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Variational approach to the calculation of g_A

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ABSTRACT

A long standing problem in lattice QCD has been the discrepancy between the experimental and calculated values for the axial charge of the nucleon, $g_A \equiv G_A(Q^2 = 0)$. Though finite volume effects have been shown to be large, it has also been suggested that excited state effects may also play a significant role in suppressing the value of g_A. In this work, we apply a variational method to generate operators that couple predominantly to the ground state, thus systematically removing excited state contamination from the extraction of g_A. The utility and success of this approach is manifest in the early onset of ground state saturation and the early onset of a clear plateau in the correlation function ratio proportional to g_A. Through a comparison with results obtained via traditional methods, we show how excited state effects can suppress g_A by as much as 8% if sources are not properly tuned or source–sink separations are insufficiently large.

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1. Introduction

In recent years, lattice calculations have taken a tremendous step towards simulating QCD at the physical point. Algorithmic and technological developments have allowed simulations to probe at or near physical quark masses on increasingly larger volumes, with finer lattice spacings and vastly increased statistics. Calculations of the ground state spectrum have yielded results consistent to within a few percent of their physical values with well controlled systematic errors [1,2]. Naturally the next step has been to strive for this level of precision for the matrix elements of these states. Despite the remarkable consistency between lattice and experimental data for the pion form factor $F_\pi(Q^2)$, a complete description of other hadronic states, particularly the nucleon, has proven to be remarkably challenging [3,4].

The most notable shortfall is for the nucleon axial charge, $g_A \equiv G_A(Q^2 = 0)$. In principle g_A should be relatively simple to calculate. Being an iso-vector quantity, disconnected loop contributions are absent and as we have direct access to $G_A(0)$, we circumvent the need for extrapolations in Q^2. Unfortunately, the lattice values for g_A to date have been consistently lower than the experimental value by as much as 10–15% [5]. In an effort to account for these discrepancies, several studies have carefully examined the systematic errors present in the calculation [6–19]. In this Letter we will focus on the role of excited state effects.

Recently there has been an increased effort to understand and reduce the impact of excited states on form factor calculations. In computing these quantities, it is well understood that to ensure excited state contributions to the correlation function are sufficiently suppressed, one needs large Euclidean time separations between operators. To choose a suitable time separation one should identify the time slices where the correlation functions take on their asymptotic form. For the two commonly used sequential source techniques, this is a relatively simple procedure for the fixed current method. One simply chooses a current insertion time, t_C, once the asymptotic behaviour is observed in the two-point correlator. Results are extracted from the data once the asymptotic behaviour is observed in the three-point correlator.

For the fixed sink method, one requires knowledge of the asymptotic behaviour of the three-point correlator *a priori*. Unfortunately, the temptation to use earlier sink times in order to obtain more precise results is inescapable. These results can suffer from excited state contaminations, even if a plateau is observed with t_C. In Refs. [10,16], it was found that for certain matrix elements, e.g. $\langle x \rangle$, the source–sink separations often used in the literature were not sufficiently large to suppress excited state effects. Nonetheless, as we move ever closer to the physical point one is naturally forced to choose earlier sink times as the signal degrades much quicker.

To counter this issue, new techniques are being devised to try and control the sub-leading terms to the three point correlator. The use of the summation method [18,20] has shown improvement upon the conventional approach, but the underlying excited states contributions are still present. It is not hard to imagine situations where these still impact significantly and alter the final result.

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In this Letter we take a somewhat different approach. Rather than reduce the impact of excited states through Euclidean time evolution, we seek to separate them out from the ground state at the source and sink. Drawing upon the techniques developed for excited state spectroscopy calculations, we will use the variational approach to construct interpolating fields that couple with individual energy eigenstates and use these to isolate the desired matrix elements [21,22]. An analogous approach has been presented in [23,24] for the study of $B^* \to B\pi$ transitions and in [25] for the study of the axial charges of nucleon excited states. Here we apply it specifically to g_A to remove excited state contributions.

