Physics Letters B 726 (2013) 488–492

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Self-dual Maxwell–Chern–Simons solitons from a Lorentz-violating model

Rodolfo Casana *, Lucas Sourrouille

Departamento de Física, Universidade Federal do Maranhão, 65080-805 São Luís, Maranhão, Brazil

ARTICLE INFO

Article history:
Received 3 July 2013
Received in revised form 16 August 2013
Accepted 20 August 2013
Available online 29 August 2013
Editor: M. Cvetič

Keywords:
Chern–Simons-like gauge theory
Topological solitons
Lorentz symmetry violation

ABSTRACT

Self-dual abelian Higgs system, involving both the Maxwell and Chern–Simons terms are obtained from Carroll–Field–Jackiw theory by dimensional reduction. Bogomol'nyi-type equations are studied from theoretical and numerical point of view. In particular, we show that the solutions of these equations are Nielsen–Olesen vortices with electric charge.

1. Introduction

Lorentz and CPT violation has recently received substantial attention as a potential signature for underlying physics, possibly arising from the Planck scale. The most simple Lorentz- and CPT-breaking field theory consists of a three-dimensional Chern–Simons term embedded in Maxwell's four-dimensional classical electrodynamics [1]. Such model, usually known as Carroll–Field–Jackiw, is part of the Standard-Model Extension (SME), the general field-theory framework for Lorentz and CPT tests, which contains the Standard Model of particle physics and general relativity as limiting cases [2]. Recent studies in the context of the CPT-odd sector of SME, have been carried out in different areas such as radiative corrections [3], nontrivial spacetime topology [4], causality [5], supersymmetry [6], the cosmic microwave background [7], general relativity [8] and topological defects, specifically monopole structures was presented in [9] and, the first investigation about BPS vortex solutions in the presence of CPT-even Lorentz-violating terms of the SME was performed in Ref. [10].

On the other hand, it is well known that the Higgs models with the Maxwell term, support topologically stable vortex solutions [11]. With a specific choice of coupling constants, minimum energy vortex configurations satisfy first order differential equations [12]. When this occur the model presents another interesting feature, which says us that the model is the bosonic sector of a supersymmetric theory [13].

The two-dimensional matter field interacting with gauge fields whose dynamics is governed by a Chern–Simons term support soliton solutions [14]. When self-interactions are suitably chosen vortex configurations satisfy the Bogomol'nyi-type equations with a specific sixth order potential [15]. Another important feature of the Chern–Simons gauge field is that inherits its dynamics from the matter fields to which it is coupled, so it may be either relativistic [15] or non-relativistic [16]. In addition the soliton solutions are of topological and non-topological nature [17]. The relativistic Chern–Simons–Higgs model described in Ref. [15], was generalized to include a Maxwell term for the gauge field in Ref. [18]. There, the authors, in order to maintain a notion of self-duality, in which a lower bound for the energy is saturated by solutions to a set of self-duality equations, introduce an additional neutral scalar field. Also, a generalized version of Maxwell–Chern–Simons–Higgs BPS vortices, in the framework of $(1+2)$-dimensional effective field theories, was introduced in Ref. [19].

In the present work, we are interested in studying BPS vortices in a context of a CPT-odd and Lorentz-violating abelian Higgs system, involving both the Maxwell and Chern–Simons terms, which is obtained from Carroll–Field–Jackiw theory by dimensional reduction. In particular, we will show that this model support self-dual vortex solutions, which are different from those obtained in Ref. [18]. The difference lies in the fact that in our model the neutral scalar field appears naturally from the dimensional reduction, so that the model is self-dual without the requirement to

* Corresponding author.
E-mail address: rodolfo.casana@gmail.com (R. Casana).

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.08.051

introduce an additional neutral field such as in Ref. [18]. In addition we show that the vortex solution are identical in form to the Nielsen–Olesen vortices, although, in our model, the vortex possess electric charge.

2. The theoretical framework

Let us start by considering the Lorentz and CPT-violating Maxwell–Chern–Simons theory, proposed by Carroll, Field, and Jackiw [1]

$$S=\int d^4x\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{1}{4}p_\rho\epsilon^{\rho\sigma\mu\nu}A_\sigma F_{\mu\nu}-A^\nu J_\nu\right), \tag{1}$$

where p_α is a four-vector couples to electromagnetic field, which determines a preferred direction in spacetime violating Lorentz as well as CPT symmetry, and $F_{\mu\nu}$ is the stress tensor defined by $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. Here, the term $A^\nu J_\nu$ represents the coupling between the gauge field and an external current. The metric tensor is $g^{\mu\nu}=(1,-1,-1,-1)$ and $\epsilon^{\alpha\beta\mu\nu}$ is the totally antisymmetric Levi-Civita tensor such that $\epsilon^{0123}=1$.

The Lagrangian (1) leads to the following equations of motion

$$\partial_\mu F^{\mu\nu}+p_\mu\tilde{F}^{\mu\nu}=J^\nu, \tag{2}$$

where $\tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual electromagnetic tensor.

In the following we consider a model composed by the gauge field in (1) coupled to a Higgs field

$$S=\int d^4x\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{1}{4}p_\alpha\epsilon^{\alpha\beta\mu\nu}A_\beta F_{\mu\nu}+|D_\mu\phi|^2-V\big(|\phi|\big)\right], \tag{3}$$

where Greek indexes run on 0, 1, 2, 3, $D_\mu=\partial_\mu-ieA_\mu$ is the covariant derivative and $V(|\phi|)$ is a self-interacting potential to be determined below.

