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Thermodynamics of rotating charged dilaton black holes in an external magnetic field

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ABSTRACT

In the present Letter we study the long-standing problem for the thermodynamics of magnetized dilaton black holes. For this purpose we construct an exact solution describing a rotating charged dilaton black hole immersed in an external magnetic field and discuss its basic properties. We derive a Smarr-like relation and the thermodynamics first law for these magnetized black holes. The novelty in the thermodynamics of the magnetized black holes is the appearance of new terms proportional to the magnetic momentum of the black holes in the Smarr-like relation and the first law.

1. Introduction

Black holes influenced by external fields are interesting subject to study and have stimulated a lot of research during the last three decades. Black holes immersed in external magnetic field (the so-called magnetized black holes) are of particular interest to astrophysics where the magnetic field plays important role in the physical processes taking place in the black hole vicinity [1–26]. The study of black holes in external magnetic field results in finding interesting astrophysical effects as the charge accretion and the flux expulsion from extreme black holes [1,3,10–13,16,23]. From a theoretical point of view, it is very interesting to investigate how the external fields influence the black hole thermodynamics — more specifically, how the external magnetic field affects it.

The expectation was that a thermodynamic description of the black holes in external magnetic field would include also the value of the background magnetic field as a further parameter. However, more detailed studies showed that this was not the case for static electrically uncharged solutions in four dimensions and some electrically uncharged rotating black holes in higher dimensions — the external magnetic field only distorts the horizon geometry without affecting the thermodynamics [17,18,21]. In a recent paper [26] the author studied the thermodynamics of some charged dilaton black holes in external magnetic field and he found that even in this case the black hole thermodynamics is not affected by the external magnetic field.

A natural and a long-standing problem is whether this remains true in the rotating case. More or less surprisingly this problem has not been investigated systematically in the literature even for the magnetized Kerr–Newman solution in Einstein–Maxwell gravity. One possible reason for this is the fact that the magnetized Kerr–Newman solution is rather complicated.

The main purpose of the present Letter is to study the long-standing problem for the thermodynamics of the magnetized rotating and electrically charged black holes. More precisely we study the thermodynamics of electrically charged and rotating black holes in Einstein–Maxwell-dilaton (EMd) gravity. For this purpose we construct an exact solution describing a rotating charged dilaton black hole with a dilaton coupling parameter $\alpha = \sqrt{3}$ immersed in an external magnetic field and discuss its basic properties. Then we show how the external magnetic field affects the thermodynamics by deriving a Smarr-like relation and the first law. The novelty in the thermodynamics of the magnetized black holes is the appearance of new terms proportional to the magnetic momentum of the black holes in the Smarr-like relation and the first law.

2. Exact rotating charged dilaton black hole in an external magnetic field

The EMd gravity is described by the following field equations

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$$R_{\mu\nu} = 2\nabla_\mu\varphi\nabla_\nu\varphi + 2e^{-2\alpha\varphi}\left(F_{\mu\sigma}F_\nu{}^\sigma - \frac{g_{\mu\nu}}{4}F_{\rho\sigma}F^{\rho\sigma}\right), \tag{1}$$

$$\nabla_\mu\left(e^{-2\alpha\varphi}F^{\mu\nu}\right) = 0 = \nabla_{[\mu}F_{\nu\sigma]}, \tag{2}$$

$$\nabla_\mu\nabla^\mu\varphi = -\frac{\alpha}{2}e^{-2\alpha\varphi}F_{\rho\sigma}F^{\rho\sigma}, \tag{3}$$

where ∇_μ and $R_{\mu\nu}$ are the Levi-Civita connection and the Ricci tensor with respect to the spacetime metric $g_{\mu\nu}$. $F_{\mu\nu}$ is the Maxwell tensor and the dilaton field is denoted by φ, with α being the dilaton coupling parameter governing the coupling strength of the dilaton to the electromagnetic field.

