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Massive photons and Dirac monopoles: Electric condensate and magnetic confinement

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ABSTRACT

We use the generalized Julia–Toulouse approach (GJTA) for condensation of topological currents (charges or defects) to argue that massive photons can coexist consistently with Dirac monopoles. The Proca theory is obtained here via GJTA as a low energy effective theory describing an electric condensate and the mass of the vector boson is responsible for generating a Meissner effect which confines the magnetic defects in monopole–antimonopole pairs connected by physical open magnetic vortices described by Dirac brane invariants, instead of Dirac strings.

1. Introduction

In his seminal work [1], Dirac established a theory of magnetic monopoles interacting with massless vector bosons, from which emerged a possible explanation for the electric charge quantization observed in Nature: the mere existence of a monopole would imply in the quantization of the electric charge in multiples of the inverse of the magnetic charge, what is based on the consistency condition for the magnetic Dirac string to be unobservable at the quantum level. Since then, the physics involving Dirac monopoles has been proved to be useful also to investigate other physical scenarios [2–4].

Our aim in this work is to generalize the Dirac's non-minimal prescription for the case where the vector bosons are massive, with the hope to clarify some misunderstandings found in the literature, like the claims that Dirac monopoles and massive photons cannot coexist and that the Dirac strings would become observable when the vector bosons are massive [5].

One of the main points involved in this issue regards the fact that the Dirac theory of monopoles was developed in the context of massless vector bosons and its extension to the case of massive photons is not immediate. Another key point refers to the very general observation that a massive photon generates a Meissner effect, which confines magnetic probe sources. Together with these observations, one must also keep in mind that, since the Dirac strings are unphysical artifacts used to introduce monopoles in a theory with a single gauge potential defined over the whole spacetime, except at the location of the world-surfaces of these strings, there are no physical processes that could turn these Dirac branes into observables: this point is in fact a consistency condition that must be always satisfied in order to keep the consistency of the formalism. These basic observations can be gathered together through the use of a generalization of the so-called Julia–Toulouse approach for condensation of topological currents (charges or defects).

The original Julia–Toulouse approach [6,7] is a prescription used to construct a low energy effective theory for a system with condensed charges or defects, having previous knowledge of the model describing the system in the regime where these sources are dilutely distributed through the space and also of the symmetries expected for the regime where the charges or the defects condense. Based mainly on [6,7], and taking also into account the ideas developed in [2,8] regarding the formulation of ensembles of charges and defects, we introduced in [9,10] a generalization of the Julia–Toulouse approach, whose main feature is a careful treatment of a local symmetry which we call as the Dirac brane symmetry, which is independent of the usual gauge symmetry [2], and consists in the freedom of deforming the Dirac strings without any observable consequences. In what follows, we are going to call this generalized prescription as the generalized Julia–Toulouse approach (GJTA).

In the present work we shall follow a very general strategy to obtain a consistent formulation of the Proca theory in the presence

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of external monopoles. We begin with the Maxwell theory in the presence of diluted electric charges and introduce external magnetic defects through the original Dirac's non-minimal substitution, which can be safely applied to massless gauge theories. We then use the GJTA to construct the Proca theory in the regime where the electric charges condense, getting the correct definition of the massive electrodynamics in the presence of Dirac monopoles. Through this process, we shall see that due to the Dirac's veto [1], the Dirac branes are effectively removed from the formalism in the electric condensed regime, giving place to physical open magnetic vortices with a monopole–antimonopole pair in their ends. These open vortices are described by Dirac brane invariants corresponding to the confining magnetic flux tubes. In particular, since the magnetic probe sources are confined in this scenario due to the Meissner effect associated to the mass acquired by the vector boson as a result of the electric condensation process, it is impossible to introduce isolated magnetic defects into the massive electrodynamics, the only possibility being the introduction of mesonic monopole–antimonopole pairs (as far as we know, this conclusion was firstly explicitly pointed out in [11]). However, contrary to the usual claim found in the literature [2,4,5,7,11,12], the monopoles with opposite magnetic charges in these pairs are not connected by Dirac strings, but instead, they are connected by physical confining magnetic flux tubes described by Dirac brane invariants and we show how these structures emerge in the formalism by taking the Dirac brane symmetry carefully into account.

2. Massive electrodynamics and Dirac monopoles

We are going to work in $(3+1)$-dimensional Minkowski spacetime $\mathbb{R}^{1,3}$ and make use of natural units with $c=\hbar=1$.

The partition function of the Maxwell theory in the presence of diluted electric charges and magnetic monopoles is given by:

$$Z_d[J_1,j_1]=\int_{G.F.}\mathcal{D}A_1\exp\left\{i\int_{\mathbb{R}^{1,3}}\left[-\frac{1}{2}(dA_1-g*\chi_2)\wedge*(dA_1-g*\chi_2)-eA_1\wedge*J_1\right]\right\},\tag{1}$$

where $J_1=\delta\Sigma_2$ is the topological electric current which localizes the world-line of the electric charge e, the physical boundary of the world-surface of the electric Dirac string localized by the Chern–Kernel Σ_2 and $j_1=\delta\chi_2$ is the topological magnetic current which localizes the world-line of the magnetic charge g, the physical boundary of the world-surface of the magnetic Dirac string localized by the Chern–Kernel χ_2. The acronym "G.F." stands for some "gauge fixing" procedure that must be used at some stage of the calculations.

As discussed in [10], the magnetic Dirac brane symmetry corresponds to the local invariance of (1) under deformations of the magnetic Dirac branes that keep fixed their physical boundaries corresponding to the monopole currents and also satisfies the Dirac's veto [1,13], which prohibits the magnetic Dirac branes of crossing the electric world-lines. This local symmetry implies in the Dirac charge quantization condition [1,2], $eg=2\pi n$, $n\in\mathbb{Z}$, as a consistency condition for the invisibility of the Dirac branes, which are unphysical.

Let us work with the electromagnetic dual of (1). For this sake, we make use of the master representation of (1):

$$Z_d[J_1,j_1]=\int_{G.F.}\mathcal{D}A_1\mathcal{D}G_2\exp\left\{i\int_{\mathbb{R}^{1,3}}\left[\frac{1}{2}G_2\wedge*G_2-G_2\wedge*(dA_1-g*\chi_2)-eA_1\wedge*J_1\right]\right\},\tag{2}$$

from which we can return to the original representation (1) after integrating out the auxiliary field G_2. Instead of this, we integrate out the gauge field A_1 in (2), obtaining the dual representation:

$$\begin{aligned}Z_d[J_1,j_1]&=\int\mathcal{D}G_2\delta[d*G_2+e*J_1]\times\exp\left\{i\int_{\mathbb{R}^{1,3}}\left[\frac{1}{2}G_2\wedge*G_2-gG_2\wedge\chi_2\right]\right\}\\&=\int_{G.F.}\mathcal{D}C_1\exp\left\{i\int_{\mathbb{R}^{1,3}}\left[-\frac{1}{2}(dC_1-e*\Sigma_2)\wedge*(dC_1-e*\Sigma_2)+gC_1\wedge*j_1-eg*\Sigma_2\wedge*\chi_2\right]\right\}\\&=\int_{G.F.}\mathcal{D}C_1\exp\left\{i\int_{\mathbb{R}^{1,3}}\left[-\frac{1}{2}(dC_1-e*\Sigma_2)\wedge*(dC_1-e*\Sigma_2)+gC_1\wedge*j_1\right]\right\},\end{aligned}\tag{3}$$

where the dual gauge field C_1 has emerged by solving the functional constraint $d*G_2=-e*J_1\Rightarrow *G_2=dC_1-e*\Sigma_2$ and, in passing to the last line of (3), we used that $-eg\int_{\mathbb{R}^{1,3}}*\Sigma_2\wedge*\chi_2=-egN$, where N is an integer corresponding to the intersection number between the electric and magnetic Dirac branes, such that, due to the Dirac charge quantization condition, the complex exponential of this term gives 1 and makes no contribution in the partition function [2,14]. The dual representation (3) is physically equivalent to the original representation (1), but here the couplings are inverted: the dual gauge field couples minimally to the monopole currents and non-minimally to the electric charges. Hence, from the point of view of the dual gauge field, the electric Dirac branes are seen as defects, being C_1 and dC_1 singular over these branes. Notice, however, that the non-minimal coupling $(dC_1-e*\Sigma_2)$, which represents the physical electromagnetic fields, is regular everywhere, since the singularity of dC_1 is exactly canceled out by the singular term $*\Sigma_2$ [2,4].

At this point, we are ready to apply the GJTA and consider the effects of a electric charge condensation in this system. The condensation of electric charges is represented here by a proliferation of the electric world-lines, which implies in a proliferation of the electric Dirac branes from which these world-lines are boundaries. Due to the proliferation of the electric Dirac branes, the dual gauge field becomes ill-defined in almost the whole space and its degrees of freedom are not adequate to describe the system in the electric condensed regime. However, the non-minimal coupling remains regular everywhere. The GJTA in this picture consists in taking the regular non-minimal coupling as a new field describing the low energy excitations of the electric condensate [7]:

$$(dC_1-e*\Sigma_2)\overset{\text{cond.}}{\longmapsto}mH_2,\tag{4}$$

where m is a phenomenological mass scale associated to the electric condensate. Notice that the prescription (4) effectively promotes a dynamical term for the massless 1-form gauge field C_1 describing the system in the diluted regime to a mass term for the 2-form Kalb–Ramond field H_2 describing the system in the condensed regime: this *rank-jumping* of the field describing the excitations of the theory and the associated *mass gap generation* constitute a signature of the *condensation of topological currents* in the picture where the condensing currents are non-minimally coupled to the gauge field describing the theory in the diluted regime [6,7,9,14].