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Vacuum waves

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ABSTRACT

An exact solution of the five-dimensional field equations is studied which describes waves in the classical Einstein vacuum. While the solution is essentially 5D in nature, the waves exist in ordinary 3D space. They should not be confused with standard gravitational waves, since their phase velocity can exceed that of light. They resemble de Broglie waves, and may give insight to wave-particle duality.

1. Introduction

Five-dimensional relativity is the simplest extension of general relativity that might be expected to show the low-energy effects of even higher dimensions connected with the physics of elementary particles. Below, I will examine the properties of an exact solution of the 5D field equations which is of special interest. It describes the classical Einstein vacuum, with periodic perturbations which exist in ordinary 3D space (Section 2). These waves have unusual properties, so it is difficult to classify them. They require the presence of a vacuum and have phase velocities which can exceed the speed of light, implying that they are not gravitational waves but more resemble de Broglie waves. If these vacuum waves were present in the early universe or persist today, they may provide a way to test for an extra dimension.

Extra dimensions are widely regarded as the best available route to the unification of gravity with the interactions of particles. Modern Kaluza–Klein theory exists in two versions, namely Space–Time–Matter theory and Membrane theory. Both employ a 5D manifold with a non-compactified extra dimension. In STM theory, the terms in the field equations are grouped so that those involving the extra dimension form an effective or induced energy-momentum tensor which satisfies Einstein's equations, thereby explaining the origin of matter in terms of geometry. In M theory, the extra dimension is used to split the manifold into parts on either side of a singular hypersurface where matter is concentrated, thereby explaining the properties of particles. The theories are similar mathematically, and because they both reduce in the appropriate limit to 4D general relativity they are acceptable observationally [1]. However, it is not enough to merely reproduce acceptability, and new phenomena should be sought relevant to the fifth dimension. Vacuum waves appear to be such a phenomenon.

2. An exact solution with vacuum waves

The field equations of five-dimensional relativity are commonly taken to be given by the Ricci tensor as $R_{AB} = 0$ ($A, B = 0, 123, 4$ for time, ordinary space and the extra coordinate). These 5D equations actually contain Einstein's 4D equations, by Campbell's embedding theorem [1]. It is a corollary of this theorem that when the 5D metric takes the so-called canonical form, it embeds all solutions of general relativity which are empty of ordinary matter but contain finite vacuum energy as measured by the cosmological constant [2]. The latter parameter Λ is positive or negative, depending on whether the extra dimension is spacelike or timelike. For both cases, the equation of state of the vacuum in terms of its effective energy density and pressure is $p_v = -\rho_v = -\Lambda/8\pi$. Here units have been chosen which render the speed of light c and the gravitational constant G unity, though these quantities and Planck's constant h will be made explicit later in order to aid physical understanding. The extra coordinate will be labelled $x^4 = l$, to avoid confusion with the usual coordinates of spacetime $x^\alpha = t, xyz$. Other notation is standard.

There are many solutions known of the 5D field equations $R_{AB} = 0$, including several in canonical form [3–5]. The 5D Schwarzschild–de Sitter solution is one such, which ensures the agreement of 5D relativity with the classical tests of Einstein's theory. But for present purposes, consider the following solution:

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$$dS^2 = \frac{l^2}{L^2}\left\{dt^2 - \exp\left[\pm\frac{2i}{L}(t+\alpha x)\right]dx^2 - \exp\left[\pm\frac{2i}{L}(t+\beta y)\right]dy^2 - \exp\left[\pm\frac{2i}{L}(t+\gamma z)\right]dz^2\right\} + dl^2. \tag{1}$$

This describes a wave propagating through ordinary 3D space, where the frequency $f = 1/L$ is fixed by the solution. The wave-numbers $k_x = \alpha/L$, $k_y = \beta/L$, $k_z = \gamma/L$ along the x, y, z axes are written in terms of the dimensionless constants α, β, γ which are arbitrary. The phase velocity of the wave along (say) the x-axis is c/α, and is also arbitrary. The constant length L in (1) is likewise arbitrary in a mathematical sense, but plays on important physical role. It was originally introduced to the canonical metric as a measure of the scale of the 4D potential [2]. But in vacuum spacetimes, this is given by the 4D curvature which is provided by the cosmological constant Λ. By evaluating the Einstein tensor for the 4D part of (1), it may be shown by some algebra that the equations of general relativity are satisfied, with a cosmological constant $\Lambda = -3/L^2$. Accordingly, (1) describes waves travelling in a classical vacuum with positive pressure.

Other properties of (1) may be revealed by using one of the software packages currently available. In fact, while (1) was found originally by solving the field equations by hand, the quickest way to verify it is by computer.

