can be viewed as a propagating shock front [5]. For the expanding configuration, energy is transferred from the location of the shock front, where the energy density diverges, to the region behind it.

We consider generalized theories that describe branes embedded in a flat bulk spacetime with one additional spatial dimension. The leading contribution to the action is given by the volume swept by the brane (the worldvolume), expressed in terms of the induced metric. It is invariant under arbitrary changes of the brane worldvolume coordinates. We eliminate this gauge freedom by identifying the brane coordinates with certain bulk coordinates (static gauge). The remaining bulk coordinate becomes a scalar field of the worldvolume theory, with dynamics governed by the DBI action. More complicated terms can also be included in the effective action. They can be expressed in terms of geometric quantities, such as the intrinsic and extrinsic curvatures of the hypersurface. In the static gauge these can be written in terms of the scalar field and its derivatives, so that we obtain a higher-derivative scalar theory with a particular structure of geometric origin [6,7]. Some of the higher-derivative terms can be considered as quantum corrections to the DBI action [8].

The focus of our analysis is on brane theories constructed so that the equation of motion does not contain field derivatives higher than the second. In this way ghost fields do not appear in the spectrum. The most general scalar–tensor theory with this property was constructed a long time ago [9], and rediscovered recently. It is characterized as the generalized Galileon (see Ref. [10] and references therein). A particular example is provided by the Galileon theory [11], which results from the Dvali–Gabadadze–Porrati (DGP) model [12] in the decoupling limit. The connection with the brane picture is made in Ref. [6], where it is shown that the Galileon theory can be reproduced in the nonrelativistic limit, starting from the effective action for the position modulus of a probe brane within a $(4+1)$-dimensional bulk.

2. The brane and Galileon theories

We consider the brane theory as formulated in Ref. [6]. The induced metric on the brane in the static gauge is

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0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.08.034

$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi\, \partial_\nu \pi$, where π denotes the extra coordinate of the bulk space. Our convention for the Minkowski metric is $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The extrinsic curvature is $K_{\mu\nu} = -\partial_\mu \partial_\nu \pi / \sqrt{1 + (\partial\pi)^2}$. We denote its trace by K. The leading terms in the brane effective action are [6]

$$S_\lambda = -\lambda \int d^4x \sqrt{-g} = -\lambda \int d^4x \sqrt{1 + (\partial\pi)^2}, \tag{1}$$

$$S_K = -M_5^3 \int d^4x \sqrt{-g} K = M_5^3 \int d^4x \big([\Pi] - \gamma^2[\phi]\big), \tag{2}$$

$$S_R = \frac{M_4^2}{2} \int d^4x \sqrt{-g} R$$
$$= \frac{M_4^2}{2} \int d^4x\, \gamma \big([\Pi]^2 - [\Pi^2] + 2\gamma^2\big([\phi^2] - [\Pi][\phi]\big)\big), \tag{3}$$

where $\gamma = 1/\sqrt{-g} = 1/\sqrt{1 + (\partial\pi)^2}$. We have adopted the notation of Ref. [6], with $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$ and square brackets representing the trace (with respect to $\eta_{\mu\nu}$) of a tensor. Also, we define $[\phi^n] \equiv \partial\pi \cdot \Pi^n \cdot \partial\pi$, so that $[\phi] = \partial^\mu \pi\, \partial_\mu \partial_\nu \pi\, \partial^\nu \pi$. We define the fundamental scale of the theory as $\Lambda = \lambda^{1/4}$. We express all dimensionful quantities in units of Λ in numerical calculations. This is equivalent to setting $\lambda = 1$.

The field equation of motion is [6]

$$\lambda\gamma\big\{[\Pi] - \gamma^2[\phi]\big\} - M_5^3 \gamma^2 \big\{[\Pi]^2 - [\Pi^2]$$
$$+ 2\gamma^2\big([\phi^2] - [\Pi][\phi]\big)\big\} - \frac{M_4^2}{2}\gamma^3\big\{[\Pi]^3 + 2[\Pi^3] - 3[\Pi][\Pi^2]$$
$$+ 3\gamma^2\big(2\big([\Pi][\phi^2] - [\phi^3]\big) - \big([\Pi]^2 - [\Pi^2]\big)[\phi]\big)\big\} = 0. \tag{4}$$

The Galileon theory [11], which results from the DGP model [12] in the decoupling limit, can also be obtained by taking the nonrelativistic limit $(\partial\pi)^2 \ll 1$ of the brane theory [6]. In this process, terms involving second derivatives of the field, such as $\Box\pi$, are not assumed to be small. If total derivatives are neglected, the leading terms in the expansion of Eqs. (1)–(3) give [6]

$$S^{NR} = \int d^4x \bigg\{ -\frac{\lambda}{2}(\partial\pi)^2 + \frac{M_5^3}{2}(\partial\pi)^2 \Box\pi$$
$$+ \frac{M_4^2}{4}(\partial\pi)^2\big((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\big)\bigg\}. \tag{5}$$

