Physics Letters B 724 (2013) 138–143

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Phenomenological high precision neutron–proton delta-shell potential

R. Navarro-Pérez, J.E. Amaro *, E. Ruiz-Arriola

Departamento de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional Universidad de Granada, E-18071 Granada, Spain

ARTICLE INFO

Article history:
Received 15 March 2013
Received in revised form 27 May 2013
Accepted 29 May 2013
Available online 1 June 2013
Editor: W. Haxton

Keywords:
Potential scattering
Pn interaction
One pion exchange

ABSTRACT

We provide a successful fit for proton–neutron scattering below pion production threshold up to LAB energies of 350 MeV. We use seven high-quality fits based on potentials with different forms as a measure of the systematic uncertainty. We represent the interaction as a sum of delta shells in configuration space below the 3 fm and a charge-dependent one-pion exchange potential above 3 fm together with magnetic effects. Special attention is payed to estimate the errors of the phenomenological interaction.

The study of the NN interaction has been a central and recurrent topic in Nuclear Physics for many years (see e.g. [1,2] and references therein). The standard approach to constrain the interaction is to analyze NN scattering data below pion production threshold and to undertake a partial wave analysis (PWA), the quality of the fit being given by the χ^2/d.o.f. value. Only by the mid 90s was it possible to fit about 4000 selected NN scattering data after discarding about further 1000 of 3σ inconsistent data with a $\chi^2/\text{d.o.f.} \lesssim 1$ and incorporating charge dependence (CD) for the One Pion Exchange (OPE) potential as well as magnetic and vacuum polarization effects [3]. This benchmark partial wave analysis (PWA) was carried out using an energy-dependent potential for the short range part for which nuclear structure calculations become hard to formulate. Thus, energy-independent *high-quality* potentials were subsequently produced with almost identical $\chi^2/\text{d.o.f.} \sim 1$ for a gradually increasing database [4–7]. While any of these potentials provides individually satisfactory fits to the available data there are no published error estimates of the potential parameters. Moreover it should also be noticed that the existing high-quality potentials are different in their specific form; they range from local to non-local in different versions of non-locality. Thus, scattering phase shifts and observable amplitudes are not identical and in fact the existing set of high-quality potentials as a whole provides a distribution of scattering observables accounting for systematic uncertainties in addition to the statistical uncertainties obtained from the fitted data for each individual potential. Given the fact that these interactions are just constrained to the elastic scattering data (and eventually to the deuteron) which go up to the pion production threshold, one is physically probing the interaction with a resolution not finer than the shortest de Broglie wavelength $\Delta\lambda = \hbar/\sqrt{M_N m_\pi} \sim 0.5$ fm. Thus, for practical purposes it may be advantageous to consider *coarse grained* interactions. This is actually the physics underlying the so-called V_{lowk} approach [8] in which an effective interaction in a restricted model space is built. By starting from *different* high-quality potentials with a common charge-dependent OPE interaction the CM-momenta above $\Lambda \sim \sqrt{M_N m_\pi}$ are eliminated by a suitable transformation and a remarkable universal interaction is obtained for $p \leqslant \Lambda$. Many of the applications of such an appealing interaction have recently been reviewed [9].

On the other hand, when switching from the NN problem to the many body nuclear problem the features and the form of the interaction are relevant in terms of computational cost and feasibility (see e.g. [10] and references therein). The lack of knowledge of a precise potential form with finer resolution than $\Delta r \sim 0.5$ fm suggests to search for a description of scattering data directly in terms of a coarse grained potential sampled at some sensible "thick points". Any sampling procedure necessarily redistributes the interaction strength and smooths the potential as compared to the zero resolution limit $\Delta r \to 0$ implicit in most potential approaches and generating the troublesome short distance cores. This requires short distance correlations in the wave function to ensure the finiteness of the energy [10]. A desirable way to sample the interaction is to provide an acceptable χ^2-fit with the minimal number of sampling points [11]; by implementing this minimal sampling we just try to avoid statistical dependence between the strengths of the potential at the chosen sampling points.

* Corresponding author.
E-mail addresses: rnavarrop@ugr.es (R. Navarro-Pérez), amaro@ugr.es (J.E. Amaro), earriola@ugr.es (E. Ruiz-Arriola).

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.05.066

Our motivation to proceed in this fashion is to make error propagation in nuclear structure calculations more direct since in the absence of statistical correlations errors may just be added in quadrature [12].

In the present work we report a first-stage form of a high-quality potential accommodating these desirable features, namely complying with existing high-quality fits and at the same time accounting for the systematic uncertainties. Our "coarse grained" potential is simple enough to undertake an analysis of the errors in its parameters as well as in the resulting phase shifts from a global fit to the experimental data. Our research is conveniently done in two stages. First we compile the phase shifts provided by the partial wave analysis (PWA) and by the high-quality potentials of Refs. [3–7] for a discrete set of energies E_i. By averaging those phase shifts for each energy and partial wave, we generate a set of "pseudodata" and "pseudo-errors" that we will use to perform a fit of our potential parameters for each partial wave, with their uncertainties. Since the pseudodata have been generated from high-quality potentials, our first-stage model, after fitting, will be a faithful representation of the experimental knowledge. A second stage of our research [14] will be to use the first-stage parameters as a starting point for a global partial wave analysis to pp and np scattering data below pion production threshold up to LAB energies of 350 MeV.

