Physics Letters B 721 (2013) 61–67

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Lepton mixing patterns from a scan of finite discrete groups

Martin Holthausen, Kher Sham Lim *, Manfred Lindner

Max-Planck Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany

ARTICLE INFO

Article history:
Received 4 January 2013
Received in revised form 14 February 2013
Accepted 25 February 2013
Available online 1 March 2013
Editor: G.F. Giudice

ABSTRACT

The recent discovery of a non-zero value of the mixing angle θ_{13} has ruled out tri-bimaximal mixing as the correct lepton mixing pattern generated by some discrete flavor symmetry (barring large next-to-leading order corrections in concrete models). In this work we assume that neutrinos are Majorana particles and perform a general scan of all finite discrete groups with order less than 1536 to obtain their predictions for lepton mixing angles. To our surprise, the scan of over one million groups only yields 3 interesting groups that give lepton mixing patterns which lie within 3-sigma of the current best global fit values. A systematic way to categorize such groups and the implications for flavor symmetry are discussed.

1. Introduction

The origin of flavor is one of the important questions of beyond the Standard Model physics. All entries of the lepton mixing matrix, or better known as the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, are of order one, with the exception of U_{e3}. Compared to the Cabibbo–Kobayashi–Maskawa (CKM) matrix whose off-diagonal entries are small, the very different form of the PMNS matrix seems to suggest a different origin of the two matrices. One popular approach to the flavor puzzle is to invoke (spontaneously broken) symmetries to describe the observed patterns. The leptonic mixing angles can be determined solely from flavor symmetry considerations (up to permutations of rows and columns of the mixing matrix). This is possible if the charged lepton and neutrino mass matrices exhibit the misaligned remnant symmetries under which charged leptons and neutrinos transform as three inequivalent singlets, as will be reviewed in the next chapter.

Assuming the remnant symmetries to be part of the original symmetry group (and not a result of an accidental symmetry) one can then determine mixing patterns from the structure of discrete symmetry groups. For review on discrete flavor symmetries and their application in model building see [1–3]. For example the symmetry group A_4 [4–10] and S_4 [11–13] can lead to the tri-bimaximal mixing pattern (TBM) by Harrison, Perkins and Scott [14,15]. With the latest global fits results [16–18] (see Table 1) on the non-zero mixing angle θ_{13} measured by DAYA-BAY [19], RENO [20] and DOUBLE-CHOOZ [21], TBM is ruled out and one is prompted to rethink the approach to lepton flavor based on discrete groups. One possibility is to build models which lead to TBM on leading order and allow for large next-to-leading order (NLO) corrections. Here the problem usually is that quite often there are many different NLO corrections, which limits the predictivity of the models. Another approach is to look for new groups that predict a different type of leptonic mixing pattern i.e. a new starting point about which models could be built. In this Letter, we shall follow the second route and therefore perform a scan of all possible finite discrete groups of the order less than 1536 with the help of the computer algebra program GAP [22–25]. To our surprise, only three finite discrete groups can yield the neutrino mixing angles allowed by the experimental constraints.

This Letter is organized as follows: in Section 2 we will present the group theoretical procedure to obtain the PMNS matrix from a finite symmetry group. This section might be skipped by readers familiar with the methodology. The method of scanning through all the groups of order less than 1536 and the relevant results are presented in Section 3 and finally we conclude in Section 4.

2. Leptonic mixing from remnant symmetries

Lepton mixing can be derived from a flavor symmetry breaking to remnant symmetries in the charged lepton and neutrino masses respectively. In concrete models, this is usually achieved via a spontaneous breaking using some scalar fields charged under this symmetry into different directions of flavor space. The charge assignments are chosen such that there are different residual symmetries in the charged lepton and neutrino sectors. The misalignment between the two residual symmetries generates the PMNS matrix [11–13,26–28]. In this method, only the structure of flavor symmetry group and its remnant symmetries are assumed and we do not consider the breaking mechanism i.e. how the required

* Corresponding author.
E-mail addresses: martin.holthausen@mpi-hd.mpg.de (M. Holthausen), khersham.lim@mpi-hd.mpg.de (K.S. Lim), lindner@mpi-hd.mpg.de (M. Lindner).

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.02.047

Table 1
Global fit of neutrino oscillation parameters (for normal ordering of neutrino masses) adapted from [17]. The errors of the best fit values indicate the one sigma ranges. In the global fit there are two nearly degenerate minima at $\sin^2\theta_{23} = 0.430^{+.031}_{-.030}$, see Fig. 1.

	Δm^2_{21} [10^{-5} eV^2]	$\lvert\Delta m^2_{31}\rvert$ [10^{-3} eV^2]	$\sin^2\theta_{12}$ [10^{-1}]	$\sin^2\theta_{23}$ [10^{-1}]	$\sin^2\theta_{13}$ [10^{-2}]	δ [π]
best fit	$7.62^{+.19}_{-.19}$	$2.55^{+.06}_{-.09}$	$3.20^{+.16}_{-.17}$	$6.13^{+.22}_{-.40}$	$2.46^{+.29}_{-.28}$	$0.8^{+1.2}_{-.8}$
3σ range	7.12–8.20	2.31–2.74	2.7–3.7	3.6–6.8	1.7–3.3	0–2

vacuum alignment needed to achieve the remnant symmetries is dynamically realized.

