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Quark mass dependence of the $X(3872)$ binding energy

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ABSTRACT

We explore the quark-mass dependence of the pole position of the $X(3872)$ state within the molecular picture. The calculations are performed within the framework of a nonrelativistic Faddeev-type three-body equation for the $D\bar{D}\pi$ system in the $J^{PC} = 1^{++}$ channel. The πD interaction is parametrised via a D^* pole, and a three-body force is included to render the equations well defined. Its strength is adjusted such that the $X(3872)$ appears as a $D\bar{D}^*$ bound state 0.5 MeV below the neutral threshold. We find that the trajectory of the $X(3872)$ depends strongly on the assumed quark-mass dependence of the short-range interactions which can be determined in future lattice QCD calculations. At the same time we are able to provide nontrivial information on the chiral extrapolation in the X channel.

1. Introduction

A decade ago, a new era in hadronic spectroscopy began with the observation by the Belle Collaboration of the charmonium-like meson $X(3872)$ [1] which was quickly confirmed by many other collaborations, see Ref. [2] for a recent review article. In fact, such a state was predicted to exist as a $D\bar{D}^*$ bound system, analogous to the deuteron, long before its discovery, based on scattering calculations using a one-pion-exchange (OPE) potential [3,4]. This state possesses a number of intriguing features which make it an attractive object for both experimental and theoretical studies. In particular, it is seen with approximately equal branching fractions in the modes $\pi^+\pi^- J/\psi$ [5] and $\pi^+\pi^-\pi^0 J/\psi$ [6], which indicates a strong isospin violation in the X decay. Recently, the quantum numbers of the $X(3872)$ were measured to be 1^{++} [7], consistent with its being an S-wave $D\bar{D}^{*}$[1] bound system [8–11]. This interpretation finds support especially in the very small binding energy, $E_B < 1$ MeV, with respect to the $D^0\bar{D}^{*0}$ channel [12].

While some authors claim that the iterated OPE potential alone is strong enough to form the $X(3872)$ [13–16], others come to an opposite conclusion, see, for example, [17,18]. Different (complementary or alternative to the OPE) short-range $D\bar{D}^*$ mechanisms in the X are considered, for instance, in [19–22]. It has to be noticed, however, that pions are treated as static in most models for the X involving OPE. Furthermore, as stressed in [23], since the X resides only 7 MeV above the $D\bar{D}\pi$ threshold, the presence of the corresponding cut might weaken the pion potential considerably.

In order to better understand the role of the OPE interaction including that of the three-body cut as well as to use a treatment with minimal bias, the $X(3872)$ was studied in [24] within a three-body scattering Faddeev-type formalism for the $D\bar{D}\pi$ system. The πD interactions were parametrised via the D^* pole. In addition to the OPE, a $D\bar{D}^*$ contact term with a strength C_0 was also included to parametrise unknown short-distance physics. The resulting interaction was iterated to all orders through the solution of the integral equation. The equations were solved in momentum space using a sharp cut-off prescription. The scale-dependence of the contact term was determined by requiring the equations to have a pole at a fixed binding energy of 0.5 MeV for a wide range of different cut-offs Λ. It was found, in particular, that C_0 varies strongly with Λ taking values between $\pm\infty$. Interestingly, for $\Lambda \simeq 1$ GeV, the contact term turns out to be very small which might explain the results of [13–16]. However, it follows from [24] that these findings are scheme-dependent. The low-energy dynamics of the $D\bar{D}^*$ system was also investigated within a pionless EFT [25] as well as the so-called X-EFT approach (see, for example, [26,27]) that assumes that pions can be treated perturbatively.

In this Letter we apply the formalism developed in [24] to explore the behaviour of the binding energy E_B as the quark masses

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[1] This shorthand notation is used for the proper C-parity eigenstate.

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go away from their physical value. Since the pion mass enters our calculations explicitly and the pion mass squared is directly proportional to the quark mass then, equivalently, in the following we talk about the pion mass dependence of E_B. To this end, we treat all masses and coupling constants as functions of the pion mass m_π and define their physical values (labelled as "ph") as those which correspond to the physical pion mass $m_\pi^{\rm ph}$ or, equivalently, the physical values of the light quark masses. We refer to this case as to the physical limit or physical point. We then perform an expansion of all such quantities in terms of the parameter $\delta m_\pi/\mathrm{M}$, where the small scale is $\delta m_\pi = m_\pi - m_\pi^{\rm ph}$, while the large scale M is given by a typical hadronic scale such as, for instance, the D- or D^*-meson mass or the chiral symmetry breaking scale $\sim 4\pi f_\pi$, with f_π the pion decay constant. In either case, M appears to be of order 1 GeV so that keeping both the leading term in the expansion $\propto \delta m_\pi^2/\mathrm{M}^2$ and the leading term in the chiral expansion of the $D\bar{D}^*$ potential, as formulated above, is justified for $\delta m_\pi \sim m_\pi^{\rm ph}$. We assume that this framework might be still applicable in the case of larger pion masses of the order of 300-400 MeV, which is the typical value used in today's lattice QCD calculations in the charm sector [28].

It should be stressed that in the approach used here, besides the known m_π-dependences of the masses and coupling constants, we also need to include a dependence of the short-range interaction C_0 with an *a priori* unknown strength, in analogy to the investigations in the nucleon–nucleon (NN) sector [29,30].[2] The inclusion of m_π-dependence of the short-range interaction is necessary not only because of power counting arguments, but also demanded by the renormalisation group: a Λ- and m_π-dependent short-ranged operator is needed to absorb the Λ-dependence we encounter when varying the pion mass in the other quantities and especially in the pion propagator. Our formalism seems more complicated than that of [16], where it is claimed that a model-independent, parameter-free prediction can be given for the pion mass dependence of the X. However, it is our understanding that the result of [16] is regularisation scheme as well as scale-dependent.

