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# Hexadecapole degree of freedom in $^{94}$Mo

R.J. Casperson [a,b,*], V. Werner [b], S. Heinze [c]

[a] Lawrence Livermore National Laboratory, Livermore, CA 94551, USA
[b] Wright Nuclear Structure Laboratory, Yale University, 272 Whitney Ave., New Haven, CT 06520-8124, USA
[c] Institute of Nuclear Physics, University of Cologne, Zülpicherstrasse 77, 50937 Cologne, Germany

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ABSTRACT

The weakly collective nucleus $^{94}$Mo has been considered an excellent example of quadrupole vibrational mixed-symmetry structure. Algebraic models such as the interacting boson model-2 have been largely unsuccessful in describing the structure of the $4_2^+$ state, due to the strong M1 transition between the $4_2^+$ and $4_1^+$ states, and the low energy of the $4_2^+$ state. In this Letter, we show that introducing $g$-bosons into the interacting boson model-2 allows for the first time a description of the electromagnetic decay properties of the low-lying $4^+$ states in $^{94}$Mo within an algebraic model. The hexadecapole degree of freedom is shown to be important for the description of the collective low-energy structure of nuclei in this region.



Collectivity in nuclei, its manifestation over a broad range of nuclei, and its emergence from underlying microscopic structures, remains one of the most important topics in contemporary nuclear structure physics. A vast number of nuclear models have emerged that describe collective structures from various degrees of microscopic origins, or that are entirely based on semi-classical geometrical approaches. Nuclei near shell closures offer a means of studying the relation of microscopic nucleon configurations, and their mixing into collective states. An important degree of freedom in even–even nuclei is isospin, since the proton–neutron degree of freedom adds states in addition to those where protons and neutrons are indistinguishable in the nuclear wave functions, as well as isovector transitions between states of differing isospin content.

The interacting boson model (IBM) is an algebraic model that can be treated as a truncation of the nuclear shell model, and has been widely and successfully employed for describing collective even–even nuclei [1–3]. In its original form, it truncates the shell model valence space to $L(j^2) = 0$ or 2 configurations, formed by two identical nucleons in a given orbital, hence, coupling to $s$ or $d$-bosons (sd-IBM-1). The sd-IBM-2 version of the model differentiates between protons and neutrons and predicts so-called mixed-symmetry (MS) states, which differ from the fully-symmetric (FS) states of the sd-IBM-1 in $F$-spin quantum number, the bosonic analog of isospin for fermions. Since quadrupole interactions dominate, usually the sd-IBM-2 Hamiltonian is truncated at the quadrupole level, and excited states can be expressed in terms of quadrupole-phonon excitations ($Q$-phonons) of either FS or MS character. The distinctive feature of MS states are strong M1 transitions to their symmetric counterparts with the same number of phonons. A review on MS states can be found in [4].

$Q$-phonon type excitations have been studied not only within the IBM, but also within microscopic models such as the quasiparticle phonon model (QPM) [5–8] and the shell model [9,10]. An excellent laboratory for the study of phonon excitation schemes in near-spherical nuclei has been found in the $N = 52$ isotopes, in particular $^{94}$Mo. In comprehensive studies, summarized in [11], the one-phonon mixed-symmetry $2^+$ state, as well as most members of the two-phonon $F = F_{max} - 1$ MS multiplet have been observed. E2 and M1 transition strengths were found to be in good agreement with the $\gamma$-soft limit of the sd-IBM-2, however, level spacings in the $\gamma$-soft limit are overpredicted. Energy data suggest a rather vibrational structure.

Despite the seeming conflicts in energies versus transition strengths in the structural interpretation of $^{94}$Mo, this nucleus is the best example for FS and MS multi-phonon structures to date. Nevertheless, one characteristic of $^{94}$Mo that cannot be described in the sd-IBM-2 without sacrificing the description of the quadrupole-collective FS and MS states, is the strong M1 transition between the lowest $4^+$ states of $B(M1; 4_2^+ \to 4_1^+) = 1.23(20)\,\mu_N^2$ [11], the strongest known in this mass region. In [11], Fransen et al. speculated that this strong M1 transition, along with a collective $B(E2; 4_2^+ \to 2_1^+)$ transition, could be described by including $g$-bosons in the IBM. In this work, we include this hexadecapole

* Corresponding author at: Lawrence Livermore National Laboratory, Livermore, CA 94551, USA.
*E-mail address:* casperson1@llnl.gov (R.J. Casperson).