Perhaps the most striking feature of (1) is that it allows phase velocities that are, in a formal sense, greater than the speed of light. This because, as noted above, the phase speed is c/α and α can be arbitrarily small. It should be recalled that this does not necessarily conflict with causality, as long as the waves concerned do not carry conventional information [6]; and there is a (somewhat unusual) interpretation of the Lorentz transformations due to Rindler that allows such speeds [7]. The fact that the waves described by (1) can be superluminal implies that they are not conventional gravitational waves of the kind found in 4D general relativity. This is confirmed by noting that the waves in (1) depend for their existence on the presence of vacuum energy with its typical equation of state ($p_v = -\rho_v$), whereas standard gravitational waves propagate through truly empty space ($p = \rho = 0$). Also, in the quantum version of classical Kaluza–Klein theory, the particles associated with the scalar field have spin 0, while in Einstein's theory the graviton has spin 2. It should be recalled that the solution (1) requires $\Lambda < 0$ with vacuum density $\rho_v < 0$, and such a medium is unlike any form of ordinary matter. All waves in ordinary matter have velocities given by an expression of the sort $w = \sqrt{K/\rho}$, where K is a constant that depends on microscopic physics (the bulk modulus for fluids and the shear modulus for solids). This is discussed in standard texts, a few of which point out that a value $K < 0$ would imply that an increase in the ambient pressure causes an increase in the volume of an element of the material, or equivalently a decrease in the density. This behaviour is the opposite of that observed in ordinary matter. However, it is exactly the behaviour consistent with the equation of state $p_v = -\rho_v c^2$ of the vacuum when the pressure is positive and the density is negative. In understanding the possibility of superluminal velocities as given by the solution (1) of 5D relativity, it is also useful to consider 4D wave mechanics. There, a wave with phase speed w has associated with it a particle with ordinary speed v, and the application of Planck's law implies that the two are related by $vw = c^2$. Then if $v < c$ it follows necessarily that $w > c$. In fact, the relation $vw = c^2$ is named after de Broglie and is well known in quantum theory. Accordingly, it is logical to inquire into the connection between (1) and de Broglie waves.

3. Vacuum waves and de Broglie waves

In the preceding section, an exact 5D solution (1) was given which is characterized by waves which in ordinary 3D space resembles de Broglie waves. The question arises of whether this correspondence can be made exact, and if so what other implications arise for wave mechanics.

The theoretical basis of de Broglie waves was stated by him in 1924. He postulated that any massive particle has associated with it a wave, in analogy with a photon's electromagnetic wave, in a way that respects the Lorentz transformations and Planck's law. The direction of motion of the particle is the same as the normal to the wave-front. Let the mass and velocity of the particle be m, v and the frequency and velocity of the wave be f, w. Then in terms of the 4-vectors for the particle and the wave, there is a match of the magnitude of the quantities concerned, via de Broglie's equation: $m(v, 1) = hf(1/w, 1/c^2)$. Equating components in de Broglie's equation gives $mv = hf/w$ and $m = hf/c^2$. The second of these relations is simply a statement of the equality of the energy of the particle and the energy of the wave: $E = mc^2 = hf$. The first relation, when divided by the second, gives $vw = c^2$.

To see how this relation compares with the wave of the solution (1), consider the element of proper distance in the latter along the x-axis, given by $d\bar{x} = \exp[i(ft + k_x x)]\,dx$. The frequency is $f = c/L$. By Planck's law, the energy of the wave is equivalent to the mass m of the associated particle, $E = hf = hc/L = mc^2$, so $L = h/mc$. That is, the size of the 4D potential well in (1) equals the Compton wavelength of the particle. The momentum of the particle $p_x = mv_x$ is inversely proportional to the wavelength, so the wave-number k_x is directly proportional to p_x, and can be written in the correct dimensional form as $k_x = (mc/h)(v_x/c) = v_x/cL$. Recalling that the frequency is $f = c/L$, the phase velocity of the wave along the x-axis is $w_x = f/k_x = (c/L)(cL/v_x) = c^2/v_x$. Thus along each axis, the velocities of the particle and the wave are connected by de Broglie's relation $vw = c^2$.

In standard wave mechanics, the problematical nature of the medium which supports the waves is effectively avoided, by introducing a complex wave function ψ that is abstract and makes no direct reference to conventional properties of matter. The theory employs operators, which act on the time and space parts of the 4D metric to produce the energy and momenta of a particle, whose mass is then given by a relation in ψ called the Klein–Gordon equation. Since wave mechanics is used to successfully model the interactions of particles, it is instructive to see how fares an alternative approach based on a 5D metric like (1) above. That metric has a 4D part which is scaled by a length L that was identified as the Compton wavelength h/mc of a test particle with spacetime coordinates x^γ. The 4D part of the 5D metric defines the line element of spacetime as usual via $ds^2 = g_{\alpha\beta}(x^\gamma)\,dx^\alpha\,dx^\beta$. Then it is possible to define a dimensionless action, which can be used to obtain a wave function:

$$I \equiv \int ds/L = \int (h/mc)^{-1}\,ds, \quad \psi = \exp(iI). \tag{2}$$

The first derivative of this gives

$$p_\alpha = (h/i\psi)\partial\psi/\partial x^\alpha, \tag{3}$$

where the 4-momenta are defined as usual ($p^\alpha \equiv mcu^\alpha = mc\,dx^\alpha/ds$). The second derivative, taken covariantly if the spacetime is curved, splits into a real part and an imaginary part. One of these gives $p^\beta_{;\beta} = 0$, the standard conservation law for the momenta. The other part gives