Physics Letters B 723 (2013) 224–228

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Gauge-flation: Inflation from non-Abelian gauge fields

A. Maleknejad [a,b,*], M.M. Sheikh-Jabbari [b]

[a] Department of Physics, Alzahra University, P.O. Box 19938, Tehran 91167, Iran
[b] School of Physics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran

ARTICLE INFO

Article history:
Received 25 December 2012
Received in revised form 20 March 2013
Accepted 1 May 2013
Available online 3 May 2013
Editor: S. Dodelson

ABSTRACT

Inflationary models are usually based on dynamics of one or more scalar fields coupled to gravity. In this work we present a new class of inflationary models, *gauge-flation* or non-Abelian gauge field inflation, where slow-roll inflation is driven by a non-Abelian gauge field. This class of models is based on a gauge field theory with a generic non-Abelian gauge group minimally coupled to gravity. We then focus on a particular gauge-flation model by specifying the action for the gauge theory. This model has two parameters which can be determined using the current cosmological data and has the prospect of being tested by Planck satellite data. Moreover, the values of these parameters are within the natural range of parameters in generic grand unified theories of particle physics.

Inflationary Universe paradigm [1], the idea that early Universe has undergone an inflationary (accelerated expansion) phase, has appeared very successful in reproducing the current cosmological data through the ΛCDM model [2]. Many models of inflation have been proposed and studied so far, e.g. see [3], which are all compatible with the current data. Inflationary models are generically single or multi scalar field theories with standard or non-standard kinetic terms and a potential term, which are minimally or non-minimally coupled to gravity. Generically, in these models inflationary period is driven by a "slowly rolling" scalar field (inflaton field) whose kinetic energy remains small compared to the potential terms.

Toward the end of inflation the kinetic term becomes comparable to the potential energy, and inflaton field(s) start a (fast) oscillation around the minimum of their potential losing their energy to other fields present in the theory, the (p)reheating period. The energy of the inflaton field(s) should eventually be transferred to standard model particles, reheating, where standard FRW cosmologies take over. Therefore, to have a successful cosmology model one should embed the model into particle physics models. With the current data the scale of inflation (or Hubble parameter H during inflation) is not restricted well enough, it can range from 10^{14} GeV to the Bing Bang Nucleosynthesis scale 1 MeV. However, larger H, $H \gtrsim 10$ GeV, is preferred within the slow-roll inflationary models with preliminary particle or high energy physics considerations. It is hence natural to tune the inflationary model within the existing particle physics models suitable for similar energy scales.

Most of successful inflationary scenarios so far use scalar field(s) as the inflaton, because turning on time dependent scalar fields does not spoil the homogeneity and isotropy of the cosmology. Although it is relatively easy to write down a potential respecting the slow-roll dynamics conditions, it is generically not easy to argue for such potentials and their stability against quantum corrections within particle physics models. For example, the Higgs sector in the ordinary electroweak standard model minimally coupled to Einstein gravity does not support a successful inflationary model, e.g. see [4]. The situation within beyond standard model theories seems not to be better.

Vector gauge fields are commonplace in all particle physics models. However, their naive usage in constructing inflationary models is in clash with the homogeneity and isotropy of the background. It has been argued that this obstacle may be overcome by introducing many vector fields which contribute to the inflation, such that the anisotropy induced by them all average out [5]. Alternatively one may introduce three orthogonal vector fields and retain rotational invariance by identifying each of these fields with a specific direction in space [5]. Nonetheless, it was shown that it is not possible to get a successful vector inflation model in a *gauge invariant* setting [5]. Lack of gauge invariance, once quantum fluctuations are considered may lead to instability of the background and may eventually invalidate the background classical inflationary dynamics analysis [6].

Here, we construct a new class of vector inflation models and to avoid the above mentioned possible instability issue we work in the framework of gauge field theories. In addition, to remove the incompatibility with isotropy resulting from gauge fields we

* Corresponding author at: Department of Physics, Alzahra University, P.O. Box 19938, Tehran 91167, Iran.
E-mail address: azade@ipm.ir (A. Maleknejad).