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**Table 1**
Dominant components of wave functions of $4_1^+$ and $4_2^+$ states from shell model calculations for $^{94}$Mo [14]. Components are given in percentages, and the configurations listed are the largest components in both wave functions.

| Component | $4_1^+$ | $4_2^+$ |
|---|---|---|
| $\pi(g_{9/2}^2)_0 \times \nu(d_{5/2}^2)_4$ | 20% | 23% |
| $\pi(p_{1/2}^{-2}g_{9/2}^4)_0 \times \nu(d_{5/2}^2)_4$ | 10% | 12% |
| $\pi(g_{9/2}^2)_4 \times \nu(d_{5/2}^2)_0$ | 13% | 21% |
| $\pi(p_{1/2}^{-2}g_{9/2}^4)_4 \times \nu(d_{5/2}^2)_0$ | 6% | 4% |

degree of freedom in the IBM-2, which is typically truncated in collective models, and show that within the sdg-IBM-2 the strong $4_2^+ \to 4_1^+$ transition can be accounted for. This M1 transition is shown to arise from the exchange of a MS to a FS $g$-boson configuration, and hence, has its origin in the hexadecapole degree of freedom. Earlier approaches involving $g$-bosons in the IBM focused on deformed regions [12], or involved IBM-1 only, focusing on transitional regions. Hexadecapole excitations have been much discussed and were suggested to exist at relatively low energies [13], but with the richness of the sdg-IBM group chains, the choice of physically relevant Hamiltonians in the large sdg-IBM model space poses a serious challenge.

First hints for the potential role of $g$-bosons in the lowest-lying $4^+$ states of $^{94}$Mo have already been found in shell model calculations [14] using a $^{88}$Sr core. The dominant components in the wave functions of the $4_{1,2}^+$ states, as given in Table 1, were found to be seniority $\nu = 2$, $j = 4$ configurations, which by definition are $g$-boson configurations within the IBM truncation. The M1 transition strength between the two lowest $4^+$ states was calculated to be $1.79\,\mu_N^2$. Even though relative phases between proton and neutron configurations are not given in [14], a recalculation reveals that the main proton and neutron configurations have the same relative sign for the $4_1^+$ state, but opposite signs for the $4_2^+$ state, indicative of FS and MS configurations in these states, respectively. Analogous results were obtained in similar calculations for the $^{92,94}$Zr isotopes [15,16]. Hence, the approach of employing the sdg-IBM-2 within the present work is in accordance with, and in fact motivated by these shell model calculations.

The sdg-IBM-1 is an algebraic model with states and operators that are constructed from boson creation and annihilation operators. The $s$, $d$, and $g$-bosons are defined by $L = 0$, $L = 2$, and $L = 4$, which have 1, 5, and 9 dimensions respectively. The model therefore has 15 dimensions, and the algebraic group that describes them is U(15). The boson operators are $s^\dagger$, $d_\mu^\dagger$, $g_\nu^\dagger$, $s$, $d_\mu$, and $g_\nu$ where $\mu = -2\ldots 2$ and $\nu = -4\ldots 4$. These operators satisfy the Bose commutation relations, but in order to calculate matrix elements of these operators, the annihilation operators need to be modified in order to obtain spherical tensors:

$$\tilde{s} = s \qquad \tilde{d}_\mu = (-1)^\mu d_{-\mu} \qquad \tilde{g}_\nu = (-1)^\nu g_{-\nu} \tag{1}$$

The distinction between proton and neutron bosons in the sdg-IBM-2 results in twice as many boson operators, and a 30 dimensional algebra. There are symmetries that can be exploited to simplify the lattice of algebras, but it is first helpful to consider the transition from the sd-IBM-1 to the sd-IBM-2. Typically, when attempting to describe a nucleus within the sd-IBM-1, the structure of the nucleus is first compared to the spectra generated by the three dynamical symmetries: U(5), O(6), and SU(3). These symmetries have clear geometric analogues in nuclei: U(5) is vibrational, O(6) corresponds to a deformed gamma-soft nucleus, and SU(3) has the structure of a rigid rotor. Due to the lack of clear physical meaning for the Casimir operators of the dynamical symmetries of the sdg-IBM-2, we chose to begin with a transitional Hamiltonian in multipole form, that describes the space spanning the $U_{\pi+\nu}(5)$ and $O_{\pi+\nu}(6)$ dynamical symmetries of the sd-IBM-2 [1], conserving $F$-spin:

$$\hat{H} = c\left((1-\zeta)(\hat{n}_{d_\pi} + \hat{n}_{d_\nu}) - \frac{\zeta}{4N}(\hat{Q}_\pi + \hat{Q}_\nu)\cdot(\hat{Q}_\pi + \hat{Q}_\nu) + \lambda\hat{M}\right)$$
$$\hat{n}_{d_\rho} = d_\rho^\dagger \cdot \tilde{d}_\rho \qquad \hat{Q}_\rho = \left[s_\rho^\dagger \tilde{d}_\rho\right]^{(2)} + \left[d_\rho^\dagger \tilde{s}_\rho\right]^{(2)}$$
$$\hat{M} = F_{\max}(F_{\max}+1) - \hat{F}^+\hat{F}^- - \hat{F}^0\hat{F}^0 + \hat{F}^0 \tag{2}$$