This Letter is organized as follows. In Section 2 we will examine the variational method in the context of excited state spectroscopy and then outline how this method can be applied to the calculation of hadronic matrix elements. Section 3 outlines the details of this calculation. In Section 4 we present our results and compare our variational method with the traditional, single-operator approach to the calculation of g_A. Section 5 is a cost–benefit discussion for the variational method. Finally we provide our concluding remarks in Section 6.

2. Variational method for matrix elements

The 'variational method' [26,27] is a well established approach for determining the excited state hadron spectrum. It is based on the creation of a matrix of correlation functions in which different superpositions of excited state contributions are linearly combined to isolate the energy eigenstates. A diversity of excited state superpositions is central to the success of this method.

Starting from a basis of operators $\{\chi_i(x) \mid i = 1, \ldots, N\}$, we construct a correlation matrix of two-point correlation functions,

$$G_{ij}(\vec{p}; t; \Gamma) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \operatorname{tr}\big(\Gamma \langle \Omega | \chi_i(x) \bar{\chi}_j(0) | \Omega \rangle\big). \tag{1}$$

Due to the discrete nature of the lattice, we can decompose these correlation functions into a discrete sum over energy eigenstates,

$$G_{ij}(\vec{p}, t; \Gamma) = \sum_{\alpha} e^{-E_\alpha(\vec{p})t} Z_i^\alpha(\vec{p}) \bar{Z}_j^\alpha(\vec{p}) \operatorname{tr}\left(\frac{\Gamma(\not{p} + m_\alpha)}{2E_\alpha(\vec{p})}\right), \tag{2}$$

where the parameters $Z_i^\alpha(\vec{p})$ are the coupling strengths of the interpolators $\chi_i(x)$ with the energy eigenstate of mass m_α and Γ projects out the desired parity. We choose new operators $\phi^\alpha(x)$ to be linear combinations

$$\phi^\alpha(x) = \sum_i v_i^\alpha \chi_i(x), \qquad \bar{\phi}^\alpha(x) = \sum_j u_j^\alpha \bar{\chi}_j(x), \tag{3}$$

with a suitable choice of coefficients v_i^α and u_j^α, such that these interpolators couple to a single energy eigenstate,

$$\langle \Omega | \phi^\beta(0) | \alpha, p, s \rangle = \delta^{\alpha\beta} \mathcal{Z}_\alpha(\vec{p}) \sqrt{\frac{m_\alpha}{E_\alpha(\vec{p})}} u(p, s). \tag{4}$$

From Eqs. (2) and (4) we find that the necessary values for v_i^α and u_j^α are the solutions of the following eigenvalue equations

$$v_i^\alpha(\vec{p}) \big[G(\vec{p}, t_0 + \Delta t) \big(G(\vec{p}, t_0) \big)^{-1} \big]_{ij} = c^\alpha v_j^\alpha(\vec{p}), \tag{5}$$

$$\big[\big(G(\vec{p}, t_0) \big)^{-1} G(\vec{p}, t_0 + \Delta t) \big]_{ij} u_j^\alpha(\vec{p}) = c^\alpha u_i^\alpha(\vec{p}), \tag{6}$$

where the eigenvalue $c^\alpha = e^{-m_\alpha \Delta t}$.

It is important to note that both (5) and (6) are evaluated for a given momentum $\vec{p}$ and so the diagonalisation condition is only satisfied when we project with the relevant coefficients as follows:

$$v_i^\alpha(\vec{p}) G_{ij}(\vec{p}, t; \Gamma) u_j^\beta(\vec{p}) \propto \delta^{\alpha\beta}. \tag{7}$$

Thus the two-point correlation function for the state $|\alpha, p\rangle$ is

$$G^\alpha(\vec{p}, t; \Gamma) \equiv v_i^\alpha(\vec{p}) G_{ij}(\vec{p}, t; \Gamma) u_j^\alpha(\vec{p}). \tag{8}$$

We can extract the mass m_α from $G^\alpha(\vec{p} = 0, t)$ in the standard way.

