Physics Letters B 726 (2013) 461–466

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Anisotropic branes

Souvik Banerjee *, Samrat Bhowmick, Sudipta Mukherji

Institute of Physics, Bhubaneswar, 751 005, India

ARTICLE INFO

Article history:
Received 4 February 2013
Received in revised form 8 July 2013
Accepted 16 August 2013
Available online 23 August 2013
Editor: L. Alvarez-Gaumé

ABSTRACT

We present a class of anisotropic brane configurations which shows BKL oscillations near their cosmological singularities. Near horizon limits of these solutions represent Kasner space embedded in AdS background. Dynamical probe branes in these geometries inherit anisotropies from the background. Amusingly, for a probe M5 brane, we find that there exists a parameter region where three of its world-volume directions expand while the rest contract.

1. Introduction

AdS/CFT correspondence proposes an equivalence between the string theory in AdS space and a conformal field theory on its boundary. One of the remarkable features of this correspondence is the fact that it relates a strongly coupled theory with a weakly coupled one. Consequently, it provides us with a way to tame the non-perturbative region of one by performing computations on its dual. Due to strong gravitational fluctuations, physics around cosmological singularities is dominated by non-perturbative effects and one hopes that the AdS/CFT correspondence would shed some light into it. Indeed, in recent years, we have witnessed several important investigations where attempts were made to find the signatures of these singularities in their gauge theory duals. Expectation is that the dual theory evolution might be able to provide a sensible quantum description of these singularities. Successes have been varied, please see Refs. [1–5].

Inspired by this line of developments, in this Letter, we search for D brane solutions in ten-dimensional type IIB theory where the world-volume metric expands anisotropically and show instabilities within their supergravity descriptions. We find that appropriately tuning the five form field strength, it is possible to construct a D3 brane with four-dimensional Kasner-like world-volume. Along with a time-like singularity at $r=0$, the metric shows an additional cosmological singularity at $t=0$. Perturbation around $t=0$ generates an analogue of Belinskii–Lifshitz–Khalatnikov (BKL) oscillations. The near horizon geometry of this brane reduces to that of a Kasner-universe in AdS space plus a five sphere along with an appropriate five form field strength. We further probe the geometry by a dynamical D3 brane whose world-volume inherits its anisotropic expansion/contraction along with a BKL like oscillation. Similar solutions can be constructed even within eleven-dimensional supergravity. As an illustrative example, we discuss the case of M5 brane. The near horizon geometry is now a six-dimensional Kasner space. A dynamical probe M5 brane in this space-time again acquires an anisotropic expansion in some directions and contraction along some. Amusingly, we find that it is possible to tune parameters in such a manner that three directions expand and the rest contract. Close to the cosmological singularities supergravity descriptions of all these solutions are expected to break down. We hope that the gauge theory description would shed some light on the physics near the singularities.

2. D3 brane with anisotropic time-dependent world-volume

Besides the static D branes of odd space dimensions, the IIB string theory admits *time-dependent* branes. Consider, for example, the case of D3 brane. The equations of motion following from the relevant part of standard IIB supergravity action[1]

$$S_{IIB} = -\frac{1}{16\pi G_{10}} \times \int d^{10}x \sqrt{-g}\left(R - \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - \frac{1}{2\times 5!}F_5^2\right) \tag{1}$$

have the following forms:

$$R^{\mu}_{\nu} = \frac{1}{2}\partial^{\mu}\phi\partial_{\nu}\phi + \frac{1}{2\times 5!}\left(5F^{\mu\xi_2\ldots\xi_5}F_{\nu\xi_2\ldots\xi_5} - \frac{1}{2}\delta^{\mu}_{\nu}F_5^2\right),$$
$$\partial_{\mu}\left(\sqrt{g}F^{\mu\xi_2\ldots\xi_5}\right) = 0,$$
$$\nabla^2\phi = 0. \tag{2}$$

* Corresponding author.
E-mail addresses: souvik@iopb.res.in (S. Banerjee), samrat@iopb.res.in (S. Bhowmick), mukherji@iopb.res.in (S. Mukherji).

[1] The self-duality condition of the 5-form field strength is to be imposed at the level of equation of motion.

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.08.041

These equations are solved by

$$ds^2 = \left(1+\frac{l^4}{r^4}\right)^{-\frac{1}{2}}\left[-dt^2+t^{2\alpha}\,dx^2+t^{2\beta}\,dy^2+t^{2\gamma}\,dz^2\right]$$
$$+\left(1+\frac{l^4}{r^4}\right)^{\frac{1}{2}}\left[dr^2+r^2\,d\Omega_5^2\right],$$
$$F_{txyzr} = \frac{2\sqrt{2}l^4t^{\alpha+\beta+\gamma}r^3}{(l^4+r^4)^2}, \qquad F_{ijklm} = \sqrt{-g}\epsilon_{txyzrijklm}F^{txyzr},$$
$$\phi = 0, \tag{3}$$

provided

$$\alpha+\beta+\gamma = 1 \quad \text{and} \quad \alpha^2+\beta^2+\gamma^2 = 1. \tag{4}$$

Here, i, j, k, l, m are the indices on S^5. The numbers α, β, γ can be organized in an increasing order $\alpha < \beta < \gamma$ and they vary in the range

$$-\frac{1}{3} \leqslant \alpha \leqslant 0, \qquad 0 \leqslant \beta \leqslant \frac{2}{3}, \quad \text{and} \quad \frac{2}{3} \leqslant \gamma \leqslant 1. \tag{5}$$

These numbers can also be parametrized as

$$\alpha(u) = \frac{-u}{1+u+u^2}, \qquad \beta(u) = \frac{1+u}{1+u+u^2},$$
$$\gamma(u) = \frac{u+u^2}{1+u+u^2}, \tag{6}$$

where the Lifshitz–Khalatnikov parameter $u \geqslant 1$. Further, values $u < 1$ lead to the same range as

$$\alpha\left(\frac{1}{u}\right) = \alpha(u), \qquad \beta\left(\frac{1}{u}\right) = \beta(u), \qquad \gamma\left(\frac{1}{u}\right) = \gamma(u). \tag{7}$$

The five form charge can be calculated by integrating $*F_5$ over the transverse space and it turns out to be time independent.

