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Yukawa coupling beta-functions in the Standard Model at three loops

A.V. Bednyakov [a,*], A.F. Pikelner [a], V.N. Velizhanin [b]

[a] Joint Institute for Nuclear Research, 141980 Dubna, Russia
[b] Theoretical Physics Department, Petersburg Nuclear Physics Institute, Orlova Roscha, Gatchina, 188300 St. Petersburg, Russia

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ABSTRACT

We present the results for three-loop beta-functions for Yukawa couplings of heavy Standard Model fermions calculated within the unbroken phase of the model. The calculation is carried out with the help of the MINCER program in a general linear gauge, and the final result is independent of the gauge-fixing parameters. In order to calculate three-point functions, we made use of infrared rearrangement (IRR) trick. Due to the chiral structure of the SM a careful treatment of loops with fermions is required to perform the calculation. It turns out that gauge anomaly cancellation in the SM allows us to obtain the result by means of the semi-naive treatment of γ_5.

The Yukawa couplings being the fundamental parameters of the Standard Model (SM) Lagrangian describe the interactions of quarks and leptons with the Higgs field. Having in mind the discovery of the Higgs boson candidate [1,2] one can hope that some day the values of Yukawa couplings will be deduced directly from the experimental data (see, e.g., the discussion presented in Ref. [3]). In order to obtain a very precise SM prediction for the running Yukawa couplings at some high-energy scale, one usually uses known masses of quarks and leptons [4] since it is the Yukawa interactions that give the fundamental fermions their masses after spontaneous electroweak symmetry breaking. Due to the observed hierarchy of the SM fermion masses the corresponding values are usually defined at different scales, so one inevitably makes use of renormalization group equations (RGE) to connect these scales. The so-called threshold (matching) corrections (see Refs. [5,6] for the case of running masses and Yukawa couplings) are also very important for extractions of the running SM parameters defined in the minimal subtraction ($\overline{\text{MS}}$) scheme, in which counter-terms and beta-functions have a very simple polynomial structure. It also should be mentioned that contrary to leptons, quarks are not observed as free particles, so the pole mass which is usually associated with the physical mass of a particle, although being a gauge-invariant and infrared finite quantity [7,8], suffers from the so-called renormalon ambiguity [9,10]. This intrinsic uncertainty of the quark pole mass is estimated to be of the order of $\Lambda_{QCD} \simeq 200$ MeV. For the top quark one usually neglects this uncertainty since the corresponding mass $M_t = 175$ GeV is much bigger than Λ_{QCD}. Moreover, it is generally believed that due to the short lifetime the t-quark does not have time to hadronize, so the notion of the pole mass can be used in this case. According to the recent studies of the relation between the $\overline{\text{MS}}$-running mass of the top quark and the corresponding pole mass, electroweak corrections can become important and for the observed value of the Higgs boson mass can severely cancel the QCD contribution [6]. As a consequence, theoretical uncertainty in determination of the top quark Yukawa coupling is reduced, thus calling for more precise determination of the corresponding RGEs. For all the other quarks one usually uses the $\overline{\text{MS}}$ masses defined initially in the context of QCD (see, e.g., Ref. [11] and references therein).

The SM Higgs boson with $M_h = 125$ GeV decays predominately into the $b\bar{b}$ pairs. In spite of the fact that this decay mode is very hard to observe due to the enormous QCD background it is obvious that the precise value of the corresponding Yukawa coupling is required to test whether the SM correctly describes Nature at the electroweak scale. If one considers leptonic decays of the Higgs boson, the most promising decay channel is the tree-level decay to a tau-anti-tau pair. These facts somehow motivate our study of the three-loop contribution to the corresponding Yukawa beta-functions.

Moreover, we would like to stress here that a separate study of the high-energy behavior of the SM can also be of great

* Corresponding author.
E-mail addresses: bednya@theor.jinr.ru (A.V. Bednyakov), andrey.pikelner@cern.ch (A.F. Pikelner), velizh@thd.pnpi.spb.ru (V.N. Velizhanin).