In this Letter we present the results of the first-stage fit which is interesting in its own right, since the existing systematic errors relevant for Nuclear Physics can already be extracted. Actually, the second-stage fit is currently being done and will be presented in a forthcoming publication [14] where the specific statistical errors will be considered. Note that the first-stage parameters are not checked against the experimental database, but to the pseudodata. Therefore calling it already "high-quality" could seem not justified a priory. At this point we should anticipate that, as presented in Refs. [13,14], a χ^2/d.o.f. value can be computed with the experimental data and the first-stage optimal parameters. The optimal value turns out to be very close to one (χ^2/d.o.f. $= 1.12$), when about 4-digits are taken into account, although errors deduced directly from experimental data turn out to be much smaller than the systematic errors addressed here. The reason is that here we are fitting pseudodata constructed from a family of high-quality potentials, and the resulting potential inherits some of its characteristics. This fact makes the first-stage fit to pseudodata highly interesting and useful despite its simplicity. Our procedure allows to undertake an analysis of the systematic errors in the first-stage parameters. We do this by assuming that the systematic error inherent to any specific choice of the potential form corresponds to individual uncorrelated pseudodata.[1] Hence we may invoke the central limit theorem to undertake the traditional statistical treatment to the mean average and the corresponding standard deviation of Refs. [3–7] without any further ado. We will use this compilation as our pseudo-database.

A convenient representation to sample the short distance component of the NN interaction was already suggested by Aviles [15] almost 40 years ago in terms of delta shells which for any partial wave ${}^{2S}(l',l)_J$ we take as

$$V^{JS}_{l,l'}(r) = \frac{1}{2\mu_{np}} \sum_{n=1}^{N} (\lambda_n)^{JS}_{l,l'} \delta(r - r_n) \quad r \leqslant r_c \tag{1}$$

with μ_{np} the reduced np mass and $r_c = 3$ fm. In the spirit of Refs. [3–7] for $r \geqslant r_c$ we use the well-known long-distance tail of the NN potential

$$V(\vec{r}) = V_{EM}(\vec{r}) + V_{OPE}(\vec{r}), \quad r > r_c, \tag{2}$$

where V_{EM} is the electromagnetic potential of Ref. [5], and V_{OPE} is the one-pion exchange potential.

The solution of the corresponding Schrödinger equation in coupled channels is straightforward; for any $r_n < r < r_{n+1}$ with $r_N < r_c$ we have free solutions and log-derivatives are discontinuous at the r_n-points so that one generates an accumulated S-matrix at any sampling point providing a discrete version of Calogero's variable phase equation [16]. Although this potential is formally local, the fact that we are coarse graining the interaction enables to encode efficiently nonlocalities operating below the finest resolution Δr. Of course, once we admit that the interaction below r_c is unknown there is no advantage in prolonging the well-known charge-dependent OPE tail and other electromagnetic effects for $r < r_c$. The low energy expansion of the discrete variable phase equations was used in Ref. [17] to determine threshold parameters in all partial waves. The relation to the well-known Nyquist theorem of sampling a signal with a given bandwidth has been discussed in Ref. [18]. Some of the advantages of using this simple potential for nuclear structure calculations as well as the connection to the $V_{\rm lowk}$ approach have been spelled out already [19].

To generate the pseudodata we use the standard LAB energy values usually listed in the high-quality potentials, namely $E_{\rm LAB} = 1, 5, 10, 25, 50, 100, 150, 200, 250, 300, 350$ MeV and fit to the mean phase-shift values at those energies with an error equal to the standard deviation. In this way we account for the systematic errors due to the different representations of the potentials [3–7]. We find that they are generally larger than those quoted by the original PWA where only statistical uncertainties where explicitly discussed for a *fixed* potential form [3]. With these pseudodata sets and the given energies we undertake a phase-shift fit and determine errors using the standard covariance matrix.

As expected from Nyquist theorem, we need at most $N = 5$ sampling points which for simplicity are taken to be equidistant with $\Delta r = 0.6$ fm between the origin and $r_c = 3$ fm. This is the minimal number which provides an acceptable fit to the pseudodata compiled from Refs. [3–7]. Our results for the np phase shifts for all partial waves with total angular momentum $J \leqslant 5$ are depicted in Fig. 1. The fitting parameters $(\lambda_n)^{JS}_{l,l'}$ entering the delta-shell potentials, Eq. (1), are listed in Table 1 with their deduced uncertainties. Of course, a definitive assessment on systematic errors would require testing *all possible* potential forms. Thus, the errors will generally be larger than those estimated here. We find that correlations among the different $(\lambda_n)^{JS}_{l,l'}$ values within a given partial wave channel are unimportant, and hence these parameters are essentially independent from each other. This is a direct consequence of our strategy to minimize the number of sampling points. We find that introducing more points or equivalently reducing Δr generates unnecessary correlations. Also, lowering the value of r_c below 3 fm, requires overlapping the short-distance potential, Eq. (1), with the OPE plus em corrections. Note that in order to encompass all different forms of potentials, 48 parameters are needed in contrast to the about 40 independent parameters found in the original 1990 and subsequent models.

We determine the deuteron properties by solving the bound state problem in the ${}^3S_1$–${}^3D_1$ channel using the corresponding parameters listed in Table 1. The predictions are presented in Table 2 where our quoted errors are obtained from propagating Table 1.

[1] We have actually checked that, within the corresponding statistical uncertainty, the absence of correlations among the different partial waves of the six high-quality potentials by a direct evaluation of the correlation coefficient (see Ref. [11] for a definition) holds for every single LAB energy below 350 MeV.