In our convention, the extremal D3 brane is represented by $\alpha = \beta = \gamma = 0$ and is not continuously connected to the above solution. Unlike extremal D3 brane, this solution breaks all the supersymmetries of IIB theory due to its explicit time dependence. The Kretschmann scalar for the metric is given by

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{16(-\alpha^2(l^4+r^4)^6+\alpha^3(l^4+r^4)^6-5l^8r^4(l^8+12r^8)t^4)}{r^4(l^4+r^4)^5t^4}. \tag{8}$$

In writing the above equation, we have used the condition (4). It has a time-like singularity at $r = 0$ at any finite time. It, further, has a cosmological singularity at $t = 0$.

In the large r limit, equations in (3) reduce to a four-dimensional Kasner solution plus a flat six-dimensional part. Within the Bianchi classification of homogeneous spaces, the Kasner metric corresponds to choosing all three of the structure constants to be zero. A generic perturbation near the singularity breaks these constraints generating Belinskii–Lifshitz–Khalatnikov (BKL) oscillations [6]. To briefly illustrate the BKL oscillation, appropriately generalized to our context, we replace the world-volume metric on the brane by type IX homogeneous space.

To this end, let us consider the brane configuration of the form

$$ds^2 = \left(1+\frac{l^4}{r^4}\right)^{-\frac{1}{2}}\left[-dt^2+\left(a(t)^2l_il_j+b(t)^2m_im_j\right.\right.$$
$$\left.\left.+c(t)^2n_in_j\right)dx^i\,dx^j\right]+\left(1+\frac{l^4}{r^4}\right)^{\frac{1}{2}}\left[dr^2+r^2\,d\Omega_5^2\right] \tag{9}$$

with the anti-symmetric five form field and the scalar

$$F_{txyzr} = \frac{2\sqrt{2}r^3l^4a(t)b(t)c(t)\sin(x)}{(r^4+l^4)^2},$$
$$F_{ijklm} = \sqrt{-g}\epsilon_{txyzrijklm}F^{txyzr}, \tag{10}$$
$$\phi = 0. \tag{11}$$

Here l_i, m_i, n_i are frame vectors. For IX metric, all the three structure constants are 1 and the simplest choice for the frame vectors is

$$l_i = (\sin x, -\cos z\sin x, 0),$$
$$m_i = (\cos x, \sin z\sin x, 0),$$
$$n_i = (0, \cos x, 1). \tag{12}$$

The coordinates run through values in the ranges $0 \leqslant x \leqslant \pi$, $0 \leqslant y \leqslant 2\pi$, $0 \leqslant z \leqslant 4\pi$.[2] The above configuration (9)–(11) is a solution provided they satisfy IIB equations of motion (2). This requirement leads to the following differential equations for a, b and c.

$$\frac{(a_tbc)_t}{abc} = \frac{1}{2a^2b^2c^2}\left[\left(b^2-c^2\right)^2-a^4\right],$$
$$\frac{(ab_tc)_t}{abc} = \frac{1}{2a^2b^2c^2}\left[\left(c^2-a^2\right)^2-b^4\right],$$
$$\frac{(abc_t)_t}{abc} = \frac{1}{2a^2b^2c^2}\left[\left(a^2-b^2\right)^2-c^4\right],$$
$$\frac{a_{tt}}{a}+\frac{b_{tt}}{b}+\frac{c_{tt}}{c} = 0, \tag{13}$$

where the subscript indicates derivative with respect to t.[3] These are exactly the equations responsible for generating standard BKL oscillations. Consequently, the brane world-volume metric will oscillate with negative powers of t oscillating from one direction to another. In the next paragraph, for the sake of completeness, we give a brief analysis of this oscillation.

To proceed, first we notice that if all the expressions on the right-hand side of (13) are small in some region, the system will have a Kasner-like regime with

$$a \sim t^{\alpha}, \qquad b \sim t^{\beta}, \qquad c \sim t^{\gamma}, \tag{14}$$

where α, β, γ satisfy constraint as in (4). However, now since α is negative, close to $t = 0$, a^4 term in the right-hand sides of (13) will start dominating. It is useful to write these equations in terms of new variables defined as

$$a = e^p, \qquad b = e^q, \qquad c = e^s, \qquad e^{p+q+s}\,d\tau = dt. \tag{15}$$

In the vicinity of $t = 0$, (13) reduces to

$$p_{\tau\tau} = -\frac{1}{4}e^{4p}, \qquad q_{\tau\tau} = s_{\tau\tau} = \frac{1}{2}e^{4p}, \tag{16}$$

where the subscript τ indicates derivative with respect to τ. The solution of these equations should describe the evolution of world-volume metric from the initial state of Kasner metric. In terms of the new variables, this is equivalent to

$$p_\tau = \alpha, \qquad q_\tau = \beta, \qquad s_\tau = \gamma. \tag{17}$$

[2] In all our discussion, we will closely follow [7]. [9] also has a lucid review of BKL oscillations for types VIII and IX spaces.

[3] For type I spaces, in which Kasner metric belongs, the right-hand sides of all the equations in (13) would have been zero. This is due to the fact that all the structure constants are zero for type I spaces.