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On quantization of the SU(2) Skyrmions

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ABSTRACT

There are two known approaches for quantizing the SU(2) Skyrme model, the semiclassical and canonical quantization. The semiclassical approach does not take into account the non-commutativity of velocity of quantum coordinates and the stability of the semiclassical soliton is conveniently ensured by the symmetry breaking term. The canonical quantum approach leads to quantum mass correction that is not obtained in the semiclassical approach. In this Letter we argue that these two approaches are not equivalent and lead to different results. We show that the resulting profile functions have the same asymptotic behaviour, however their shape in the region close to the origin is different.

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1. Introduction

The Skyrme topological soliton model is a nonlinear field theory, with localized finite energy soliton solutions [1,2]. The first comprehensive phenomenological application of the model to baryons was the semiclassical calculation of the static properties of the nucleon [3]. The low-energy QCD in the large color limit [4] is argued to describe baryons as solitons in the weakly coupled phase of mesons which was the original idea of [1] and [3]. The original model was defined for a unitary field $U(\mathbf{x}, t)$ that belongs to the fundamental representation of SU(2). The boundary constraint $U \to \mathbb{1}$ as $|\mathbf{x}| \to \infty$ implies that the unitary field represents a mapping from $S^3 \to S^3$. The integer valued winding number which classifies the solitonic sectors of the model was interpreted to be the baryon number. The semiclassical quantization of model has proven to be useful for baryon phenomenology. However the semiclassically-treated SU(2) model was shown to have an instability in calculating the energy functional [5,6]. The stability and correct asymptotic behaviour of solutions can be achieved by introducing an additional symmetry breaking term. The alternative stabilization of the quantum SU(2) Skyrme model has been obtained by quantizing the soliton quantum canonically in collective coordinate approach [7,8]. The non-commutativity of canonical momenta in a Hamiltonian system leads to non-commutativity of velocities of the canonical coordinates (collective coordinates) which cannot be ignored. It was shown that the procedure of the canonical quantization contributes to the appearance of new terms in the explicit form of the Lagrangian of the model. These terms are interpreted as quantum corrections to the mass of the soliton ('quantum mass corrections') that restore the stability of the solitons that is lost in the semiclassical approach [8]. The purpose of the present Letter is to show that the quantum mass corrections of the soliton are important in ensuring the stability of the quantum solitons and realize Skyrme's original conjecture that 'the mass (of the meson m_π) may arise as a self-consistent quantal effect. This point will not be followed here, but when, for calculation purposes, we want to allow phenomenologically for a finite mass this will be done by adding to L a term (proportional to m_π^2)' [2]. We find stable quantum solitons by varying the complete quantum energy functional for nucleon. The stability is ensured by the consequence of iterative calculations. The shapes of quantum solitons with different stabilizing terms are demonstrated in Fig. 2. We do not consider quantum soliton with quantum numbers of Δ resonances because there are no known stable solutions for the quantum SU(2) Skyrme model defined in fundamental representation. The stable quantum solutions for Δ exist in the generalized SU(2) Skyrme model which is defined for higher representations [8] or in the SU(3) Skyrme model [9]. After some preliminary definitions in Section 2 below, the main part of this Letter is organized as follows. In Section 3 the quantum Skyrme model is constructed *ab initio* in the collective coordinates framework and canonicaly quantized. The structure of energy functional is derived. Section 4 contains numerical results and summarizing discussion.

2. Classical Skyrmion

The SU(2) Skyrme model is conveniently defined via the chirally symmetric Lagrangian density

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$$\mathcal{L}_{\mathrm{Sk}} = -\frac{f_\pi^2}{4}\operatorname{Tr}\{\mathbf{R}_\mu \mathbf{R}^\mu\} + \frac{1}{32e^2}\operatorname{Tr}\{[\mathbf{R}_\mu, \mathbf{R}_\nu][\mathbf{R}^\mu, \mathbf{R}^\nu]\}, \tag{1}$$

which is written in terms of the su(2)-valued right chiral current $\mathbf{R}_\mu = (\partial_\mu U)U^\dagger$. Here f_π and e are model parameters, whose values are constrained by fitting with the experimental data. The chiral symmetry breaking term of Lagrangian density is defined by

$$\mathcal{L}_{\mathrm{SB}} = -\mathcal{M}_{\mathrm{SB}} = -\frac{f_\pi^2}{4} m_0^2 \operatorname{Tr}\{U + U^\dagger - 2\cdot \mathbb{1}\}, \tag{2}$$

where m_0 is the third model parameter.

The classical static soliton (Skyrmion) is obtained by employing the spherically symmetric hedgehog ansatz

$$U_0(\hat{x}, F(r)) = \exp i(\sigma\cdot\hat{x})F(r), \tag{3}$$

where σ are Pauli matrices and $\hat{x}$ is the unit vector. With this ansatz the classical Lagrangian density reduces to the following simple form

$$\begin{aligned}\mathcal{L}_{\mathrm{cl}}(F(r)) &= -\mathcal{M}_{\mathrm{Sk}}(F(r)) - \mathcal{M}_{\mathrm{SB}}(F(r)) \\ &= -\frac{f_\pi^2}{2}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right) \\ &\quad - \frac{1}{2e^2}\frac{\sin^2 F}{r^2}\left(2F'^2 + \frac{\sin^2 F}{r^2}\right) \\ &\quad - f_\pi^2 m_0^2 (1-\cos F),\end{aligned} \tag{4}$$

which defines the energy of the Skyrmion,

$$E_0 = -4\pi \int \mathcal{L}_{\mathrm{cl}}(F(r)) r^2\,\mathrm{d}r.$$

Variation of (4) leads to a differential equation for the profile function $F(r)$. The standard boundary conditions for a Skyrmion are $F(0) = \pi$, $F(\infty) = 0$.