where $F_{\max} = \frac{1}{2}(N_\pi + N_\nu)$, $\hat{F}^+ = s_\pi^\dagger \tilde{s}_\nu + d_\pi^\dagger \cdot \tilde{d}_\nu$, $\hat{F}^- = s_\nu^\dagger \tilde{s}_\pi + d_\nu^\dagger \cdot \tilde{d}_\pi$, and $\hat{F}^0 = s_\pi^\dagger \tilde{s}_\pi - s_\nu^\dagger \tilde{s}_\nu + d_\pi^\dagger \cdot \tilde{d}_\pi - d_\nu^\dagger \cdot \tilde{d}_\nu$. The $d$-boson number operator $\hat{n}_{d_\rho}$ corresponds to vibrational structure, the quadrupole term is deformation-driving, and the Majorana operator $\hat{M}$ shifts the energies of MS states. By adjusting $\zeta$ between 0 and 1, the structure of the level scheme transitions from spherical to deformed. The parameter $c$ sets the energy scale, and $N$ is the total boson number.

The Hamiltonian used in the sdg-IBM-2 calculations for $^{94}$Mo modifies the vibrational and rotational terms of the Hamiltonian to include $g$-boson components:

$$\hat{H} = c\Big((1-\zeta)\big(\hat{n}_{d_\pi} + \hat{n}_{d_\nu} + \alpha(\hat{n}_{g_\pi} + \hat{n}_{g_\nu})\big)$$
$$- \frac{\zeta}{4N}(\hat{Q}_\pi + \hat{Q}_\nu)\cdot(\hat{Q}_\pi + \hat{Q}_\nu) + \lambda_{sd}\hat{M}_{sd} + \lambda_{sg}\hat{M}_{sg}\Big)$$
$$\hat{Q}_\rho = \left[s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger \tilde{s}_\rho\right]^{(2)} + \beta\left[d_\rho^\dagger \tilde{g}_\rho + g_\rho^\dagger \tilde{d}_\rho\right]^{(2)} + \chi_d\left[d_\rho^\dagger \tilde{d}_\rho\right]^{(2)}$$
$$+ \chi_g\left[g_\rho^\dagger \tilde{g}_\rho\right]^{(2)}$$
$$\hat{n}_{d_\rho} = s_\rho^\dagger \tilde{s}_\rho \qquad \hat{n}_{d_\rho} = d_\rho^\dagger \cdot \tilde{d}_\rho \qquad \hat{n}_{g_\rho} = g_\rho^\dagger \cdot \tilde{g}_\rho \tag{3}$$

The operator $\hat{M}_{sd}$ is identical to the Majorana operator from the sd-IBM-2, and $\hat{M}_{sg}$ is similar, but with the $d$-boson operators replaced with $g$-boson operators. The full Majorana operator should include $s$, $d$, and $g$-boson operators, but these two operators give more flexibility for adjusting states based on their proton–neutron $d$ and $g$-boson symmetry. The parameters $\chi_d$ and $\chi_g$ control the rigidity of the quadrupole deformation, and will be set to 0 for the description of $^{94}$Mo. This corresponds to the conservation of $d$-parity [17], resulting in strict selection rules for M1 transitions, forcing, for example, the $1_1^+$ to $2_1^+$ M1 transition strength to equal 0. The $n_{g_\pi} + n_{g_\nu}$ terms correspond to the $g$-boson vibrational structure, and the parameter $\alpha$ defines the energy of the hexadecapole boson relative to the quadrupole boson. The parameter $\beta$ adjusts the $d$–$g$ interaction in the quadrupole operator.

The M1 and E2 transition operators are defined to be

$$\hat{T}(E2) = e_{B_\pi}\hat{Q}_\pi + e_{B_\nu}\hat{Q}_\nu$$
$$\hat{T}(M1) = \sqrt{\frac{3}{4\pi}}(g_{d\pi}\hat{L}_{d\pi} + g_{d\nu}\hat{L}_{d\nu} + g_{g\pi}\hat{L}_{g\pi} + g_{g\nu}\hat{L}_{g\nu}) \tag{4}$$

where $\hat{L_{d\rho}} = \sqrt{10}[d_\rho^\dagger \tilde{d}_\rho]^{(1)}$, $\hat{L_{g\rho}} = \sqrt{60}[g_\rho^\dagger \tilde{g}_\rho]^{(1)}$, $e_{B_\rho}$ are effective boson charges, and $g_{d\rho}$ and $g_{g\rho}$ are the $g$-factors for $d$ and $g$-bosons respectively. $e_{B_\pi}$ is a free parameter, setting the scale for E2 transitions, and $e_{B_\nu}$ has been fixed to 0. The proton $d$ and $g$-boson $g$-factors set the scale for M1 transitions, and have been set equal, in order to further reduce the number of free parameters. The neutron $d$ and $g$-boson $g$-factors have both been fixed to 0. This adds two additional parameters to the calculation, which results in a total of eight parameters. The consistent-Q formalism [18] is applied, and the same quadrupole operators as in the Hamiltonian are applied in the E2 transition operator, hence, explicitly including both $d$ and $g$-boson components.