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importance. One can use RGE to find the scale where New Physics should enter the game, e.g., to unify the interactions or stabilize the Higgs potential [12–17].

One- and two-loop results for SM beta-functions have been known for quite a long time [18–31] and are summarized in [32].

Not long ago full three-loop gauge coupling beta-functions were calculated [33,34]. The beta-functions for the Higgs self-coupling and top Yukawa coupling were also considered at three loops [35]. However, in Ref. [35], all the electroweak couplings were neglected together with the Yukawa couplings of other SM fermions. In this Letter, we provide the full analytical result for the three-loop beta-functions of the strongest Yukawa couplings y_t, y_b, y_τ for the three heaviest SM fermions (top, bottom, and tau). We take into account all the interactions of the SM.

Let us briefly define our notation. The full Lagrangian of the unbroken SM which was used in this calculation is given in our previous paper [34]. However, we do not keep the full flavor structure of Yukawa interactions but use the following simple Lagrangian which describes fermion-Higgs interactions

$$\mathcal{L}_{\text{Yukawa}} = -y_t\big(\bar{Q}\,\Phi^c\big)t_R - y_b\big(\bar{Q}\,\Phi\big)b_R - y_\tau\big(\bar{L}\Phi\big)\tau_R + \text{h.c.} \tag{1}$$

with $Q = (t,b)_L$, and $L = (\nu_\tau,\tau)_L$ being SU(2) doublets of left-handed fermions of the third generation, u_R, t_R, and τ_R are the corresponding right-handed counter parts.[1] The Higgs doublet Φ with $Y_W = 1$ has the following decomposition in terms of the component fields:

$$\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(h + i\chi) \end{pmatrix}, \qquad \Phi^c = i\sigma^2\Phi^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}}(h - i\chi) \\ -\phi^- \end{pmatrix}. \tag{2}$$

Here a charge-conjugated Higgs doublet is introduced Φ^c with $Y_W = -1$.

For loop calculations it is convenient to define the following quantities:

$$a_i = \left(\frac{5}{3}\frac{g_1^2}{16\pi^2}, \frac{g_2^2}{16\pi^2}, \frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{y_b^2}{16\pi^2}, \frac{y_\tau^2}{16\pi^2}, \frac{\lambda}{16\pi^2}, \xi_G, \xi_W, \xi_G\right), \tag{3}$$

where we use the SU(5) normalization of the U(1) gauge coupling g_1. We also stress that the calculation is carried out in a general linear R_ξ gauge, so the renormalization of all three gauge-fixing parameters ξ_G, ξ_W, and ξ_B, given in Ref. [34], is required.

The Yukawa beta-functions are extracted from the corresponding renormalization constants which relate bare couplings to the renormalized ones in the $\overline{\text{MS}}$-scheme. The latter are found with the help of the following formulae:

$$Z_{y_f} = \frac{Z_{ffh}}{\sqrt{Z_{f_L}Z_{f_R}Z_h}} \quad \text{or} \quad Z_{y_f} = \frac{Z_{ff\chi}}{\sqrt{Z_{f_L}Z_{f_R}Z_\chi}}, \quad f = t, b, \tau, \tag{4}$$

where Z_{ffh} and $Z_{ff\chi}$ are the renormalization constants for the three-point vertices involving two fermions f and the Higgs h or the would-be Goldstone boson χ, respectively. The renormalization constants Z_{f_L}, Z_{f_R} for left- and right-handed fermions, and Z_χ and Z_h for the neutral components of the Higgs doublet are obtained from the corresponding self-energy diagrams.

In order to extract a three-loop contribution to the considered renormalization constants, it is sufficient to know the two-loop results for the gauge and Yukawa couplings and the one-loop expression for the Higgs self-interaction.