Specifically, we are interested in exploring the solitonic structure of the model obtained from (3) via dimensional reduction. In particular we are motivated by searching of a model with Maxwell–Chern–Simons self-dual vortex solutions different from those present in Ref. [18]. In order to analyze the $(2+1)$-dimensional problem it is natural to consider a dimensional reduction of the action by assuming that the fields do not depend on one of the spatial coordinates, say x_3. Renaming A_3 as N, leads to an action that can be written as [20]

$$\begin{aligned}S=\int d^3x\Big[&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{1}{4}p_3\epsilon^{\beta\mu\nu}A_\beta F_{\mu\nu}+\frac{1}{2}(\partial_\mu N)\\&-\frac{1}{2}\epsilon^{\mu\rho\sigma}p_\mu N\partial_\rho A_\sigma-\frac{1}{2}\epsilon^{\mu\rho\sigma}p_\mu A_\rho\partial_\sigma N\\&+|D_\mu\phi|^2-e^2N^2|\phi|^2-V\big(|\phi|\big)\Big],\end{aligned} \tag{4}$$

where now Greek's indexes are 0, 1, 2 and the coefficient p_3 play the role of Chern–Simons' parameter. The Gauss law is

$$\partial_iF_{0i}+p_3F_{12}+\epsilon_{ij}p_i\partial_jN=eJ_0, \tag{5}$$

where $J_0=i[\phi(D_0\phi)^*-\phi^*D_0\phi]$ is the conserved matter current. Integrating this equation, over the entire plane, we obtain the important consequence that any object with charge $Q=e\int d^2x\,\rho$ also carries magnetic flux $\Phi=\int B\,d^2x$ [21]:

$$\Phi=-\frac{1}{\kappa}Q. \tag{6}$$

Here, we are interested in time-independent soliton solutions that ensure the finiteness of the action (4). These are the stationary points of the energy which for the static field configuration reads

$$\begin{aligned}E=\int d^3x\Big[&\frac{1}{2}F_{i0}^2+\frac{1}{2}F_{12}^2+|D_0\phi|^2+|D_i\phi|^2+\frac{1}{2}(\partial_0N)^2\\&+\frac{1}{2}(\partial_iN)^2+e^2N^2|\phi|^2+Np_0F_{12}\\&+\epsilon_{ij}Np_i\partial_jA_0+V\big(|\phi|\big)\Big].\end{aligned} \tag{7}$$

Since we are motivated by the desire to find self-dual soliton solution, we will choose $p_1=p_2=0$. For this particular choice, the Gauss law (5) takes the simple form

$$\partial_iF_{0i}+p_3F_{12}=eJ_0. \tag{8}$$

This is just the Gauss law of the model proposed in Ref. [18]. After integration by parts and using the Gauss law, the energy functional can be rewritten as

$$\begin{aligned}E=\int d^3x\Big\{&\frac{1}{2}[F_{i0}\pm\partial_iN]^2+|D_i\phi|^2+|D_0\phi\mp ie\phi N|^2\\&+\frac{1}{2}(\partial_0N)^2+\frac{1}{2}F_{12}^2+N(p_0\pm p_3)F_{12}+V\big(|\phi|\big)\Big\}.\end{aligned} \tag{9}$$

Here, the form of the potential $V(|\phi|)$ that we choose is motivated by the desire to find self-dual soliton solution and coincides with the symmetry breaking potential of the Higgs model

$$V\big(|\phi|\big)=\frac{\lambda^2}{2}\big(|\phi|^2-v^2\big)^2. \tag{10}$$

To proceed, we need a fundamental identity

$$|D_i\phi|^2=\big|(D_1\pm iD_2)\phi\big|^2\pm eF_{12}|\phi|^2\pm\frac{1}{2}\epsilon^{ij}\partial_iJ_j. \tag{11}$$

Using this identity and choosing $p_0=\mp p_3$ the energy may rewritten as

$$\begin{aligned}E=\int d^3x\Big(&\frac{1}{2}[F_{i0}\pm\partial_iN]^2+|D_0\phi\mp ie\phi N|^2\\&+\big|(D_1\pm iD_2)\phi\big|^2+\frac{1}{2}\big[F_{12}\pm e\big(|\phi|^2-v^2\big)\big]^2\\&\pm ev^2F_{12}+\left(\frac{\lambda}{4}-\frac{e^2}{2}\right)\big(|\phi|^2-v^2\big)^2+\frac{1}{2}(\partial_0N)^2\Big).\end{aligned} \tag{12}$$

When the symmetry breaking coupling constant λ is such that

$$\lambda=2e^2, \tag{13}$$

i.e. when the self-dual point of the Abelian Higgs model is satisfied, the energy (12) reduce to a sum of square terms which are bounded below by a multiple of the magnitude of the magnetic flux:

$$E\geqslant ev^2|\Phi|. \tag{14}$$

Here, the magnetic flux is determined by the requirement of finite energy. This implies that the covariant derivative must vanish asymptotically, which fixes the behavior of the gauge field A_i. Then we have

$$\Phi=\int d^2x\,B=\frac{2\pi}{e}n, \tag{15}$$

where n is a topological invariant which takes only integer values. The bound is saturated by the fields satisfying the Gauss law and $\partial_0N=0$. This way, the first-order self-duality equations: