Physics Letters B 726 (2013) 544–548

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Massive gravity acausality redux

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ARTICLE INFO

Article history:
Received 28 June 2013
Accepted 3 September 2013
Available online 11 September 2013
Editor: M. Cvetič

ABSTRACT

Massive gravity (mGR) is a $5(=2s+1)$ degree of freedom, finite range extension of GR. However, amongst other problems, it is plagued by superluminal propagation, first uncovered via a second order shock analysis. First order mGR shock structures have also been studied, but the existence of superluminal propagation in that context was left open. We present here a concordance of these methods, by an explicit (first order) characteristic matrix computation, which confirms mGR's superluminal propagation as well as acausality.

1. Introduction

A natural physical question is whether gravity is necessarily infinite range—like its non-abelian Yang–Mills (YM) counterpart—or whether "nearby", massive, extensions are also permitted, at least as effective theories within a certain domain of validity. This question was first studied at linearized level almost 80 years ago by Fierz and Pauli (FP) [1], who constructed a massive spin $s=2$ model with the required $2s+1=5$ degrees of freedom (DoF). Even this was nontrivial, as the "natural" DoF count would be six–the number of components of the symmetric 3-tensor h_{ij} governing the kinetic, linearized Einstein, action. Indeed (up to field redefinitions) only one mass combination, $m^2(h_{\mu\nu}\bar{g}^{\nu\rho}h_{\rho\sigma}\bar{g}^{\sigma\mu} - h_{\mu\nu}\bar{g}^{\mu\nu}h_{\rho\sigma}\bar{g}^{\rho\sigma})$, accomplishes this so long as the fiducial metric $\bar{g}_{\mu\nu}$ is Einstein ($G_{\mu\nu}(\bar{g}) \propto \bar{g}_{\mu\nu}$) [2]. The (observationally necessary) extension to the non-linear domain, with the full scalar curvature $R(g_{\mu\nu})$ kinetic term and mass terms built from an arbitrary (diffeomorphism invariant) combination of the dynamical metric $g_{\mu\nu}$ and the fixed (but now potentially arbitrary) background $\bar{g}_{\mu\nu}$, proved more elusive.

Further developments began about halfway since the time of FP, but almost immediately ground to a halt because it was shown that, for generic mass terms, a sixth, ghost, excitation necessarily develops beyond linear, FP, order [3]. This was catastrophic because the ghost arises *within* the effective theory's supposed domain of validity, reducing it to nil. It took the subsequent four decades to discover that exactly three mass terms evade this no-go result. One of these was discovered in [4] based upon the bimetric model of [5]. Much later, that mass term and two others were uncovered in mGR's decoupling limit [6]. Absence of the "bulk" ghost mode was finally proven in [7]. Predictably, it was time for the next blow to strike: The very mass terms that avoided the ghost replaced that woe with superluminal–tachyonic modes, discovered by analyzing second order shocks [8]. This result was perhaps not surprising[1] since superluminal behavior had already been uncovered in the model's Stückelberg sector and decoupling limit [10] as well as in a spherically symmetric analysis on Friedmann–Lemaître–Robertson–Walker (FLRW) backgrounds [11]. Concordantly, unstable cosmological solutions were discovered [12] (similar pathologies also arise in other non-linear gravity models, such as $f(T)$ [13] and Poincaré gauge gravity [14]). Moreover, mGR also seems not to allow static black hole solutions [15].

The characteristics of mGR were subsequently studied in [16] in a certain first order formulation where a (generically) maximal rank characteristic matrix was found. However, a study of zeros of this matrix and thus superluminality was postponed in that work, which focused on the relationship between first order shocks and the second order shocks of [8]. In this work, we exhibit further superluminal behavior in the first order setting and clarify the relation between the various superluminal modes and acausality. We also give a compact computation and formula for the (pathological)

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[1] It also follows a similar pattern of massive higher spin inconsistencies when these models interact with background fields; see old results for $s=3/2, 2$ in both E/M and GR backgrounds [9].

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.09.001

mGR characteristic matrix by employing vierbeine and spin connections. A toy scalar field example is given in the discussion, which further illuminates our findings. The power of the characteristic method [9] is that there is no need to wait the thirty odd years it took for Gödel to discover closed timelike curves in GR, but rather acausality can be detected without directly solving the mGR field equations.[2] Moreover the causal inconsistencies we find are local, as opposed to the non-local Gödel type acausal anomalies of GR. Our conclusion is that mGR is *unphysical*, leaving GR on its isolated consistency pedestal.

2. Massive gravity

The model's field equation is

$$G_{\mu\nu}(g) = \tau_{\mu\nu}(f,g) := \Lambda g_{\mu\nu} - m^2(f_{\mu\nu} - g_{\mu\nu} f), \tag{1}$$

where the metric $g_{\mu\nu}$ is dynamical and $G_{\mu\nu}(g)$ is its Einstein tensor. The rank two tensor

$$f_{\mu\nu} := f_\mu{}^m e_{\nu m}$$

is built from the vierbein $e_\mu{}^m$ of the dynamical metric $g_{\mu\nu}$ and a non-dynamical vierbein $f_\mu{}^m$ of a non-dynamical *background/fiducial* metric $\bar{g}_{\mu\nu}$. All index manipulations will be performed using the dynamical metric and vierbein, in particular $f := f_\mu{}^m e^\mu{}_m$. The inverse background vierbein is denoted by $\ell^\mu{}_m$.

Of the three permitted bulk ghost-free mass terms, we focus on the above, simplest, possibility (linear in the fiducial vierbein); of the other two, one is known to have tachyonic behavior as well [17], while the last is—formally—open because its covariant constraint form, if any, is as yet unknown [18].

The parameter m is the FP mass when the theory is linearized around an Einstein background $\bar{g}_{\mu\nu}$ with cosmological constant $\bar{\Lambda}$. Requiring a good linearization (without constant terms in the linear equations of motion) demands the further parameter condition $\Lambda - \bar{\Lambda} + 3m^2 = 0$ (in particular flat backgrounds are achieved by tuning the *parameter* $\Lambda = -3m^2$). As a consequence of Eq. (1), the vierbein obeys the symmetry constraint

$$f_{[\mu}{}^m e_{\nu]m} = 0. \tag{2}$$

3. First order formulation

To perform a first order shock and characteristic surface analysis we first write the system in a first order formulation in the usual way. The dynamical metric $g_{\mu\nu}$ is replaced by the vierbein $e_\mu{}^m$ (with $g_{\mu\nu} = e_\mu{}^m \eta_{mn} e_\nu{}^n$), and an off-shell spin connection $\omega_\mu{}^m{}_n$ determined by the torsion-free condition built into the "Palatini" first order action,

$$\partial_{[\mu} e_{\nu]}{}^m + \omega_{[\mu|}{}^m{}_n e_{|\nu]}{}^n = 0. \tag{3}$$

The standard Bianchi identities for the Riemann tensor then become first order integrability conditions

$$R_{\mu\nu\rho\sigma}(e,\omega) - R_{\rho\sigma\mu\nu}(e,\omega) = 0 = R_{[\mu\nu\rho]\sigma}(\omega,e). \tag{4}$$

Note that there is no need to impose the condition $\nabla_{[\mu} R_{\nu\rho]\sigma\kappa} = 0$ because it holds identically for any ω. The field equations imply that the Einstein tensor obeys $G(e,\omega)_{\mu\nu} = G(e,\omega)_{\nu\mu}$ and, in turn, the symmetry constraint (2). The latter's curl gives a further integrability condition

$$f_{[\mu}{}^\sigma K_{\nu\rho]\sigma} = 0 \tag{5}$$

where the contorsion,

$$K_\mu{}^m{}_n := \omega_\mu{}^m{}_n - \omega(f)_\mu{}^m{}_n,$$

measures the failure of parallelograms of one (torsion-free) connection to close with respect to the other and will play a crucial role in further developments.

Going beyond kinematics, dynamics are generated by the first order evolution equation

$$G_{\mu\nu}(e,\omega) - \Lambda g_{\mu\nu} + m^2(f_{\mu\nu} - g_{\mu\nu} f) = 0, \tag{6}$$

where $G(e,\omega)$ is obtained from the Riemann tensor $R(\omega) = d\omega + \omega \wedge \omega$ in the usual way.

So far the choice of couplings $\tau_{\mu\nu}$ has not been invoked. The covariant vector and scalar constraints (whose existence was verified in [18]) responsible for the ultimate ghost-free, $5 = 2s+1$, $s = 2$ DoF count, depend in an essential way on this choice.[3] They have been calculated explicitly in [8] and read

$$0 = \nabla^\mu[G_{\mu\nu} - \tau_{\mu\nu}] = m^2 e^\mu{}_m K_\mu{}^m{}_n e_\nu{}^n =: m^2 K_\nu, \tag{7}$$

$$\begin{aligned} 0 &= \frac{1}{m^2}\nabla_\rho\big(\ell^{\rho\nu}\nabla^\mu[G_{\mu\nu} - \tau_{\mu\nu}]\big) + \frac{1}{2} g^{\mu\nu}[G_{\mu\nu} - \tau_{\mu\nu}] \\ &= -\frac{3m^2}{2} f - \frac{1}{2}\big[e^\mu{}_n e^\nu{}_m \bar{R}_{\mu\nu}{}^{mn} + 4\Lambda\big] \\ &\quad + \frac{1}{2}\big[K_{\mu\nu\rho} K^{\nu\rho\mu} + K_\mu K^\mu\big]. \end{aligned} \tag{8}$$

Note that the term $K_\mu K^\mu$ in the scalar constraint can be dropped since it is the square of the vector one (7).

4. Shocks

We investigate first order shocks by positing

$$\big[\partial_\alpha e_\mu{}^m\big]_\Sigma = \xi_\alpha \mathcal{E}_\mu{}^m, \qquad \big[\partial_\alpha \omega_\mu{}^m{}_n\big]_\Sigma = \xi_\alpha \Omega_\mu{}^m{}_n.$$

Since we wish to study superluminal propagation, we take the normal ξ to be timelike: $\xi^\mu g_{\mu\nu}\xi^\nu = -1$. For compactness of notation, we denote the contraction of ξ on an index of any tensor by an "o", so $\xi.V := V_o$, where we use a lower dot to denote tensor contraction, to avoid confusion with the usual vector dot product. Also, the operator $\top_\mu^\nu := \delta_\mu^\nu + \xi_\mu \xi^\nu$ is a projector; we will denote its action on tensors by Latin indices, for example

$$V_i := \top_i^\nu V_\nu \quad \Rightarrow \quad V_\mu V^\mu = V_i V^i - V_o V_o.$$

We split our shock analysis into two parts: First, we deal with the consequences of the "kinematical" equations, namely Eqs. (3)–(4), and then turn to the dynamical equation, Eq. (6) and its constraints given by Eqs. (2), (5), (7), (8). These will give algebraic conditions on the shock profiles $\mathcal{E}_\mu{}^m$ and $\Omega_\mu{}^m{}_n$; there would be causal consistency only if these conditions forced all shock profiles to vanish.

Firstly, we observe that the discontinuity in the torsion-free condition (3) implies

$$\xi_{[\mu}\mathcal{E}_{\nu]\rho} = 0.$$

Multiplying by ξ^μ we find

$$\mathcal{E}_{\mu\nu} = -\xi_\mu \mathcal{E}_{o\nu}.$$

[2] Actually, in [11], solutions with infinitely rapid propagation—in open FLRW backgrounds—were explicitly given; these are likely to include examples of acausal structures, though their energy scale is as yet unclear.

[3] To be precise, a vector constraint exists for *any* algebraic coupling $\tau_{\mu\nu}$, but the condition it imposes on fields is τ-dependent. The very existence of a scalar constraint hinges on the exact choice of τ.