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Equivalence between Born–Infeld tachyon and effective real scalar field theories for brane structures in warped geometry

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ABSTRACT

An equivalence between Born–Infeld and effective real scalar field theories for brane structures is built in some specific warped space–time scenarios. Once the equations of motion for tachyon fields related to the Born–Infeld action are written as first-order equations, a simple analytical connection with a particular class of real scalar field superpotentials can be found. This equivalence leads to the conclusion that, for a certain class of superpotentials, both systems can support identical thick brane solutions as well as brane structures described through localized energy densities, $T_{00}(y)$, in the 5th dimension, y. Our results indicate that thick brane solutions realized by the Born–Infeld cosmology can be connected to real scalar field brane scenarios which can be used to effectively map the tachyon condensation mechanism.

1. Introduction

Brane cosmology driven by scalar fields has been recurrently studied in order to address the cosmological constant and hierarchy problems [1,2], as well as symmetry breaking issues [3] (see also Ref. [4] for the projection on the brane of vector and tensor fields in the bulk space). The first ideas for brane world scenarios assumed a warped 4-dimensional brane universe embedded in a higher-dimensional bulk space, where the brane corresponds to a localized delta function on the extra-dimensional coordinate [5]. Brane world scenarios also have been discussed in the context of realizing 4-dimensional gravity on a domain wall in 5-dimensional space–time [5,6], with extensions to domain walls in gravity coupled to scalars [7,8] and to time-evolving cosmological models [9] (see also Ref. [6] and references therein).

The brane scenario examined here is related to generic solutions of the 5-dimensional Born–Infeld field theories of the form

$$S = \int dx^5 \sqrt{\det g_{AB}}\left[-\frac{1}{4}R - U(\varphi)\sqrt{1 - g^{AB}\partial_A\varphi\partial_B\varphi}\right], \qquad (1)$$

where R is the scalar curvature, and g_{AB} denotes the metric tensor, with A and B running from 0 to 4. The field φ is a tachyon field and $U(\varphi)$ is its potential, with dimensional constants absorbed by a suitable field normalization. From this action, it has been conjectured that the dynamics of a Born–Infeld tachyon field in a background of an unstable D-brane system can be perturbatively described by the dynamics of an effective real scalar field [10]. According to such an assumption, tachyon calculations would be reliable only in the approximation where φ derivatives can be truncated beyond the quadratic order [11].

The perturbative truncation leads to an effective action driven by a real scalar field, χ, coupled to 5-dimensional gravity, given by

$$S_{\rm eff} = \int dx^5 \sqrt{\det g_{AB}}\left[-\frac{1}{4}R + \frac{1}{2}g_{AB}\partial^A\chi\partial^B\chi - V(\chi)\right], \qquad (2)$$

which gives rise to several possibilities for investigating the related tachyon field dynamics. In quantum field theories, a tachyon field can be realized by the instability of the quantum vacuum, described by the quantum state displaced from a local maximum of an effective potential like $V(\chi)$. In the effective real scalar field scenario, the tachyon field would follow a spontaneous symmetry breaking (SSB) that implies into a process dubbed as tachyon condensation [12,13]. Given its remarkable applications in brane world models, tachyon condensation is argued to play an important role also in string theory (see e.g. Refs. [14,15]). Tachyon condensation can also reproduce the results of a collision process similar to a kink–antikink or to a soliton–antisoliton annihilation that drives the system to the SSB vacuum after complete annihilation. In this

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context, the Big-Bang has been hypothesized to be due to such a brane–antibrane collision. Notice that branes defined as classical solutions of tachyonic potentials naturally arise in systems with rolling tachyons on unstable branes [16]. The resulting vacuum state after annihilation exhibits the remaining lower-dimensional branes as relics of tachyon condensation [17] that (re)produce the effects of cosmic strings in brane cosmology [18–20].

Real scalar field models coupled to gravity lead also to analytical solutions of gravitating defect structures which allow for the inclusion of thick branes used in several brane cosmology scenarios. Thick domain walls, for instance, are often associated to integrable models. In general, potentials associated to single real scalar field support BPS type solutions [21,22] of first-order differential equations. In this case, the equations result into topological defects that admit an internal structure.

However, there has been no consensus about how reliably effective real scalar field models can describe the Born–Infeld tachyonic dynamics [23], despite of the importance of real scalar fields in describing brane structures in warped geometry [7,24–29].

Therefore, the brane model discussed in this Letter treats Born–Infeld tachyon fields without any build in association with the real scalar field (cf. Eq. (2)). Assuming that the equations of motion for the Born–Infeld tachyon fields can be mapped by superpotential parameters constrained by first-order equations, analogously to the procedure of mapping BPS solutions into real scalar fields, one is able to find exact solutions for the tachyon field, φ. In addition, a fruitful connection between tachyon and real scalar field superpotentials can be established. The resulting brane scenario exhibits an exact equivalence between Born–Infeld tachyon and real scalar field dynamics in 5 dimensions, which is reproduced by a unique warp-factor and leads to the same localized energy densities.