We find that, as expected, the pion exchange itself gets weaker with increasing pion mass. Meanwhile, the m_π-dependence of the counter term can now either enhance this feature, thus leading to a rapid disappearance of the $X(3872)$ pole as m_π is increased, or, on the contrary, weaken the effect even that much, that the X binding increases with increasing pion mass. In any case, the pion mass dependence $E_B(m_\pi)$ turns out to be nontrivial. Thus our work provides important insights for the chiral extrapolation in the $X(3872)$ sector. Furthermore, with the help of our findings and from the pion mass dependence of the $X(3872)$ pole once it is available from lattice simulations, we will eventually be able to extract valuable information on the physics of the interaction that leads to the formation of the $X(3872)$.

2. Scattering equations

In this section we outline briefly our theoretical formulation following closely the lines of [24]. The lowest-order $D^*D\pi$ interaction Lagrangian has the form [26]

$$\mathcal{L} = \frac{g_c}{\sqrt{2} f_\pi}\left(\boldsymbol{D}^{*\dagger}\cdot\nabla\pi^a\tau^a D + D^\dagger\tau^a\nabla\pi^a\cdot\boldsymbol{D}^*\right),$$

[2] See also a recent calculation of [31] where the pion mass dependence of NN contact interactions was determined by resonance saturation using unitarised chiral perturbation theory combined with lattice QCD results.

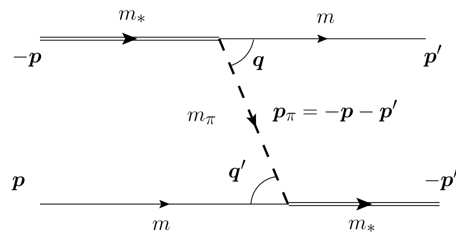

Fig. 1. Kinematics of the $D\bar{D}^*$ scattering due to the OPE. Double lines denote D^*'s, single lines denote D's, while the dashed line stands for the pion.

$$\pi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}, \tag{1}$$

where g_c is the dimensionless $D^*D\pi$ coupling constant. The latter can be fixed from the $D^{*0}\to D^0\pi^0$ width via the relation

$$\Gamma\left(D^{*0}\to D^0\pi^0\right) \equiv \Gamma_* = \frac{g_c^2 m_0}{24\pi f_\pi^2 m_{*0}}\left[2\mu_q\left(D^0\pi^0\right)(m_{*0}-m_0-m_{\pi^0})\right]^{3/2}, \tag{2}$$

where the reduced mass is defined as $\mu_q(XY) = m_X m_Y/(m_X + m_Y)$. Here and in what follows, m_0, m_c, m_{*0}, m_{*c}, m_{π^0}, and m_{π^c} are the masses of the neutral and charged D mesons, D^* mesons, and pions, respectively.

In the physical limit one has [12] $f_\pi^{\rm ph} = 92.4$ MeV,

$$m_{\pi^0}^{\rm ph} = 134.98\ \mathrm{MeV},\quad m_{\pi^c}^{\rm ph} = 139.57\ \mathrm{MeV},$$
$$m_0^{\rm ph} = 1864.84\ \mathrm{MeV},\quad m_c^{\rm ph} = 1869.62\ \mathrm{MeV},$$
$$m_{*0}^{\rm ph} = 2006.97\ \mathrm{MeV},\quad m_{*c}^{\rm ph} = 2010.27\ \mathrm{MeV},$$

while the value $\Gamma_*^{\rm ph} = 42$ keV can be deduced from the data for the charged D^* decay modes [12]. Then relation (2) gives $g_c^{\rm ph} = 0.62$.

Using the notation of [24], the P-wave $D^*D\pi$ vertex can be written as

$$v_{D^*D\pi}(\boldsymbol{q}) = g\boldsymbol{\epsilon}\cdot\boldsymbol{q}, \tag{3}$$

where $\boldsymbol{\epsilon}$ is the D^* polarisation vector and $\boldsymbol{q}$ is the relative momentum in the $D\pi$ system. The vertex coupling g is related to the dimensionless constant g_c from the Lagrangian (1) as

$$g = \frac{g_c}{(4\pi)^{3/2} f_\pi}\left(\frac{m_0}{m_{*0}\mu_q(D^0\pi^0)}\right)^{1/2}, \tag{4}$$

and its physical value is [24]

$$g^{\rm ph} = 1.29\cdot 10^{-5}\ \mathrm{MeV}^{-3/2}. \tag{5}$$

The OPE potential visualised in Fig. 1 is

$$V^{nn'}(\boldsymbol{p},\boldsymbol{p}') = -g^2\frac{(\boldsymbol{p}'+\alpha\boldsymbol{p})_n(\boldsymbol{p}+\alpha\boldsymbol{p}')_{n'}}{D_3(\boldsymbol{p},\boldsymbol{p}')},\quad \alpha = \frac{m}{m+m_\pi}, \tag{6}$$

where the indices n, n' are contracted with the corresponding indices of the D^* polarisation vectors. The inverse three-body propagator reads

$$D_3(\boldsymbol{p},\boldsymbol{p}') = 2m + m_\pi + \frac{p^2}{2m} + \frac{p'^2}{2m} + \frac{(\boldsymbol{p}+\boldsymbol{p}')^2}{2m_\pi} - M - i0. \tag{7}$$

The OPE potential (6) interrelates the four D-meson channels defined as