The relation between the bare and renormalized parameters can be written in the following way

$$a_{k,\text{Bare}}\,\mu^{-2\rho_k\epsilon} = Z_{a_k}a_k(\mu) = a_k + \sum_{n=1}^{\infty} c_k^{(n)}\frac{1}{\epsilon^n}, \tag{5}$$

where $\rho_k = 1/2$ for the gauge and Yukawa constants, $\rho_k = 1$ for the scalar quartic coupling constant, and $\rho_k = 0$ for the gauge-fixing parameters. The bare couplings are defined within the dimensionally regularized [36] theory with $D = 4 - 2\epsilon$. The four-dimensional beta-functions, denoted by β_i, are defined via[2]

$$\beta_i(a_k) = \left.\frac{da_i(\mu,\epsilon)}{d\ln\mu^2}\right|_{\epsilon=0}, \qquad \beta_i = \beta_i^{(1)} + \beta_i^{(2)} + \beta_i^{(3)} + \cdots \tag{6}$$

with $\beta_i^{(l)}$ being the l-loop contribution to the beta-function for a_i. The expression for β_i can be extracted from the corresponding renormalization constants (5) with the help of

$$\beta_i = \sum_l \rho_l a_l \frac{\partial c_i^{(1)}}{\partial a_l} - \rho_i c_i^{(1)}. \tag{7}$$

Here, again, a_i stands for both the gauge couplings and the gauge-fixing.

It should be noted that the divergent part of the considered three-point functions should resemble the tree-level structure

$$\Delta\mathcal{L} = -\frac{y_f}{\sqrt{2}}\bar{f}fh - i\frac{y_f}{\sqrt{2}}\bar{f}\gamma_5 f\chi. \tag{8}$$

Since we separately consider the contributions to the $\bar{f}_L f_R\phi$ and $\bar{f}_R f_L\phi$ vertices ($\phi = h, \chi$), we have to be sure that the corresponding divergencies sum up to give the unit matrix in the case of the Higgs boson or γ_5 in the case of χ. This serves as an additional self-consistency check of our result.

It turns out that due to the gauge symmetry the Higgs field h and the would-be Goldstone boson χ renormalize in the same way so that $Z_\chi = Z_h$. Moreover, the same reasoning can be applied to the considered Yukawa vertices giving $Z_{ffh} = Z_{ff\chi}$. This fact was also checked by explicit calculation at the three-loop level.

Actually, it is not trivial to satisfy these two requirements (to conserve the chiral structure of the Lagrangian and do not break the gauge invariance) at three loops. Both the issues are related to the γ_5 problem present in dimensionally regularized theories. It is known from the literature (see, e.g., Ref. [37] and recent explicit calculation [35]) that the traces with an odd number of γ_5 appearing for the first time in the three-loop diagrams require special treatment. We closely follow the semi-naive approach presented in Ref. [38]. First of all, we anticommute γ_5 with other matrices and use $\gamma_5^2 = 1$. In the "even" traces all γ_5 are contracted with each other, so the corresponding expressions can be treated naively in dimensional regularization. In "odd" traces we are left with only one γ_5. These traces are evaluated as in four dimensions and produce totally antisymmetric tensors via the relation

$$\text{tr}\big(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\big) = -4i\epsilon^{\mu\nu\rho\sigma} \tag{9}$$

with $\epsilon^{0123} = 1$.

Since we are using both the γ_5 anticommutativity and the four-dimensional relation (9), the cyclicity of the trace should be relinquished [39]. Due to this, a comment is in order about our positioning of γ_5 within the "odd" traces. It is known that different prescriptions give rise to an ambiguity of order $D - 4$. In our calculation the kinematical structure of a diagram, including the starting point for a closed fermion loop, is fixed automatically by

[1] Here we assume that neutrinos are massless.

[2] For the Yukawa couplings it is also convenient to consider the running of the coupling itself. It is obvious that $\beta_{y_i} = (\beta_i/a_i)y_i/2$ with $i = t, b, \tau$.