In what follows we shall call χ a real scalar field, even when considering that its associated action may approach a tachyonic action that circumstantially results into a condensation mechanism and associated instabilities. We shall bear in mind that we seek for an analytical correspondence between the Born–Infeld tachyon with the real scalar field in order to obtain two equivalent brane world scenarios.

The framework for discussing a single real scalar field coupled to gravity in the brane scenario follows previous discussions [7,26–29]. The correspondence between the Born–Infeld tachyon and the real scalar field is obtained through a set of first-order equations. Novel integrable models that admit thick brane solutions to the Born–Infeld action through twin warp factors bound from above are also discussed.

2. Real scalar fields

Let us start considering a 5-dimensional space–time warped in 4 dimensions. In order to ensure the Poincaré invariance in 4 dimensions, the space–time metric is written as follows:

$$ds^2 = g_{AB}\,dx^A\,dx^B = e^{2A(y)}\eta_{\mu\nu}\,dx^\mu\,dx^\nu - dy^2, \tag{3}$$

where $\eta_{\mu\nu} \equiv \{+1,-1,-1,-1\}$, μ and ν run from 0 to 3, $y \equiv x_4$ is the infinite extra-dimension coordinate (varying from $-\infty$ to ∞) such that the normal to surfaces of constant y lie orthogonal to the brane, $e^{2A(y)}$ is the warp factor.[3]

Considering the real scalar field action, Eq. (2), one can compute the stress-energy tensor

$$T^{\chi}_{AB} = \partial_A\chi\,\partial_B\chi + g_{AB}V(\chi) - \frac{1}{2}g_{AB}g^{MN}\partial_M\chi\,\partial_N\chi, \tag{4}$$

which, supposing that both the scalar field and the warp factor dynamics depend only on the extra coordinate, y, leads to an explicit dependence of the energy density in terms of the field, χ, and of its first derivative, $d\chi/dy$, as

$$T^{\chi}_{00}(y) = \left[\frac{1}{2}\left(\frac{d\chi}{dy}\right)^2 + V(\chi)\right]e^{2A(y)}. \tag{5}$$

With the same constraints on χ about the dependence on y, the equations of motion arising from the above action are

$$\frac{d^2\chi}{dy^2} + 4\frac{dA}{dy}\frac{d\chi}{dy} - \frac{d}{d\chi}V(\chi) = 0, \tag{6}$$

by varying the action with respect to the scalar field, χ, and

$$\frac{3}{2}\frac{d^2A}{dy^2} = -\left(\frac{d\chi}{dy}\right)^2, \tag{7}$$

by varying the action with respect to the metric, or equivalently to A, which can be manipulated to yield

$$3\left(\frac{dA}{dy}\right)^2 = \frac{1}{2}\left(\frac{d\chi}{dy}\right)^2 - V(\chi). \tag{8}$$

after integrating over y.

The potential for the real scalar field can be written in terms of a *superpotential*, w, in a specialized form as

$$V(\chi) = \frac{1}{8}\left(\frac{dw}{d\chi}\right)^2 - \frac{1}{3}w^2, \tag{9}$$

which has been often discussed in the context of thick brane solutions with a single scalar field [7,27,28,30,31]. It has the advantage of simplifying the above equations through first-order equations

$$\frac{d\chi}{dy} = \frac{1}{2}\frac{dw}{d\chi}, \tag{10}$$

and

$$\frac{dA}{dy} = -\frac{1}{3}w, \tag{11}$$

for which analytical solutions can be immediately obtained through simple integrations. for which analytical solutions can be immediately obtained through simple integrations. In particular, it was first discussed in the context of supergravity on domain walls [32] and its corresponding generalization to non-supersymmetric domain walls in various dimensions [7,33]. Another method through which one endows the scalar field dependence on the extra dimension and obtains the metric function and the potential through the field equations have been discussed [34,35] (see also Ref. [36] and references therein).

From Eq. (9) follows the energy density expressed as

$$T^{\chi}_{00}(y) = \left[\frac{1}{4}\left(\frac{dw}{d\chi}\right)^2 - \frac{1}{3}w^2\right]e^{2A(y)}. \tag{12}$$

As will be discussed next, an analogous first-order formulation for tachyon fields can be carried out.

3. Born–Infeld tachyon fields

The action for a tachyon field, φ, coupled to 5-dimensional gravity is given by Eq. (1), in the geometry described by Eq. (3).

[3] For the purpose of our calculations, we have suppressed brane tension terms (tensionless brane). It can be assumed that tension terms are absorbed by the metric (see, for instance Eqs. (24) and (25) from Ref. [36] and Refs. [41,42] where the real scalar field Lagrangian is discussed in the context of thick brane solutions).