The asymptotically flat, rotating and electrically charged dilaton black holes with $\alpha = \sqrt{3}$ are described by the following exact solution to the EMd equations [27]

$$\begin{aligned} ds_0^2 = {} & \left[H\left(r^2+a^2\right) + a^2H^{-1}Z\sin^2\theta\right]\sin^2\theta\left(d\phi - \frac{a}{\sqrt{1-\upsilon^2}}\frac{H^{-1}Z}{[H(r^2+a^2)+a^2H^{-1}Z\sin^2\theta]}dt\right)^2 \\ & - \left(H^{-1}(1-Z) + \frac{a^2}{1-\upsilon^2}\frac{H^{-2}Z^2\sin^2\theta}{[H(r^2+a^2)+a^2H^{-1}Z\sin^2\theta]}\right)dt^2 + H\frac{\Sigma}{\Delta}dr^2 + H\Sigma\, d\theta^2, \end{aligned} \tag{4}$$

$$A_t^0 = \frac{\upsilon}{2(1-\upsilon^2)}H^{-2}Z, \tag{5}$$

$$A_\phi^0 = -\frac{a\upsilon}{2\sqrt{1-\upsilon^2}}H^{-2}Z\sin^2\theta, \tag{6}$$

$$e^{\frac{2}{3}\sqrt{3}\varphi_0} = H^{-1}, \tag{7}$$

where the functions H, Z, Σ and Δ are given by

$$H = \sqrt{\frac{1-\upsilon^2+\upsilon^2 Z}{1-\upsilon^2}}, \quad Z = \frac{2mr}{\Sigma}, \tag{8}$$

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2mr + a^2. \tag{9}$$

The solution parameters υ, m and a are related to the mass M_0, the angular momentum J_0 and the charge Q_0 of the black hole via the formulae

$$M_0 = m\left(1 + \frac{1}{2}\frac{\upsilon^2}{1-\upsilon^2}\right), \qquad J_0 = \frac{ma}{\sqrt{1-\upsilon^2}}, \qquad Q_0 = \frac{m\upsilon}{1-\upsilon^2}. \tag{10}$$

The event horizon lies at

$$r_+ = m + \sqrt{m^2 - a^2}, \tag{11}$$

which is the greater root of $\Delta = 0$ and the angular velocity of the horizon is

$$\Omega_H^0 = \sqrt{1-\upsilon^2}\frac{a}{r_+^2 + a^2}. \tag{12}$$

Another very important characteristic of the black hole, which will play a crucial role in the thermodynamics of the magnetized black holes, is the magnetic momentum μ defined from the asymptotic behavior of A_ϕ^0 for $r \to \infty$, namely

$$A_\phi^0 \to -\mu\frac{\sin^2\theta}{r} \tag{13}$$

and given by

$$\mu = \frac{ma\upsilon}{\sqrt{1-\upsilon^2}} = \upsilon J_0 = \sqrt{1-\upsilon^2}aQ_0. \tag{14}$$

In order to construct the exact solution describing rotating and charged dilaton black holes in an external magnetic field we shall follow the following scheme. We consider the 4D rotating charged dilaton black holes with a metric

$$ds_0^2 = g_{\mu\nu}^0\, dx^\mu\, dx^\nu = X_0\left(d\phi + W^0\, dt\right)^2 + X_0^{-1}\gamma_{ab}\, dx^a\, dx^b, \tag{15}$$

scalar field φ_0 and gauge potential A_μ^0. Its Kaluza–Klein uplifting to a 5D vacuum Einstein solution is given by

$$ds_5^2 = e^{\frac{2}{3}\sqrt{3}\varphi_0}\, ds_0^2 + e^{-\frac{4}{3}\sqrt{3}\varphi_0}\left(dx_5 + 2A_\mu^0\, dx^\mu\right)^2. \tag{16}$$

Then we perform a twisted Kaluza–Klein reduction along the Killing field $V = B\frac{\partial}{\partial\phi} + \frac{\partial}{\partial x_5}$, which gives the following 4D solution to the EMd equations with $\alpha = \sqrt{3}$:

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