The term of highest order in the Galileon theory, omitted here, can be obtained by including in the brane action the Gibbons–Hawking–York term associated with the Gauss–Bonnet term of $(4+1)$-dimensional gravity. The theory can be put in more conventional form by defining $\lambda = \Lambda^4$ and employing the scalar field $\Lambda^2\pi$. As before, we express all quantities in units of Λ in numerical calculations. The field equation of motion is

$$\lambda[\Pi] - M_5^3\big([\Pi]^2 - [\Pi^2]\big)$$
$$- \frac{M_4^2}{2}\big([\Pi]^3 + 2[\Pi^3] - 3[\Pi][\Pi^2]\big) = 0, \tag{6}$$

to be compared with Eq. (4). The relation between the brane and Galileon theories implies the existence of solutions similar to the ones we described for the brane theory.

Before presenting solutions of the equations of motion, we must clarify an important point in their interpretation. We shall study configurations of two parallel branes, possibly connected by a throat. The form of the term (2), which is reduced to the second term of Eq. (5) in the nonrelativistic limit, breaks the reflection symmetry across an isolated brane. In a two-brane system, it would affect differently the upper and lower parts of a throat, thus breaking the reflection symmetry across the middle plane between the two branes. The origin of this counterintuitive behavior can be traced to the DGP model [12]. In that construction, the brane corresponds to the boundary of the bulk space, and the field π to a "brane-bending" mode. The two-brane configuration that we have in mind would correspond to a slab of bulk space of finite thickness, with two DGP branes on its sides. The long-distance physics will be affected by the effective compactification of the extra dimension. However, we are interested only in the range of scales that are relevant for the Galileon, which remain unaffected if we assume that the thickness of the slab of bulk space is sufficiently large. The "brane-bending" modes of the two branes are defined with respect to coordinate systems of opposite orientation along the extra dimension. If a unique coordinate system is used, one of the two modes must be shifted by the distance between the two branes and have its sign reversed. At the level of the Galileon theory, an equivalent way of describing the two-brane system is by assuming that the effective theory is given by Eq. (5), but with opposite values of the coefficient M_5^3 of the cubic term for each of the two branes. The same assumption must be made for the term (2) of the brane theory, in order to get a two-brane configuration symmetric under reflection across the middle plane.

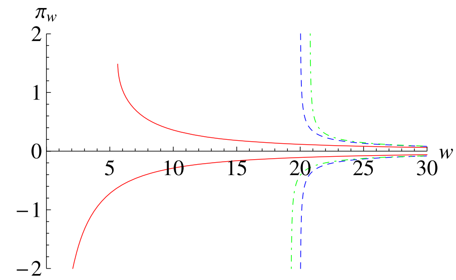

Fig. 1. The solutions (8) (dashed), (11) (dot-dashed), (18) (solid) for $\kappa = 1$, $c = 10$.

3. Solutions of the brane theory

A class of static solutions of Eq. (4) can be obtained if we make the ansatz $\pi = \pi(w)$ with $w = r^2$. Eq. (4) becomes

$$\frac{\lambda}{(1 + 4w\pi_w^2)^{3/2}}\big(3\pi_w + 8w\pi_w^3 + 2w\pi_{ww}\big)$$
$$- \frac{4M_5^3\pi_w}{(1 + 4w\pi_w^2)^2}\big(3\pi_w + 4w\pi_w^3 + 4w\pi_{ww}\big)$$
$$- \frac{12M_4^2\pi_w^2}{(1 + 4w\pi_w^2)^{5/2}}(\pi_w + 2w\pi_{ww}) = 0, \tag{7}$$

where the subscripts denote differentiation with respect to w.

For $M_5 = M_4 = 0$ the action is reduced to the DBI one. In this case, the solution of the above equation is

$$\pi_w = \pm\frac{c}{\sqrt{w^3 - 4c^2 w}}, \tag{8}$$

with $c > 0$. The two branches are depicted as dashed lines in Figs. 1 and 2 for $c = 10$. Integrating π_w with respect to w and joining the solutions smoothly at the location of the square-root singularity of Eq. (8) generates a continuous double-valued function of r, which extends from infinite r to $r_{th} = \sqrt{2c}$ and back to infinity. This catenoidal solution describes a pair of branes connected by a static throat [1,2]. The integration constant c is related