



Polymer Bose–Einstein condensates

E. Castellanos^{a,*}, G. Chacón-Acosta^b

^a Departamento de Física, Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, A.P. 14-740, México D.F. 07000, Mexico

^b Departamento de Matemáticas Aplicadas y Sistemas, Universidad Autónoma Metropolitana-Cuajimalpa, Artificios 40, México D.F. 01120, Mexico

ARTICLE INFO

Article history:

Received 23 January 2013

Accepted 2 April 2013

Available online 6 April 2013

Editor: M. Cvetič

ABSTRACT

In this work we analyze a non-interacting one-dimensional polymer Bose–Einstein condensate in a harmonic trap within the semiclassical approximation. We use an effective Hamiltonian coming from the polymer quantization that arises in loop quantum gravity. We calculate the number of particles in order to obtain the critical temperature. The Bose–Einstein functions are replaced by series, whose high order terms are related to powers of the polymer length. It is shown that the condensation temperature presents a shift respect to the standard case, for small values of the polymer scale. In typical experimental conditions, it is possible to establish a bound for λ^2 up to $\lesssim 10^{-16}$ m². To improve this bound we should decrease the frequency of the trap and also decrease the number of particles.

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1. Introduction

Several quantum gravity models suggest that the relation between energy and momentum of microscopic particles must be modified as a consequence of the quantum structure of space–time [1–9]. A deformed dispersion relation, emerges as an ideal tool in the search for possible effects related to the quantum structure of space–time. Nevertheless, the most difficult aspect in the search of experimental evidence relevant for the quantum gravity problem is the smallness of the possible effects [3,4]. Unfortunately, due to this fact, the possible bounds for the deformation parameters, open a wide range of possible magnitudes, which implies a significant challenge.

Among the models that introduce such deformations, polymer quantum systems are simple quantum mechanical models quantized in a similar way as in loop quantum gravity [10,11]. These systems provide scenarios where some characteristics of the full theory, specially the discrete nature of space, can be explored in a simpler context [12–16]. Indeed, in polymer quantum mechanics the momentum operator cannot be defined, hence a regularized operator is proposed by the introduction of the so-called polymer length scale [10,11]. Particularly, from this model an effective Hamiltonian which contains some trace of the granularity of space can be drawn [17].

The use of Bose–Einstein condensates as a tool in the search of quantum gravity effects, for instance, in the context of Lorentz

violations, or to provide phenomenological constraints on Planck scale physics, has produced several interesting research [5,6,18–25] (and references therein). In these works, the possible effects arising from Planck scale physics by looking at some modifications in the thermodynamic properties of Bose–Einstein condensates was explored. Since its observation with the help of magnetic traps [27–30], the phenomenon of Bose–Einstein condensation, from the experimental and theoretical point of view, has spurred an enormous amount of publications [30–51,58,59] (and references therein). In particular the condensates have been studied in different spatial dimensions [34,52,53,59]. Although, a one-dimensional condensate can never be reached, it is possible to obtain a quasi-one-dimensional condensate, by using extremely anisotropic traps [34, 35,54,55]. Additionally, the condensation process of a bosonic gas in one dimension, trapped in a box or in a harmonic oscillator potential, has a very particular behavior. In this situation, apparently the condensation is not possible in the thermodynamic limit if one assumes that the ground state energy is zero, or equivalently, if the associated chemical potential is zero at the condensation temperature. However, taken into account the minimum energy associated to the system, the condensation process is possible at finite temperature [34,35,49,59].

The aim of this work is to analyze the properties of a polymer one-dimensional condensate trapped in a harmonic oscillator potential, within the semiclassical approximation, in order to obtain representative bounds for the polymer length scale. We will show also that for this one-dimensional system the functional form of the relevant thermodynamic quantities, particularly, for the number of particles, can be expressed by series of the so-called Bose–Einstein functions. This series converges when the polymer scale is small enough. Finally, by the introduction of the ground state

* Corresponding author.

E-mail addresses: ecastellanos@fis.cinvestav.mx (E. Castellanos), gchacon@correo.cua.uam.mx (G. Chacón-Acosta).

energy associated to this system, we calculate the shift in the usual condensation temperature caused by the polymer scale λ^2 . These facts allow us to bound the polymer scale by using one-dimensional finite size systems.

2. Polymer quantization

Let us now present some of the main results of polymer quantization [10,11]. This quantization arises from loop quantum gravity [56]. In the *loop or polymer* representation the corresponding Hilbert space $\mathcal{H}_{\text{poly}}$ is spanned by the basis states $\{|x_j\rangle\}$, whose coefficients have a suitable fall-off [10], with the following inner product

$$\langle x_i | x_j \rangle = \delta_{i,j}, \quad (1)$$

where $\delta_{i,j}$ is the Kronecker delta. The polymer Hilbert space can be represented as $\mathcal{H}_{\text{poly}} = L^2(\mathbb{R}_d, d\mu_d)$, where $d\mu_d$ is the corresponding Haar measure, being $\mathbb{R}_d$ the real line endowed with the discrete topology, that is, the dual of the Bohr compactification of the real line [10,56]. The basic operators in this quantization are the position and translation. The position operator $\hat{x}$ acts as usual by multiplication

$$\hat{x}|x_j\rangle = x_j|x_j\rangle,$$

while the translation operator $\hat{V}(\lambda)$ moves to a position of arbitrary distance λ