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.05.001

introduce three gauge fields. We choose these gauge fields to rotate among each other by $SU(2)$ non-Abelian *gauge* transformations. Explicitly, the rotational symmetry in 3d space is retained because it is identified with the global part of the $SU(2)$ gauge symmetry. In our model we need not restrict ourselves to $SU(2)$ gauge theory and, since any non-Abelian gauge group has an $SU(2)$ subgroup, our *gauge-flation* (non-Abelian gauge field inflation) model can be embedded in non-Abelian gauge theories with arbitrary gauge group. Another advantage of using non-Abelian gauge theories is that, due to the structure of non-Abelian gauge field strength, there is always a potential induced for the combination of the gauge field components which effectively plays the role of the inflaton field.

In the above discussions we have only committed ourselves to the gauge invariance and have not fixed a specific gauge theory action. This action will be fixed on the requirement of having a successful inflationary model. We study one such gauge-flation model but gauge-flation models are expected not to be limited to this specific choice. In this Letter we consider a simple two parameter gauge-flation model and study classical inflationary trajectory for this model as well as the cosmic perturbation theory around the inflationary path. We then use the current data for constraining the parameters of our model and show that our model is compatible with the current data within a natural range for its parameters.

1. The inflationary setup

Consider a 4-dimensional $su(2)$ gauge field $A^a{}_\mu$, where $a, b, \dots$ and $\mu, \nu, \dots$ are respectively used for the indices of the gauge algebra and the space–time. We will be interested in *gauge invariant* Lagrangians $\mathcal{L}(F^a{}_{\mu\nu}, g_{\mu\nu})$ which are constructed out of metric $g_{\mu\nu}$ and the strength field F

$$F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - g\epsilon^a{}_{bc} A^b{}_\mu A^c{}_\nu, \tag{1}$$

where ϵ_{abc} is the totally antisymmetric tensor. We work with FRW inflationary background metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij}\, dx^i\, dx^j, \tag{2}$$

where indices $i, j, \dots$ label the spatial directions.

The effective inflaton field is introduced as follows: We will work in temporal gauge $A^a{}_0 = 0$ and at the background level, as in any inflationary model, we only allow for t dependent field configurations [7]

$$A^a{}_\mu = \begin{cases} \phi(t)\delta^a_i, & \mu = i, \\ 0, & \mu = 0. \end{cases} \tag{3}$$

With this choice we are actually identifying our gauge indices with the spatial indices. That is, we identify the rotation group $SO(3)$ with the global part of the gauge group, $SU(2)$. Therefore, the rotational non-invariance resulted from turning on space components of a vector is compensated by (the global part of) the gauge symmetry. $\phi(t)$ is not a genuine scalar, while

$$\psi(t) = \frac{\phi(t)}{a(t)} \tag{4}$$

is indeed a scalar. (Note that for the flat FRW metric $e^a{}_i = a(t)\delta^a_i$, where $e^a{}_i$ are the 3d triads.) The components of the field strengths in the ansatz are

$$F^a{}_{0i} = \dot\phi \delta^a_i, \qquad F^a{}_{ij} = -g\phi^2 \epsilon^a{}_{ij}. \tag{5}$$