To understand how we can utilise the variational method for use in form factor calculations, we must first identify the terms present in the three-point correlation function,

$$G_{ij}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') = \sum_{\vec{x}_1, \vec{x}_2} e^{-i\vec{p}'\cdot\vec{x}_2} e^{+i(\vec{p}' - \vec{p})\cdot\vec{x}_1} \times \operatorname{tr}\big(\Gamma' \langle \Omega | \chi_i(x_2) \mathcal{O}(x_1) \bar{\chi}_j(0) | \Omega \rangle \big), \tag{9}$$

where $\mathcal{O}(x)$ is the current operator to be inserted. Sandwiching the current between two complete sets of states we end up with three terms, the vertex amplitude, $\langle \beta, p', s' | \mathcal{O}(0) | \alpha, p, s \rangle$, and the coupling terms $\langle \Omega | \chi_i(0) | \beta, p', s' \rangle$ and $\langle \alpha, p, s | \bar{\chi}_j(0) | \Omega \rangle$,

$$\begin{aligned} &G_{ij}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') \\ &= \sum_{\alpha,\beta} e^{-E_\beta(\vec{p}')(t_2 - t_1)} e^{-E_\alpha(\vec{p})t_1} \\ &\quad \times Z_i^\beta(\vec{p}') \bar{Z}_j^\alpha(\vec{p}) \sqrt{\frac{m_\alpha m_\beta}{E_\alpha(\vec{p}) E_\beta(\vec{p}')}} \\ &\quad \times \operatorname{tr}\left(\Gamma' \sum_{s', s} u(p', s') \langle \beta, p', s' | \mathcal{O}(0) | \alpha, p, s \rangle \bar{u}(p, s) \right). \end{aligned} \tag{10}$$

The coupling parameters take the same form as they did in the calculation of the two-point correlator with two key differences. The inclusion of a current means that the initial and final momenta need not be the same. Furthermore, there also exists the possibility that the initial and final energy eigenstates are not the same. That is, the current can induce a transition between states. For this calculation the necessary expression is

$$G^\alpha(\vec{p}', \vec{p}; t_2, t_1; \Gamma') = v_i^\alpha(\vec{p}') G_{ij}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') u_j^\alpha(\vec{p}). \tag{11}$$

To isolate the matrix element from the three-point function, we construct a ratio in the standard way. In this work we shall use the ratio defined in [28]. For the state α the necessary ratio is

$$\begin{aligned} &R^\alpha(\vec{p}', \vec{p}; \Gamma', \Gamma) \\ &= \sqrt{\frac{G^\alpha(\vec{p}', \vec{p}; t_2, t_1; \Gamma') G^\alpha(\vec{p}, \vec{p}'; t_2, t_1; \Gamma')}{G^\alpha(\vec{p}, t_2; \Gamma) G^\alpha(\vec{p}', t_2; \Gamma)}}. \end{aligned} \tag{12}$$

Key to this approach is the utilisation of a basis of operators in which there is diversity in the overlap with various excited states. As there are a limited number of local bilinear operators for a given J^{PC}, a great deal of work has been made by various groups in increasing the number of available operators. Here we choose to use fermion source and sink smearing as a method of extending our operator basis, as outlined in [29,30].

3. Calculation details

For this calculation we make use of the PACS-CS $(2+1)$-flavour dynamical-QCD gauge field configurations [31] made available through the ILDG [32]. These configurations are generated using a non-perturbatively $\mathcal{O}(a)$-improved Wilson fermion action and Iwasaki gauge action. The value $\beta = 1.90$ results in a lattice spacing $a = 0.091$ fm, determined via the static quark potential. With

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