$$\hat{V}(\lambda)|x_j\rangle = |x_j - \lambda\rangle.$$

In the Schrödinger quantization, the operator $\hat{V}(\lambda)$ is weakly continuous in λ and the momentum operator is defined as its infinitesimal generator. However, in the polymer representation the translation operator $\hat{V}(\lambda)$ fails to achieve this condition, i.e., it is not weakly continuous on λ due to the discrete structure assigned to space [10]. Therefore, there is no Hermitian operator as infinitesimal generator of translations and thus, the momentum operator is not well defined. Due to this fact, any phase space function has to be regularized by the introduction of a lattice of fixed positive length. We interpret this length as a fundamental scale that is called the polymer length scale [10]. With this regularization, the polymer Hamiltonian can be defined as follows

$$\hat{H}_\lambda = \frac{\hbar^2}{2m\lambda^2} [2 - \hat{V}(\lambda) - \hat{V}(-\lambda)] + \hat{U}(x), \quad (2)$$

where $\hat{U}(x)$ is the potential term. The action of the Hamiltonian (2) decomposes the polymer Hilbert space $\mathcal{H}_{\text{poly}}$, into a continuum of separable superselected subspaces, each with support on a regular lattice $\{n\lambda + x_0 \mid n \in \mathbb{Z}\}$, with $x_0 \in [0, \lambda)$ that parameterizes a particular superselected sector [10]. Notice that the Hamiltonian (2) can be formally written as

$$\frac{\hbar^2}{2m\lambda^2} \sin^2\left(\frac{\lambda p}{\hbar}\right) + \hat{U}(x). \quad (3)$$

We can use this expression to obtain the effective Hamiltonian, simply by replacing the kinetic term with the square of the sine function.

It is worth mentioning, as was pointed out in [17], that the effective three-dimensional case present some complications in calculating analytically the integrals. Due to this fact, in the present work we restrict ourselves to the one-dimensional case.

3. Condensation temperature

Let us start with a one-dimensional effective polymer Hamiltonian [17], given by

$$H = \frac{\hbar^2}{2m\lambda^2} \sin^2\left(\frac{\lambda p_x}{\hbar}\right) + U(x) \quad (4)$$

where $U(x) = m\omega_x^2 x^2/2$ is the one-dimensional harmonic oscillator potential that model the trap, and λ is the so-called polymer length scale. Therefore, the semiclassical energy spectrum associated to (4) can be expressed as follows

$$\epsilon_p = \frac{\hbar^2}{2m\lambda^2} \sin^2\left(\frac{\lambda p_x}{\hbar}\right) + \frac{m\omega_x^2 x^2}{2}. \quad (5)$$

The associated one-dimensional spatial density can be written as [34,59]

$$n(x) = \frac{1}{2\pi\hbar} \int \frac{dp_x}{e^{\beta(\epsilon_p - \mu)} - 1}, \quad (6)$$

where, as usual, $\beta = 1/\kappa T$, being κ the Boltzmann constant, and μ is the corresponding chemical potential. Consequently, the total number of particles of the system is

$$N = \int n(x) dx. \quad (7)$$

Let us calculate the spatial density $n(x)$ for the polymer case. To do so we need to calculate the integral (6) substituting the energy (5)

$$n(x) = \frac{1}{2\pi\hbar} \int \frac{dp_x}{Z^{-1}(x) \exp\left(\frac{\beta\hbar^2}{2m\lambda^2} \sin^2\left(\frac{\lambda p_x}{\hbar}\right)\right) - 1}, \quad (8)$$

where $Z(x) = \exp[\beta(\mu - \frac{m\omega_x^2 x^2}{2})]$. The integrand of (8) can be replaced by a geometric series

$$n(x) = \frac{1}{2\pi\hbar} \int \sum_{j=1}^{\infty} (Z(x) e^{-\frac{\beta\hbar^2}{2m\lambda^2} \sin^2(\frac{\lambda p_x}{\hbar})})^j dp_x. \quad (9)$$

The sine function in the argument of the exponential can be written by using an identity, then we interchange the summation and the integration

$$n(x) = \frac{1}{2\pi\hbar} \sum_{j=1}^{\infty} Z^j(x) \int e^{-\frac{j\beta\hbar^2}{4m\lambda^2} [1 - \cos(\frac{2\lambda p_x}{\hbar})]} dp_x. \quad (10)$$

The last integral can be recognized as a modified Bessel function of first kind [60], if we introduce a regulator for the integral that corresponds to the polymer length λ [15], with the result

$$n(x) = \frac{1}{2\pi\hbar} \frac{\hbar\pi}{\lambda} \sum_{j=1}^{\infty} Z^j(x) e^{-\frac{j\beta\hbar^2}{4m\lambda^2}} I_0\left(\frac{j\beta\hbar^2}{4m\lambda^2}\right). \quad (11)$$

We obtain the total number of particles by integrating (11) over all space. The only x -dependent function is $Z(x)$, and then the corresponding integration is straightforward

$$N = \frac{1}{2} \sqrt{\frac{2\pi}{m\beta\omega_x^2}} \sum_{j=1}^{\infty} \frac{e^{-\frac{j\beta\hbar^2}{4m\lambda^2}}}{\lambda} I_0\left(\frac{j\beta\hbar^2}{4m\lambda^2}\right) \frac{e^{j\mu\beta}}{j^{1/2}}. \quad (12)$$

When $\lambda \ll 1$, we can use the asymptotic expansions of the modified Bessel functions [60,61]

$$I_0(u) \approx \frac{e^u}{\sqrt{2\pi u}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2u)^k k!} \frac{\Gamma(k + \frac{1}{2})}{\Gamma(\frac{1}{2} - k)}. \quad (13)$$

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