3. Quantization of the Skyrmion

The standard (semiclassical) approach to quantize the rotational zero modes of the Skyrmion yielding multiplets with equal spin and isospin in each multiplet was presented in [3,4]. This approach treats the soliton as a rigid body and does not take into account the non-commutativity of quantum coordinates and velocities. The canonical quantization treats the quantum variables canonically and leads to new quantum terms in the explicit of the Lagrangian that are interpreted as dynamically generated quantum mass correction. This approach was presented in [7] and further developed in [8]. The details are as follows.

The quantum field U can be written in a form with temporal and spatial parts separated explicitly

$$U(\hat{x}, F(r), \mathbf{q}(t)) = A(\mathbf{q}(t))U_0(\hat{x}, F(r))A^\dagger(\mathbf{q}(t)), \tag{5}$$

where U_0 is the classical field and $A(\mathbf{q}(t))$ is a matrix specified by three real independent parameters, generalized quantum coordinates $q^k(t)$. The Lagrangian (1) is considered quantum mechanically *ab initio*. Thus the canonical commutation relation $[p_k, q^l] = -i\delta_{kl}$ for generalized coordinates q^l and conjugate momenta p_k is required to hold. In such a way the generalized coordinates $q^k(t)$ and velocities are ought to satisfy the commutation relations

$$[\dot{q}^k, q^l] = -if^{kl}(q), \tag{6}$$

where the form of the tensor $f^{kl}(q)$ will be determined below. The temporal derivatives are calculated by employing the usual Weyl ordering, and the operator ordering is fixed by the form of the Lagrangian (1) without further ordering ambiguity. For the derivation of the canonical momenta it is sufficient to restrict the consideration to the terms of second order in velocities (the terms of first order vanish). This leads to

$$L_{\mathrm{Sk}} \approx \frac{1}{2}\dot{q}^\alpha g_{\alpha\beta}(q, F)\dot{q}^\beta + [(\dot{q})^0 - \text{order term}], \tag{7}$$

where the metric tensor takes the form

$$g_{\alpha\beta}(q, F) = -C_\alpha'^{(M)}(q)(-1)^M a(F)\delta_{M,-M'}C_\beta'^{(M')}(q). \tag{8}$$

Here $C_\alpha'^{(M)}(q)$ are functions of quantum coordinates q. Their explicit form depends on the chosen parametrization of the SU(2) group. However the explicit form does not appear anywhere in the calculations. For details on these functions we refer to [8].

The canonical commutation relations $[p_\beta, q^\alpha] = -i\delta_{\alpha\beta}$ then yield the explicit expression for the functions $f^{\alpha\beta}(q) = g^{-1}_{\alpha\beta}(q, F)$. Next, by substituting (5) into the Lagrangian density (1) and after some lengthy manipulation and integration over the space variables the complete expression of the quantum Skyrme model Lagrangian are obtained

$$L = -M_{\mathrm{cl}} - \Delta M + \frac{1}{2a(F)}\hat{J}'^2, \tag{9}$$

where ΔM is the (negative) quantum mass correction

$$\Delta M = -\frac{2\pi}{a^2(F)}\int r^2\,\mathrm{d}r \sin^2 F\left[f_\pi^2 + \frac{1}{2e^2}\left(2F'^2 + \frac{\sin^2 F}{r^2}\right)\right], \tag{10}$$

and $\hat{J}'_{(M)}$ are the angular momentum operators

$$\hat{J}'_{(M)} = \frac{i}{2}\{p_\alpha, C'^{\alpha}_{(M)}(q)\} \tag{11}$$

satisfying the standard SU(2) commutation rules, and $C'^{\alpha}_{(M)}(q)$ is the reciprocal matrix to $C_\alpha'^{(M)}(q)$. The generalized method of quantization on a curved space developed by Sugano et al. [10] allows to write the energy functional of the quantum Skyrmion for a state with fixed spin and isospin ℓ in this form

$$E(\ell, F) = M_{\mathrm{cl}}(F) + \Delta M(F) + \frac{\ell(\ell+1)}{2a(F)}, \tag{12}$$

where $a(F)$ is the quantum momentum of inertia of the Skyrmion

$$\begin{aligned}a(F) &= \frac{1}{e^3 f_\pi}\tilde{a}(F) \\ &= \frac{1}{e^3 f_\pi}\frac{8\pi}{3}\int \mathrm{d}\tilde{r}\,\tilde{r}^2 \sin^2 F\left(1 + F'^2 + \frac{1}{\tilde{r}^2}\sin^2 F\right).\end{aligned} \tag{13}$$

Notice that it differs from the mechanical moment of inertia of the classical Skyrmion. The expression (12) is the quantum version of the mass formula of the Skyrme model, which differs from the semiclassical one by the appearance of the additional (negative) quantum correction $\Delta M(F)$. The variation of the energy functional $\frac{\delta E(F)}{\delta F} = 0$ of the quantum Skyrmion for states with given ℓ leads to an integro-differential equation for the profile function $F(r)$ with the same boundary conditions as in the classical case, $F(0) = \pi$, $F(\infty) = 0$. At large distances the asymptotic solution takes the form

$$F(\tilde{r}) = k\left(\frac{\tilde{m}^2}{\tilde{r}} + \frac{1}{\tilde{r}^2}\right)\exp(-\tilde{m}\tilde{r}), \tag{14}$$

where the quantity $\tilde{m}^2$ is defined by

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