After fixing the gauge and choosing $A^a{}_0$ to be zero, system has nine other degrees of freedom, $A^a{}_i$. However, in the ansatz (3) we only keep one scalar degree of freedom. We should hence first discuss consistency of the reduction ansatz (3) with the classical dynamics of the system induced by $\mathcal{L}(F^a{}_{\mu\nu}, g_{\mu\nu})$. It is straightforward to show that the gauge field equations of motion $D_\mu \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} = 0$, where D_μ is the gauge covariant derivative, i) allow for a solution of the form (3) and, ii) once evaluated on the ansatz (3) become equivalent to the equation of motion obtained from the "reduced Lagrangian" $\mathcal{L}_{red}(\dot\phi, \phi; a(t))$,

$$\frac{d}{a^3\, dt}\left(a^3 \frac{\partial \mathcal{L}_{red}}{\partial \dot\phi}\right) - \frac{\partial \mathcal{L}_{red}}{\partial \phi} = 0, \tag{6}$$

where $\mathcal{L}_{red}$ is obtained from inserting (5) and metric (2) into the original gauge theory Lagrangian $\mathcal{L}$. Moreover, one can show that the energy momentum tensor, $T_{\mu\nu}$, computed over the FRW background (2) and the gauge field ansatz (3) takes the form of a homogeneous perfect fluid

$$T^\mu{}_\nu = diag(-\rho, P, P, P),$$

which is the same as the energy momentum tensor obtained from the reduced Lagrangian $\mathcal{L}_{red}$. That is,

$$\rho = \frac{\partial \mathcal{L}_{red}}{\partial \dot\phi}\dot\phi - \mathcal{L}_{red}, \qquad P = \frac{\partial (a^3 \mathcal{L}_{red})}{\partial a^3}. \tag{7}$$

All the above is true for any gauge invariant Lagrangian $\mathcal{L} = \mathcal{L}(F^a{}_{\mu\nu}; g_{\mu\nu})$. To have a successful inflationary model, however, we should now choose appropriate form of $\mathcal{L}$. The first obvious choice is Yang–Mills action minimally coupled to Einstein gravity. This will not lead to an inflating system with $\rho + 3P < 0$, because as a result of scaling invariance of Yang–Mills action one immediately obtains $P = \rho/3$ and that $\rho \geqslant 0$. So, we need to consider modifications to Yang–Mills. As will become clear momentarily one such appropriate choice is

$$S = \int d^4x \sqrt{-g} \times \left[-\frac{R}{2} - \frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu} + \frac{\kappa}{384}\left(\epsilon^{\mu\nu\lambda\sigma} F^a{}_{\mu\nu} F^a{}_{\lambda\sigma}\right)^2\right] \tag{8}$$

where we have set $8\pi G \equiv M_{\rm pl}^{-2} = 1$ and $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor. This specific F^4 term is chosen because the contribution of this term to the energy momentum tensor will have the equation of state $P = -\rho$, making it perfect for driving inflationary dynamics. Some other possible non-Abelian gauge field cosmological scenarios may be found in [8,9]. (To respect the weak energy condition for the F^4 term, we choose κ to be positive.) The reduced (effective) Lagrangian is obtained from evaluating (8) for the ansatz (3):

$$\mathcal{L}_{red} = \frac{3}{2}\left(\frac{\dot\phi^2}{a^2} - \frac{g^2\phi^4}{a^4} + \kappa \frac{g^2\phi^4\dot\phi^2}{a^6}\right). \tag{9}$$

Energy density ρ and pressure P are then given by

$$\rho = \rho_{YM} + \rho_\kappa, \qquad P = \frac{1}{3}\rho_{YM} - \rho_\kappa, \tag{10}$$

where

$$\rho_{YM} = \frac{3}{2}\left(\frac{\dot\phi^2}{a^2} + \frac{g^2\phi^4}{a^4}\right), \qquad \rho_\kappa = \frac{3}{2}\kappa \frac{g^2\phi^4\dot\phi^2}{a^6}. \tag{11}$$

Recalling the Friedmann equations

$$H^2 = \frac{1}{2}\left(\frac{\dot\phi^2}{a^2} + \frac{g^2\phi^4}{a^4} + \kappa\frac{g^2\phi^4\dot\phi^2}{a^6}\right),$$
$$\dot H = -\left(\frac{\dot\phi^2}{a^2} + \frac{g^2\phi^4}{a^4}\right), \tag{12}$$