The PMNS matrix is defined as

$$U_{\rm PMNS} = V_e^\dagger V_\nu \tag{1}$$

and can be determined from the unitary matrices V_e and V_ν satisfying

$$V_e^T M_e M_e^\dagger V_e^* = \mathrm{diag}\big(m_e^2, m_\mu^2, m_\tau^2\big)$$

and

$$V_\nu^T M_\nu V_\nu = \mathrm{diag}(m_1, m_2, m_3), \tag{2}$$

where the mass matrices are defined by $\mathcal{L} = e^T M_e e^c + \frac{1}{2}\nu^T M_\nu \nu$. We will now review how certain mixing patterns can be understood as a consequence of mismatched horizontal symmetries acting on the charged lepton and neutrino sectors [11–13,26–28].[1] Let us assume for this purpose that there is a (discrete) symmetry group G_f under which the left-handed lepton doublets $L = (\nu, e)^T$ transform under a faithful unitary 3-dimensional representation $\rho : G_f \to GL(3, \mathbb{C})$:

$$L \to \rho(g)L, \quad g \in G_f. \tag{3}$$

The experimental data clearly shows (i) that all lepton masses are unequal and (ii) there is mixing amongst all three mass eigenstates. Therefore this symmetry cannot be a symmetry of the entire Lagrangian but it has to be broken to different subgroups G_e and G_ν (with trivial intersection) in the charged lepton and neutrino sectors, respectively. If the fermions transform as

$$e \to \rho(g_e)e, \qquad \nu \to \rho(g_\nu)\nu, \quad g_e \in G_e,\ g_\nu \in G_\nu, \tag{4}$$

for the symmetry to hold, the mass matrices have to fulfil

$$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$$

and

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu. \tag{5}$$

Choosing G_e or G_ν to be a non-abelian group would lead to a degenerate mass spectrum, as their representations cannot be decomposed into three inequivalent 1-dimensional representations of G_e or G_ν. This scenario is not compatible with the case of three distinguished neutrino and charged lepton masses and we therefore restrict ourselves to the abelian case. We further restrict ourselves to the case of Majorana neutrinos, which implies that there cannot be a complex eigenvalue of the matrices $\rho(g_\nu)$ and they therefore satisfy $\rho(g_\nu)^2 = 1$, and we can further choose $\det\rho(g_\nu) = 1$. By further requiring three distinguishable Majorana neutrinos the group G_ν is restricted to be the Klein group $Z_2 \times Z_2$. To be able to determine (up to permutations of rows and columns) the mixing matrix from the group structure it is necessary to have all neutrinos transform as inequivalent singlets of G_ν. The same is true for the charged leptons which shows that G_e cannot be smaller than Z_3. We can now determine the mixing via the unitary matrices Ω_e, Ω_ν that satisfy

$$\Omega_e^\dagger \rho(g_e)\Omega_e = \rho(g_e)_{\rm diag}, \qquad \Omega_\nu^\dagger \rho(g_\nu)\Omega_\nu = \rho(g_\nu)_{\rm diag} \tag{6}$$

where $\rho(g)_{\rm diag}$ are diagonal unitary matrices. These conditions determine Ω_e, Ω_ν up to a diagonal phase matrix $K_{e,\nu}$ and permutation matrices $P_{e,\nu}$

$$\Omega_{e,\nu} \to \Omega_{e,\nu} K_{e,\nu} P_{e,\nu}. \tag{7}$$

It follows from Eq. (5) that up to the ambiguities of the last equation, $V_{e,\nu}$ are given by $\Omega_{e,\nu}$. This can be seen as

$$\Omega_e^T M_e M_e^\dagger \Omega_e^* = \Omega_e^T \rho^T M_e M_e^\dagger \rho^* \Omega_e^* = \rho_{\rm diag}^T \Omega_e^T M_e M_e^\dagger \Omega_e^* \rho_{\rm diag}^*$$

has to be diagonal (only a diagonal matrix is invariant when conjugated by an arbitrary phase matrix) and the phasing and permutation freedom can be used to bring it into the form $\mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$, and analogously for Ω_ν. From these group theoretical considerations we can thus determine the PMNS matrix

$$U_{\rm PMNS} = \Omega_e^\dagger \Omega_\nu \tag{8}$$

up to a permutation of rows and columns. It should not be surprising that it is not possible to uniquely pin down the mixing matrix, as it is not possible to predict lepton masses in this approach.

Let us now try to apply this machinery to some interesting cases. We have seen that the smallest residual symmetry in the charged lepton sector is given by a $G_e = \langle T \mid T^3 = E\rangle \cong Z_3$. We use a basis where the generator is given by

$$\rho(T) = T_3 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{9}$$

This matrix will be our standard 3-dimensional representation of Z_3 and the notation T_3 will be used throughout this work. It is diagonalized by

$$\Omega_e^\dagger \rho(T)\Omega_e = \mathrm{diag}\big(1, \omega^2, \omega\big)$$

and

$$\Omega_e = \Omega_T \equiv \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \tag{10}$$

and $\omega = e^{i2\pi/3}$. Having fixed the basis by choosing the Z_3 generator the way we just did, it is now essentially a question of choosing generators and studying the predicted mixing matrix. Let us first look at the case where there is only one generator S of G_ν, satisfying $\rho(S)^2 = 1$ and $\det\rho(S) = 1$:

$$\rho(S) = S_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{11}$$

Due to the degenerate eigenvalues there is a two-parameter freedom in the matrix Ω_ν and it will turn out to be useful in classifying our result later to write it as

$$\Omega_\nu^\dagger \rho(S)\Omega_\nu = \mathrm{diag}(-1, 1, -1) \quad \text{with } \Omega_\nu = \Omega_U U_{13}(\theta, \delta) \tag{12}$$

[1] We here follow the presentation and convention in [26,27].